Rational ergodicity of "discrepancy" skew products and the asymptotics of affine random walks.

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The talk is based on the joint work with Michael Bromberg and Hitoshi Nakada

Abstract: It is not hard to show that if $(X(n) : n = 1, 2, \ldots)$ is given by a RAT (random affine transformation) of the form $X(n+1) = a(n+1)X(n) + b(n+1)$ where $\{(a(n), b(n)) : n = 1, 2, \ldots\}$ are iid rvs taking values in $\{-1, +1\} \times R$ with $E(b) = 0$ and $E(b^2)$ finite, then $X(n)$ satisfies both central and local limit theorems.

We'll establish a "weak, rough, local limit theorem" for certain non-stationary, multidimensional versions of this and use this to show rational ergodicity of "discrepancy" skew products $T : X = [0, 1) \times Z \to X$ by $T(x, z) = (x + A \mod 1, z + D(x))$ where A is irrational and the "discrepancy function" $D = 1$ on $[0, \frac{1}{2})$ and $D = -1$ on $[\frac{1}{2}, 1)$.

For each A there is a RAT sequence depending on the "minus-sign" continued fraction expansion of A. For badly approximated A, this RAT sequence satisfies the "weak, rough, local limit theorem", whence bounded rational ergodicity of T_A. This was shown with Michael Keane for quadratic A.

Regional proximality and the Veech relation in minimal flows

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Let (X, T) be a flow, with X compact Hausdorff. A classical theorem of Gottschalk and Ellis is that the equicontinuous structure relation S_{eq} is generated by RP, the regionally proximal relation. If (X, T) is minimal, then in many cases (including T abelian and (X, T) distal) RP is an equivalence relation, so $S_{eq} = RP$. A remarkable result of Veech is that if T is abelian and X is a metric space, the apparently simpler relation V, defined by $(x, y) \in V$ if there is a $z \in X$ and a sequence $t_n \in T$ with $t_n x \to z$ and $t_n^{-1} z \to y$ is in fact equal to S_{eq}.

We will present an alternate proof and generalization of Veech's theorem, in collaboration with Anima Nagar and Gernot Greschonig. We will also discuss the connection of the Veech relation with the regional proximal relation of order d defined by Host, Kra, and Maass.

Generalized polynomials, Ramsey Theory and a new characterization of translations on nil-manifolds

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A classical theorem due to H. Weyl states that if P is a real polynomial such that at least one of its coefficients (other than the constant term) is irrational, then the sequence $P(n), n = 1, 2, ...$ is uniformly distributed mod 1. About 10 years ago, in joint work with A. Leibman, we demonstrated that Weyl's theorem extends to "generalized polynomials", that is, functions which are obtained from the conventional polynomials by the use of the greatest integer function, addition and multiplication. We will explain the role of dynamical systems on nil-manifolds in obtaining these results and discuss the intrinsic connection between the generalized polynomials and the polynomial extensions of Szemeredi's theorem on arithmetic progressions. We will also discuss the recent joint work with Leibman on characterization of nil-translations in terms of recurrence. We will conclude with formulating and discussing some natural open problems and conjectures.

Some remarks on dynamical system of solenoids

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We show that a solenoid is a dynamical object and express its complexity by a number of different entropy-like invariants. Some relations between these entropy-like invariants are presented. We adopt local measure entropy, introduced by M. Brin and A. Katok for a single map, to a case of a solenoid and provide lower estimations of these invariants by corresponding local measure entropies. The talk is based on joint work with Wojciech Kozlowski [1].

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Isolating zero dimensional dynamics on manifolds

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The talk is based on the joint work with Scott Schmieding.

Let h be a homeomorphism of a compact manifold M. With respect to h, a compact subset K of M is isolated (locally maximal) if it admits a neighborhood U such that $K = \bigcap_n h^n(U)$. We then say that the topological dynamical system $(h, h|K)$ is isolated by h. We consider the question: up to topological conjugacy, what zero dimensional systems can be isolated by a homeomorphism of a compact manifold? We show every homeomorphism of a compact zero dimensional metric space can be isolated in dimension greater than two. We give obstructions to isolating in dimension two, and examples beyond SFT of some zero dimensional systems which can be isolated in dimension two (a mixing strictly sofic shift; some minimal subshifts; some mixing nonexpansive positive entropy systems).

r-mixing configurations for algebraic $\mathbb{N}^d$-actions on $\mathbb{F}_p^{\mathbb{Z}}$ and limit theorems

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For $\mathbb{N}^d$-actions by algebraic endomorphisms on compact abelian groups, certain limit theorems are related to non mixing configurations, hence to the existence of solutions for "S-unit type" equations.

