Secret connections between analytic P-ideals and Banach spaces

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joint work with **Barnabas Farkas**

Ideali su ω.

- $\mathcal{I}$ is an ideal on ω;
- $\mathcal{I}$ can be treated as a subset of 2^ω(via $A \mapsto \chi_A$);
- $\mathcal{I}$ is a P-ideal if for each (A_n) from $\mathcal{I}$, there is $A \in \mathcal{I}$ such that $A_n \subseteq^* A$ for every n.

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Esempi classici.

Summable ideal:

$$\mathcal{I}_{1/n} = \{A \subseteq \omega \colon \sum_{i \in A} \frac{1}{n} < \infty\}.$$

Density ideal:

$$\mathcal{Z} = \{A \subseteq \omega \colon d(A) = 0\},$$

where

$$d(A) = \lim_{n \to \infty} \frac{|A \cap \{0, \cdots, n\}|}{n+1}.$$

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LSC sottomisura.

Consider a function $\varphi\colon \mathcal{P}(\omega) \to [0, \infty]$ such that for each A, B

- $\varphi(\emptyset) = 0$,
- $\varphi(A \cup B) \leq \varphi(A) + \varphi(B)$,
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Such functions are called LSC submeasures.

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Teorema di Solecki.

- Let φ be a LSC submeasure (taking finite values on finite sets). Define
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  - $\mathrm{Fin}(\varphi) = \{A \subseteq \omega \colon \varphi(A) < \infty\}$.
  - $\mathrm{Exh}(\varphi) = \{A \subseteq \omega \colon \lim_n \varphi(A \setminus n) = 0\}$.
- Both $\mathrm{Fin}(\varphi)$ and $\mathrm{Exh}(\varphi)$ are analytic P-ideals.
- **Theorem (Solecki)** For every analytic P-ideal there is an LSC submeasure φ such that

$$\mathcal{I} = \mathrm{Exh}(\varphi).$$

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Figure: Sławomir Solecki

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Nice extended norm = *norm* on $\mathbb{R}^\omega$ which may attain infinite values and which is compatible with the topological structure of $\mathbb{R}^\omega$.

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- $\lim_{n\to\infty} \Phi(\pi_{[0,\dots,n]}(x)) = \Phi(x)$.

$(\pi_A(x))(n) = x(n)$ for $n \in A$ and $(\pi_A(x))(n) = 0$ otherwise.

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But we will call them **nice extended norms**.

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Fin ed Exh

- Let Φ be a nice extended norm (taking finite values on sequences with finite support).
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  - $\mathrm{Fin}(\Phi) = \{x \in \mathbb{R}^\omega : \Phi(x) < \infty\}$.
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Exercise: What kind of objects are those guys?

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Spazio di Banach

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- Both $\mathrm{Fin}(\Phi)$ and $\mathrm{Exh}(\Phi)$ are Banach spaces.

Analogo di Solecki

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- **Teorema** For every Banach space X with unconditional basis there is a nice extended norm Φ such that

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Theorem (Mazur) Let $\mathcal{I}$ be an analytic P-ideal. TFAE

- $\mathcal{I}$ is F_σ.
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Theorem Let Φ be a nice extended norm. TFAE

- $\mathrm{Exh}(\Phi)$ is F_σ (in the product topology of $\mathbb{R}^\omega$);
- $\mathrm{Exh}(\Phi) = \mathrm{Fin}(\Phi)$;
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Theorem Let Φ be a nice extended norm. TFAE

- $\mathrm{Exh}(\Phi)$ is F_σ (in the product topology of $\mathbb{R}^\omega$);
- $\mathrm{Exh}(\Phi) = \mathrm{Fin}(\Phi)$;
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Analogo di Mazur

Theorem (Mazur) Let $\mathcal{I}$ be an analytic P-ideal. TFAE

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In fact, many of the above implications was known before, in a different language (Bessaga-Pełczyński, Drewnowski-Labuda, Ding and others).

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Pretensious conslusions

- Banach spaces with unconditional bases are *continuous* versions of analytic P-ideals.
- Banach spaces with unconditional bases without copies of c_0 are *continuous* versions of F_σ P-ideals.

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Sottomisure da norme

- Let Φ be a nice norm;
- Choose $x \in \mathbb{R}^\omega$;
- Let $\varphi \colon \mathcal{P}(\omega) \to [0, \infty]$ be defined by

$$\varphi(A) = \Phi(\pi_A(x));$$

- Then φ is a LSC submeasure.

A nice norm + $x \in \mathbb{R}^\omega$ gives you a natural LSC submeasure.
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Funzione peso

"A nice norm + $x \in \mathbb{R}^\omega$ = LSC submeasure".

Usually we will choose very particular "x":

$$w = (1, 1/2, 1/2, \underbrace{1/4, \cdots, 1/4}_{\text{4 times}}, \underbrace{1/8, \cdots, 1/8}_{\text{8 times}}, \cdots).$$

Why such w?

- w converges to 0;
- w is not summable;
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$\mathcal{F} =$ singletons.
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- For $F \in \mathcal{F}$ and $x \in \mathbb{R}^\omega$ let $|x|(F) = \sum_{i \in F} |x(i)|$;
- Define $\Phi(x) = \sup_{F \in \mathcal{F}} |x|(F)$;
- $\Phi_{\mathcal{F}}$ is a nice norm;
- $\varphi_{\mathcal{F}} = \Phi(\pi_\cdot(w))$ is an LSC submeasure;

$$\mathcal{F} = \{F \in [\omega]^{<\omega} \colon |F| \leq \min F + 1\} \ .$$

$\mathrm{Exh}(\Phi_{\mathcal{F}})$ = Schreier space.

$\mathrm{Fin}(\Phi_{\mathcal{F}})$ = ?.

$\mathrm{Exh}(\varphi_{\mathcal{F}})$ - density ideal.

Józef Schreier

Example: famiglie Schreier di randanti piu alti

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$\mathcal{F}$ - Schreier family of rank α.
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Example: anticatene

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$\mathrm{Exh}(\varphi_\mathcal{F})$ - trace of null.

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Molte grazie per la corteze attenzione.