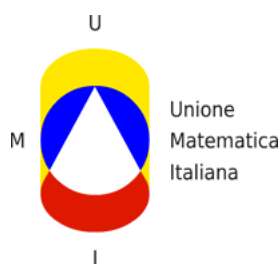


Abstracts of talks

Joint meeting of the Italian Mathematical Union, the Italian Society of Industrial and Applied Mathematics and the Polish Mathematical Society

Wrocław, 17-20 September 2018

MAIN ORGANIZERS

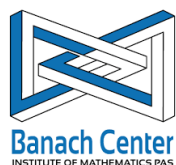


Uniwersytet
Wrocławski



Wrocław University
of Science and Technology

CO-ORGANIZERS



SPONSORS



Contents

I	Conference Information	
	Conference Schedule	13
II	Plenary Lectures	
	Plenary Speakers	17
	Guest Plenary Speaker	22
III	Sessions	
	1 Model Theory	25
	2 Number Theory	31
	3 Projective Varieties and their Arrangements	37
	4 Algebraic Geometry	43
	5 Algebraic Geometry and Interdisciplinary Applications	47
	6 Arithmetic Algebraic Geometry	53
	7 Group Theory	59
	8 Geometric Function and Mapping Theory	71
	9 Complex Analysis and Geometry	77
	10 Optimization, Microstructures, and Applications to Mechanics	81
	11 Variational and Set-valued Methods in Differential Problems	89
	12 Topological Methods and Boundary Value Problems	95
	13 Variational Problems and Nonlinear PDEs	103

14	Nonlinear Variational Methods with Applications	109
15	Advances in Kinetic Theory	115
16	Mathematical Modelling for Complex Systems: Seeking New Frontiers	125
17	Advances in Nonlinear Elliptic and Parabolic PDEs: from local to nonlocal problems	133
18	Nonlinear Partial Differential Equations and Related Function Spaces	143
19	Nonlinear Diffusion Problems	151
20	Geometric Properties of Solutions of PDEs and Nonlocal Equations	155
21	New Perspectives in Singular Hamiltonian Systems	163
22	Ergodic Theory and Topological Dynamics	167
23	Hutchinson-Barnsley Theory of Fractals	173
24	Harmonic Analysis	179
25	Nonlinear Operators, Approximation Algorithms for Operator Equations and Related Problems	187
26	Noncommutative Harmonic Analysis, Noncommutative Probability and Quantum Groups	193
27	Topology and Logic in Algebra and Functional Analysis	199
28	Operator Theory, Operator Algebras and Applications	203
29	Operator Semigroups: New Challenges and Applications	211
30	Set Theory and Topology	217
31	Computational Aspects of Applied Topology	223
32	Geometric Aspects of Applied Topology	229
33	Geometric Topology, Manifolds, and Group Actions	235
34	Stochastic Analysis and Nonlocal PDEs	245
35	Financial Mathematics	253
36	Differential Inclusions and Controlled Systems	261
37	Stochastic Models of Molecular Anomalous Diffusion and Fractional Dynamics	267
38	Probability Measures – Structures, Identification and Characterizations	273

39	Dependence Modelling and Copulas	279
40	Challenges and Methods of Modern Statistics	287
41	Recent Advances in Numerical Modeling for Differential Problems	297
42	Computational Mathematics: Discrepancy and Complexity	301
43	Quantitative Techniques for Complex Real-Life Applications	311
44	Multiscale Models in Mathematical Biology	315

IV**Poster Session**

Conference Information

THE PROGRAM COMMITTEE

Marco Andreatta	University of Trento, Italy
Alessandra Celletti	University of Rome Tor Vergata, Italy
Vittorio Coti-Zelati	University of Naples Federico II, Italy
Giorgio Fotia	Center for Advanced Studies, Research and Development in Sardinia, Italy
Grzegorz Karch	University of Wrocław, Poland
Wojciech Kryszewski	Nicolaus Copernicus University in Toruń, Poland
Jacek Miękisz	IMPAN & University of Warsaw, Poland
Giovanni Russo	University of Catania, Italy
Michał Ryznar	Wrocław University of Science and Technology, Poland
Jarosław A. Wiśniewski	University of Warsaw, Poland