In the case of algebraic $\mathbb{N}^d$-actions on $\mathbb{F}_p^{\mathbb{Z}}$, we will show how the scarcity of solutions implies the validity of some limit theorems.

Invariant measures for Cartesian powers of Chacon infinite transformation

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The talk is based on the joint work with Élise Janvresse (Amiens) and Emmanuel Roy (Paris 13)

In connection with another work on Poisson suspensions, we needed an infinite measure preserving transformation with the property that its Cartesian powers have as few invariant measures as possible. A good candidate was a rank-one transformation introduced in 1997 by Adams, Friedman and Silva, which could be seen as the analog of the classical Chacon transformation in infinite measure.

We describe all boundedly finite measures which are invariant by Cartesian powers of this infinite measure preserving version of Chacon transformation: all such ergodic measures are products of so-called *diagonal measures*, which are measures generalizing in some way the measures supported on a graph. But, unlike what happens in the finite-measure case, this class of diagonal measures is not reduced to measures supported on a graph arising from powers of the transformation: it also contains some weird invariant measures, whose marginals are singular with respect to the measure invariant by the transformation. These weird invariant measures can be fully described.

We derive from these results that the infinite Chacon transformation has trivial centralizer, and has no nontrivial factor.

Topological structures and the pointwise convergence of some averages for commuting transformations.

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The talk is based on a joint work with Wenbo Sun

In this talk I will discuss some recent results in the pointwise convergence of some averages for commuting transformations. Recently we have shown the existence of the limit of *cubic* averages arising from systems with commuting transformations. I will show the main ideas in the proof, which combine the notions of *sated* extensions by T. Austin, *magic* extensions by B. Host and ideas by W. Huang, S. Shao an X. Ye on introducing a topological structure to build a strictly ergodic model for an ergodic system. I will also discuss some partial results on the pointwise convergence of multiple averages for commuting transformations.

Joining classification on homogeneous spaces

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The talk is based on the joint work with E.Lindenstrauss

I will discuss joinings for higher rank actions on homogeneous spaces. Using the high and low entropy method E. Lindenstrauss and myself recently proved that such joinings are always algebraic. We will outline the proof and discuss the following arithmetic application of the joining classification.

Linnik (and in full generality Duke) established the equidistribution of the directions of integer points on a sphere with a given radius. Using the joining classification and a congruence condition (for two odd primes) on the square of the radius M. Aka, U. Shapira, and myself showed recently that the direction and the shape of the lattice in the orthogonal complement jointly equidistribute.

Free ergodic $\mathbb{Z}^2$-systems and complexity after Cyr and Kra

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We present the main result from the recent preprint "Free ergodic $\mathbb{Z}^2$-systems and complexity" by Van Cyr and Bryna Kra. Namely, any strictly ergodic topological model $(\mathbb{Z}^2, X)$ of (the natural extension) of the $(\times 2 \times 3)$-system, realized as a subshift, is either finite or $P(n,k) > nk/2$ for all $n, k \in \mathbb{N}$. Here the complexity function $P(n,k)$ refers to the number of $(n \times k)$-rectangular words in X. The main tool in the proof is a previous result by Cyr and Kra which gives a complexity criterion for the existence of a periodic vector for a transitive $\mathbb{Z}^2$-subshift.

Higher order regionally proximal equivalence relations for general group actions

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Joint work with Eli Glasner and XiangDong Ye.

Ellis & Gottschalk introduced the regionally proximal relation in 1960 whereas the higher order regionally proximal relations for abelian group actions were introduced by Host, Kra & Maass in 2010. The interest in these relations lies in their role in the structural theory of topological dynamical systems. For amenable minimal group actions the regionally proximal relation is an equivalence relation but for some non-amenable minimal actions it is not. When it is an equivalence relation, the quotient by the regionally proximal relation is the maximal equicontinuous factor. In this talk I will introduce a generalization of the higher order regionally proximal relations suitable for an arbitrary acting group. The surprising new main result is that these generalized relations are always equivalence relations for an arbitrary minimally acting group. Moreover the generalized regionally proximal equivalence relation of order one corresponds to the maximal (compact) abelian group factor, yielding for the first time an explicit description of this factor in the category of non-abelian minimal actions.

Smooth deformation of $\times m$ invariant sets

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I will discuss the following question: If $X \subseteq [0,1]$ is a non-trivial $\times m$-invariant set, what can we say about smooth maps which preserve it? For example, if X is minimal w.r.t. $\times m$ and f is a C^1 diffeomorphism preserving X, then its derivative is a rational power of m at every point in X. I will explain the idea of the proof and related results.

Affine embeddings and intersections of Cantor sets

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The talk is based on joint work with Dejun Feng and Hui Rao

Let $E, F \subset \mathbb{R}^d$ be two self-similar sets. Under mild conditions, we show that F can be C^1-embedded into E if and only if it can be affinely embedded into E; furthermore if F cannot be affinely embedded into E, then the Hausdorff dimension of the intersection $E \cap f(F)$ is strictly less than that of F for any C^1-diffeomorphism f on $\mathbb{R}^d$. Under certain circumstances, we prove the logarithmic commensurability between the contraction ratios of E and F if F can be affinely embedded into E. As an application, we show that

$$dim_H E \cap f(F) < \min\{dim_H E, dim_H F\}$$

when E is any Cantor-p set and F any Cantor-q set, where $p, q \geq 2$ are two integers with $\log p / \log q \notin \mathbb{Q}$. This is related to a conjecture of Furstenberg about the intersections of Cantor sets.

$\times 2$ and $\times 3$ invariant measures and entropy

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This talk, titled the same as the 1990 paper by Daniel Rudolph, will take a historical perspective and review the original result and its proof. We'll look at the tools originally brought to bear on the situation when one has an ergodic Borel probability measure on the unit circle invariant under multiplication by two relatively prime integers, and discuss the structure this yields.

Smooth K non-Bernoulli automorphisms in dimension 4.

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We give examples of smooth K, non-Bernoulli automorphisms on $\mathbb{T}^4$. They arise by taking a skew-product (with a smooth skewing function) of an Anosov map on $\mathbb{T}^2$ with a smooth Kochergin flow (with high degeneracy of the saddle) on $\mathbb{T}^2$. Joint work with Federico Rodriguez-Hertz and Kurt Vinhage.

Topological Conjugacy of Substitution Minimal Sets

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In this lecture, I shall briefly discuss my knowledge of the conjugacy classes of minimal sets arising from primitive substitutions on finite alphabets.

Bernoulli property of smooth extensions of Bernoulli shifts

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The talk is motivated by the question of Benjamin Weiss. Let T_i, $i = 0, 1$, be homeomorphisms of $I = [0, 1]$ such that $T_i = (1 - \epsilon_i)x + \epsilon_i g(x)$, $i = 0, 1$, for some reals $\epsilon_0 < 0, \epsilon_1 > 0$. Here g is $C^1(0, 1)$ homeomorphism of I , there exist $g'_+(0), g'_-(1)$ and $g(x) < x$ for $x \in (0, 1)$. Moreover , let $(\Omega, \mathcal{B}, \mu_p, \sigma)$ be the one-sided Bernoulli shift where $\Omega = \{0, 1\}^N$ and μ_p is (p, q) measure. In the space $\Omega \times I$ we define the skew products $T(\omega, x) = (\sigma(\omega), T_{\omega(0)}(x))$ and $S(\omega, x) = (\sigma(\omega), S_{\omega(0)}(x))$ where $S_i = T_i^{-1}$, $i = 0, 1$.
Let $\Lambda_i = p \log(T'_0(i)) + (1 - p) \log(T'_1(i))$ for $i = 0, 1$ and $\pi : \Omega \times I \to I$ such that $\pi(\omega, x) = x$.

Theorem 1. *Let $\Lambda_i < 0$ for $i = 0, 1$.*
1) Then there exists a measurable function $\varphi : \Omega \to I$, which is T invariant, $0 < \varphi(\omega) < 1$ and
if $x < \varphi(\omega)$, then $\lim_{n\to\infty} \pi(T^n(\omega, x)) = 0$,
if $x > \varphi(\omega)$, then $\lim_{n\to\infty} \pi(T^n(\omega, x)) = 1$ for μ_p a.e. ω.
2) S possesses invariant measure $\mu_p \times \nu$ such that $\nu(\{0\}) = \nu(\{1\}) = 0$ and the natural extension of $(S, \mu_p \times \nu)$to automorphism is Bernoulli one .

(2) gives partially positive answer to the question of Benjamin Weiss about Bernoulli properties of extensions as above. Moreover, in order to prove (1) we do not need negativity of the Schwarzian derivative of T_i, $i = 0, 1$ which is assumed in [2] and we do not use essential contraction of S assumed in [1].