THE HONORARY COMMITTEE

Adam Jezierski	Rector of University of Wrocław, Poland
Cezary Madryas	Rector of Wrocław University of Science and Technology, Poland
Ciro Ciliberto	President of the Italian Mathematical Union, Italy
Luca Formaggia	President of the Italian Society of Industrial and Applied Mathematics, Italy
Wacław Marzantowicz	President of the Polish Mathematical Society, Poland
Tomasz Jurdziński	Dean of Faculty of Mathematics and Computer Sciences University of Wrocław, Poland
Krzysztof Stempak	Dean of Faculty of Pure and Applied Mathematics Wrocław University of Science and Technology, Poland
Piotr Biler	Vice-Dean of Faculty of Mathematics and Computer Sciences University of Wrocław, Poland
Krzysztof Bogdan	Vice-Dean of Faculty of Pure and Applied Mathematics Wrocław University of Science and Technology, Poland
Adam Nowak	Branch Manager of Wrocław Branch of Institute of Mathematics Polish Academy of Science, Poland

THE HONORARY PATRONAGES

Jarosław Gowin Deputy Prime Minister of the Republic of Poland
Minister of Science and Higher Education

Italian Embassy in Warsaw
Lower Silesia Voivodeship Marshal
Voivode of Lower Silesia
Mayor of Wrocław

ORGANIZING INSTITUTIONS

Unione Matematica Italiana
Società Italiana di Matematica Applicate e Industriale
Polish Mathematical Society
University of Wrocław
Wrocław University of Science and Technology

CO-ORGANIZERS

Wrocław Branch of Institute of Mathematics Polish Academy of Science
Stefan Banach International Mathematical Center
Faculty of Mathematics, Informatics, and Mechanics of University of Warsaw

THE ORGANIZING COMMITTEE

Tomasz Grzywny Wrocław University of Science and Technology
Maciej Paluszyński University of Wrocław
Michał Balcerek Wrocław University of Science and Technology
Krzysztof Burnecki Wrocław University of Science and Technology
Edyta Kania University of Wrocław
Mariusz Olszewski Wrocław University of Science and Technology
Marcin Preisner University of Wrocław
Ziemowit Rzeszotnik University of Wrocław
Karol Szczypkowski Wrocław University of Science and Technology
Grzegorz Żurek Wrocław University of Science and Technology

Conference Schedule

MONDAY, SEPTEMBER 17

- 8:30 – 9:30** Registration
- 9:30 – 10:30** Opening ceremony
- 10:40 – 11:10** Coffee break
- 11:10 – 11:50** **Maria J. Esteban**
Functional inequalities, flows, symmetry and spectral estimates
- 12:00 – 12:40** **Rafał Łatała**
Norms of random matrices with independent entries
- 13:00 – 14:30** Lunch
- 14:30 – 16:30** Sessions
- 16:30 – 17:00** Coffee break
- 17:00 – 19:00** Sessions
- 19:00** Boat trip

TUESDAY, SEPTEMBER 18

- 9:00 – 9:40** **Giorgio Ottaviani**
The distance function from a real algebraic variety, old and new
- 9:50 – 10:30** **Marian Mrozek**
Combinatorial topological dynamics
- 10:40 – 11:10** Coffee break
- 11:10 – 11:50** **Christopher Hacon**
Boundedness of varieties of general type
- 12:00 – 12:40** **Jacek Świątkowski**
New results in the topological classification of Gromov boundaries of hyperbolic groups
- 13:00 – 14:30** Lunch
- 14:30 – 16:30** Sessions
- 16:30 – 17:00** Coffee break
- 17:00 – 19:00** Sessions
- 20:00** Concert