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Dynamics of $\mathcal{B}$-free sets and Toeplitz sequences

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Given a set $\mathcal{B}$ of natural numbers greater than 1, the set $\mathcal{F}_\mathcal{B} = \mathbb{Z} \setminus \bigcup_{b\in\mathcal{B}} b\mathbb{Z}$ is called the set of $\mathcal{B}$-free numbers. Looking at this set as a point in $\{0, 1\}^\mathbb{Z}$, the corresponding subshift is called the $\mathcal{B}$-free subshift. I will present some recent results on $\mathcal{B}$-free subshifts and show some relations with the theory of Toeplitz dynamical systems. The talk is based on some joint results with A. Bartnicka, S. Kasjan, J. Kułaga-Przymus and B. Weiss.

x2-x3: the cellular automata analogues

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In this talk we will review results involving the analogous x2 x3 conjecture in the cellular automata context, *i.e.* using as a main example Ledrappier's cellular automaton, but from this example explore classes of expansive or positively expansive cellular automata where the conjecture could be true.

Freedom versus interaction in a dynamical system

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The talk is based on joint work with Benjamin Wilson.

The topological and measure-theoretic entropies of a symbolic dynamical system are measures of the freedom to choose the next symbol in a string, given what has been produced so far. The complexity function of a sequence or subshift counts the number of words of each length, and the topological entropy is its exponential growth rate. More generally, we consider the number of words that can be seen in a finite sampling window, not necessarily an interval, among all sequences in a subshift. We define a measure of freedom, which we call average sample complexity, by averaging over all windows within an interval of fixed length and then taking the exponential growth rate. When comparing possible entries at a set of sites with those on its complement, we obtain a dynamical analogue of the neural intricacy proposed by neuroscientists G. Edelman, O. Sporns, and G. Tononi, and studied in the probabilistic setting by J. Buzzi and L. Zambotti. Intricacy is a measure of the balance between freedom and interdependence; it is high when there are both local freedom and global organization, low when there is either complete freedom or complete order. There are also measure-theoretic versions based on the idea of measure-theoretic entropy, abstract definitions using open covers or separated sets, and generalizations to pressure. We provide some methods for computing these quantities and establish their relations to ordinary entropy. We show that on subshifts there exist ergodic measures that maximize average sample complexity, but numerical studies indicate that typically these maximizing measures are not Markov of any order and are supported on small subshifts.

Poisson character of some ergodic random measures

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In a recent work, we showed that, under integrability assumptions, an ergodic simple point process (particles with no multiplicities) driven by some particular infinite measure preserving transformation (like the infinite Chacon transformation) has to be a mixture of Poisson point processes. In this talk, we elaborate on this and extend our results to the case of a general random measure, where we allow multiplicities and even continuity of the realizations. This is a joint work with Elise Janvresse and Thierry de la Rue

Topological correspondence of multiple ergodic averages of nilpotent group actions

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Let (X,Γ) be a topological system, where Γ is a nilpotent group generated by $T_1,\dots,T_d$ such that for each $T\in\Gamma$, $T\neq e_\Gamma$, (X,T) is weakly mixing and minimal. For $d,k\in\mathbb{N}$, let $p_{i,j}(n), 1\leq i\leq k, 1\leq j\leq d$ be polynomials with rational coefficients taking integer values on the integers and $p_{i,j}(0)=0$. We show that if the expressions $g_i(n)=T_1^{p_{i,1}(n)}\cdots T_d^{p_{i,d}(n)}$ depend nontrivially on n for $i=1,2,\cdots,k$, and for all $i\neq j\in\{1,2,\dots,k\}$ the expressions $g_i(n)g_j(n)^{-1}$ depend nontrivially on n, then there is a residual set X_0 of X such that for all $x\in X_0$

$$\{(g_1(n)x,g_2(n)x,\dots,g_k(n)x)\in X^k : n\in\mathbb{Z}\}$$

is dense in X^k. This talk based on joint work with W. Huang and X. Ye.

Cylinder cocycle extensions of minimal rotations on monothetic groups

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The talk is based on joint work with Jan Kwiatkowski

Let X be a compact metric space and $T:X\longrightarrow X$ be a homeomorphism of X. Let $f:X\longrightarrow\mathbb{R}$ be a continuous function (called *a cocycle*). By a *cylinder transformation* we mean a homeomorphism $T_f:X\times\mathbb{R}\longrightarrow X\times\mathbb{R}$ (or rather a $\mathbb{Z}$–action generated by it) given by the formula

$$T_f(x,r)=(Tx,f(x)+r).$$

H. Poincaré addressed the problem of what types of orbits may coexist in such a system ([5]). If X is a monothetic group then the following trichotomy takes place: 1. either f has nonzero mean (equivalently all orbits are discrete) or 2. f is a coboundary (equivalently every orbit closure is homeomorphic to X) or 3. T_f is topologically transitive (has a dense orbit) ([2], [4]). Therefore 3. is the only interesting case. Since a cylindrical transformation cannot be itself minimal ([1]), it has to have non-dense orbits provided it is topologically transitive.