WEDNESDAY, SEPTEMBER 19

- 9:00 – 9:40** **Luigi Preziosi**
An overview of mathematical models for cell migration
- 9:50 – 10:30** **Anna Marciniak-Czochra**
Post-Turing tissue pattern formation: Insights from mathematical modelling
- 10:40 – 11:10** Coffee break
- 11:10 – 11:50** **Susanna Terracini**
Spiralling and other solutions in limiting profiles of competition-diffusion systems
- 12:00 – 12:40** **Marta Lewicka**
Models for Thin Prestrained Structures
- 13:00 – 14:30** Lunch
- 14:30 – 16:30** Sessions
- 16:30 – 17:00** Coffee break
- 17:00 – 19:00** Sessions
- 19:30** Banquet

THURSDAY, SEPTEMBER 20

- 9:00 – 11:00** Sessions
- 11:00 – 13:00** Poster sessions
- 13:00 – 14:30** Lunch
- 14:30 – 15:10** **Veronica Gavagna**
Francesco Maurolico, a Renaissance interpreter of Euclid
- 15:20 – 16:00** **Michael Dumbser**
New mathematical models and numerical algorithms for Newtonian and general relativistic continuum physics
- 16:10 – 16:30** Closing ceremony
- 17:00 – 19:00** Coffee break

Plenary Lectures

Plenary Speakers

NEW MATHEMATICAL MODELS AND NUMERICAL ALGORITHMS FOR NEWTONIAN AND GENERAL RELATIVISTIC CONTINUUM PHYSICS

Michael Dumbser Università di Trento

In the first part of the talk we present high order arbitrary high order accurate (ADER) finite volume and discontinuous Galerkin finite element schemes for the numerical solution of a new unified first order symmetric hyperbolic and thermodynamically compatible (SHTC) formulation of Newtonian continuum physics, including a general description of fluid and solid mechanics as well as electromagnetic fields in one single system of governing partial differential equations (PDE). The model is based on previous work of Godunov, Peshkov and Romenski (so-called GPR model) on symmetric hyperbolic and thermodynamically compatible systems.

In the second part of the talk, we show a successful extension of the GPR model to general relativity, leading to a novel and unified first order hyperbolic formulation of general relativistic continuum mechanics. The model is able to describe general relativistic solids as well as ideal and non-ideal (viscous) fluids in general relativity. Formal asymptotic expansion of the governing PDE reveals the structure of the viscous stress tensor in the asymptotic relaxation limit. The key features of the new model are its symmetric hyperbolicity and thermodynamical compatibility. The proposed PDE system is causal, covariant and has bounded signal speeds for all involved processes, including dissipative ones. Since the new model also contains elastic solids as a special case, it should be understood as an alternative to existing models for viscous relativistic fluids that are usually derived from kinetic theory and extended irreversible thermodynamics. Our new formulation is particularly well suited for implementation in already existing general relativistic hydrodynamics codes based on the usual 3+1 split. We present numerical results obtained with high order ADER schemes for inviscid and viscous relativistic flows obtained in the stiff relaxation limit of the system, as well as results for solid mechanics.

In the last part of the talk we introduce a new, provably strongly hyperbolic first order reduction of the CCZ4 formalism of the Einstein field equations of general relativity and its solution with high order ADER discontinuous Galerkin finite element schemes.

References:

- (1) *S.K. Godunov*, An interesting class of quasilinear systems, *Dokl. Akad. Nauk SSSR* **139**:521–523, 1961.
- (2) *S.K. Godunov*, E.I. Romenski, Nonstationary equations of nonlinear elasticity theory in Eulerian coordinates, *J. Appl. Mech. Tech. Phys.* 13:868–884, 1972.
- (3) *I. Peshkov*, *E. Romenski*, A hyperbolic model for viscous Newtonian flows, *Contin. Mech. Thermodyn* **28**:85–104, 2016.
- (4) *M. Dumbser*, *O. Zanotti*, *R. Loubère* and *S. Diot*, A Posteriori Subcell Limiting of the Discontinuous Galerkin Finite Element Method for Hyperbolic Conservation Laws. *Journal of Computational Physics* **278**:47–75, 2014
- (5) *M. Dumbser*, *I. Peshkov*, *E. Romenski* and *O. Zanotti*, High order ADER schemes for a unified first order hyperbolic formulation of continuum mechanics: viscous heat-conducting fluids and elastic solids. *Journal of Computational Physics* **314**:824–862, 2016
- (6) *M. Dumbser*, *F. Guercilena*, *S. Köppel*, *L. Rezzolla* and *O. Zanotti*, Conformal and covariant Z4 formulation of the Einstein equations: strongly hyperbolic first-order reduction and solution with discontinuous Galerkin schemes. *Physical Review D* **97**: 084053, 2018.