In [3] we delivered a method to construct a few classes of top. trans. cylinder transformations with a relatively large set of discrete orbits of various types (with respect to their boundedness). Using this method we show how to construct top. trans. cylindrical transformations possessing orbits of quite complicated nature. Namely ω-limit sets of some points are unions (countable or not) of locally compact Cantor sets.

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Invariant measures and filtrations

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The problem of the description of the invariant measures, or more general, the measures with given cocycle on the hyperfinite equivalence relation, must be considered in the framework of the theory of filtrations on Cantor or Borel space. For so called standard (finitely determined) filtration the list of invariant measures could be in principle reduced to some calculations. For non-standard case there is in general no chance to find appropriate parameter for ergodic invariant measures. In this setting it is very important to know whether filtration is standard (finitely determined) or not. The recent progress in the theory of filtrations will be done.

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Predictive sequences and recurrence

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A predictive sequence is a subset P of $\{n < 0\}$ such that for any finite valued zero entropy stationary ergodic process $\{X_j\}$ the random variable X_0 is a function of $\{X_j : j \in P\}$. Examples of such sequences are sets of the form $\{n < 0 : n\alpha \in U\}$ where U is a neighborhood of the identity in a compact abelian group K and α is any element of K. On the other hand any predictive sequence must be an SIP^* set and in particular must be syndetic.

The system of falling balls revisited

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The system of falling balls is a hamiltonian system of arbitrary number of particles in which hyperbolicity is present under some conditions on the masses. It was discovered some 25 years ago. Less known is the version of this system where no conditions on the masses are needed.

We will discuss the remaining roadblocks to establishing the ergodicity of the latter system.

Around the pointwise convergence of multiple ergodic averages

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In this talk, we first review the recent progress on the study of the convergence of multiple ergodic averages. Then we discuss some topological aspects of the problem, focused on the regionally proximal relation of order d along arithmetic progressions. This talk is based on joint works with Huang-Shao and Gutman-Huang-Shao.

New developments around x2 x3 conjecture
and other classical problems in Ergodic Theory
May 23–May 27,2016, Cieplice

Uniqueness and non-uniqueness of measures of maximal relative entropy

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The general setup for this talk is a given factor map π from a mixing subshift of finite type X onto a sofic subshift Y and a given ergodic measure ν on Y. The subject of this talk is to find conditions on the pair (π, ν) which guarantees uniqueness of measures (on X) of maximal relative entropy over ν. We also seek to understand cases of non-uniqueness. (This subject and the study of measures of maximal entropy on multi-dimensional subshifts of finite type seem to share following common characteristics: non-uniqueness phenomena, inability to exploit periodic points, and the conditional equidistribution property.)

Conditions for uniqueness:

	condition on π	condition on ν
classical result	regularity conditions	regular equilibrium state
recent result	class degree one	full support
new result	none	regular equilibrium state

Old classical results on this subject (by Boyle, Tuncel, Walters) obtain uniqueness by using thermodynamic ideas when both the factor code π and the image measure ν are assumed to be regular in various senses. In these results, the conditions on π are, in order of strong to weak, such as being finite-to-one, uniform, or existence of regular saturated compensation function. Conditions on ν are such as being a Markov measure, or more generally, being an equilibrium state of a regular potential function.

Without assuming any condition on π or ν, recent results (by Petersen, Quas, Shin, Allahbakhshi) show that there are at most finitely many (ergodic) measures of maximal relative entropy, by analyzing the general structure of arbitrary (possibly infinite-to-one) factor code. Uniqueness is obtained if π is assumed to be a special type of factor code, namely, a factor code whose class degree is one.

In our new results, we decompose an arbitrary factor code into a composition of class degree one factor code and finite-to-one factor code. This allows us to combine old classical results for finite-to-one case and known recent results on class degree. In this way, uniqueness is obtained when π is arbitrary and ν is an equilibrium state of a regular potential. The decomposition also allows us to better understand cases of non-uniqueness when ν is not regular.