FRANCESCO MAUROLICO, A RENAISSANCE INTERPRETER OF EUCLID

Veronica Gavagna Università di Firenze

Francesco Maurolico (1494-1575) has been one of the most interesting restorers of Greek mathematics during the Renaissance. His approach to the restoration of Classics was creative rather than philological, even in the case of Euclid's Elements. Among his extant writings we find a quite faithful "reading" of some Books of the Elements (V, VII-X), but the most innovative work is a compendium of the Euclidean text.

In 1567, in fact, Maurolico was involved by the Jesuits in a plan for arranging all the sciences in an encyclopedia structured in compendia. Such compendia were to be used in the teaching of the Collegium of Messina and, possibly, in every Collegium of the Society of Jesus. In a few months, Maurolico completed the compendia of the first ten books of the Elements and reworked a text of some years earlier (1563), dedicated to stereometry, transforming it in the compendium of Books XI and XII. To conclude the work, he then decided that the old edition of Books XIII-XV – written in 1532 – would have finally completed the whole Elementorum compendia, that unfortunately were never printed, except for Books XIII-XV (Opuscula Mathematica, 1575). Maurolico's Compendia are not really a reasoned synthesis of the Elements, but rather a reworking influenced by a deep arithmetical interpretation of the topics, from the theory of proportions to the relationships between regular polyhedra. Maurolico's reworking of Books XIII-XV, devoted to regular polyhedra, is particularly interesting for the increased number of new propositions. Maurolico's deep interest in these solids is also testified by *De impletione loci*, a work on the problem of filling space with regular polyhedra written in 1529. The goal of this writing is confuting Averroes' remark (influenced by Aristotle) on the possibility of filling space with regular tetrahedra. The novelty is that Maurolico's approach to this problem was definitely mathematical and not philosophical: he measured the dihedral angles of the regular polyhedra and tested all the suitable combinations of the solids. Finally, in his studies on regular polyhedra Maurolico emphasized the discovery of a relationship that sounds as a kind of Euler's polyhedron formula.

BOUNDEDNESS OF VARIETIES OF GENERAL TYPE

Christopher Hacon University of Utah

Complex projective varieties are subsets of complex projective space defined by a set of homogeneous polynomials. Varieties of general type are the higher dimensional analog of Riemann surfaces of genus $g \geq 2$. In this talk I will discuss recent progress on the classification of these varieties.

NORMS OF RANDOM MATRICES WITH INDEPENDENT ENTRIES

Rafał Łatała Uniwersytet Warszawski

The spectral norm of any matrix is bigger than the largest Euclidean norm of its rows and columns. We show that for Gaussian matrices with independent entries this obvious bound may be reversed in average up to a universal constant. We will also discuss similar bounds for Schatten norms and other random matrices with independent entries.

The talk is based on a joint work with Ramon van Handel (Princeton) and Pierre Youssef (Paris).

MODELS FOR THIN PRESTRAINED STRUCTURES

Marta Lewicka University of Pittsburgh

Variational methods have been extensively used in the past decades to rigorously derive nonlinear models in the description of thin elastic films. In this context, natural growth or differential swelling-shrinking lead to models where an elastic body aims at reaching a space-dependent metric. We will describe the effect of such, generically incompatible, prestrain metrics on the singular limits' bidimensional models. We will discuss metrics that vary across the specimen in both the midplate and the thin (transversal) directions. We will also cover the case of the oscillatory prestrain, exhibit its relation to the non-oscillatory case via identifying the effective metrics, and discuss the role of the Riemann curvature tensor in the limiting models.

POST-TURING TISSUE PATTERN FORMATION: INSIGHTS FROM MATHEMATICAL MODELLING

Anna Marciniak-Czochra Heidelberg University

Cells and tissue are objects of the physical world, and therefore they obey the laws of physics and chemistry, notwithstanding the molecular complexity of biological systems. What are the mathematical principles that are at play in generating such complex entities from simple laws? Understanding the role of mechanical and mechano-chemical interactions in cell processes, tissue development, regeneration and disease has become a rapidly expanding research field in the life sciences. To reveal the patterning potential of mechano-chemical interactions, we have developed two classes of mathematical models coupling dynamics of diffusing molecular signals with a model of tissue deformation. First, we derived a model based on energy minimization that leads to 4-th order partial differential equations of evolution of infinitely thin deforming tissue (pseudo-3D model) coupled with a surface reaction-diffusion equation. The second approach (full-3D model) consists of a continuous model of large tissue deformation coupled with a discrete description of spatial distribution of cells to account for active deformation of single cells. The models account for a range of mechano-chemical feedbacks, such as signalling-dependent strain, stress, or tissue compression. Numerical simulations based on the arbitrary Lagrangian-Eulerian and fully Eulerian formulations show ability of the proposed mechanisms to generate development of various spatio-temporal structures. In this study, we compare the resulting patterns of tissue invagination and evagination to those encountered in developmental biology. The new class of patterns is compared to the classical Turing patterns. Analytical and numerical challenges of the proposed models are discussed.

COMBINATORIAL TOPOLOGICAL DYNAMICS

Marian Mrozek Uniwersytet Jagielloński

Combinatorial topological dynamics is an emerging field with roots in the concept of combinatorial vector field introduced on the turn of the 20th and 21st century by American mathematician Robert Forman as a tool used by him to construct combinatorial Morse theory. I will recall the basic ideas of this theory. I will also present some recent results on combinatorial vector fields motivated by applications in sampled dynamics.

THE DISTANCE FUNCTION FROM A REAL ALGEBRAIC VARIETY, OLD AND NEW

Giorgio Ottaviani Università di Firenze

The Euclidean distance function from a conic was computed by means of invariant theory in XIX century.

The distance function from the variety of corank one matrices was computed independently by Beltrami and Jordan a few years later and gave rise to the Singular Value Decomposition. Today this function is the core of engineering applications, like “offset surface”. More generally, the distance function from a real algebraic variety is a root of an algebraic function. Having in mind applications to the spectral theory of tensors, we show a duality property of this function and we describe its lowest and highest coefficients. We show how this fits in the ED philosophy, where ED stands for “Euclidean Distance”.

AN OVERVIEW OF MATHEMATICAL MODELS FOR CELL MIGRATION

Luigi Preziosi Politecnico di Torino

Cell-extracellular matrix interaction and the mechanical properties of cell nucleus have been demonstrated to play a fundamental role in cell movement across fibre networks and micro-channels. From the point of view of application understanding this process is important to describe on one hand the spread of cancer metastases and on the other hand to optimize medical scaffold that can be use to cure chronic wounds. From the point of view of mathematics, the problem can be addressed using different methods. In fact, in the talk, I will describe several mathematical models developed to deal with such a phenomenon, starting from modelling cell adhesion mechanics to the inclusion of influence of nucleus stiffness in the motion of cells, through continuum mechanics, kinetic models and individual cell-based models.

NEW RESULTS IN THE TOPOLOGICAL CLASSIFICATION OF GROMOV BOUNDARIES OF HYPERBOLIC GROUPS

Jacek Świątkowski Uniwersytet Wrocławski

The classification of finite simple groups is one of the greatest mathematical achievements of the 21st century. In contrast with that, a classification of infinite finitely presented groups is an undecidable algorithmic problem, due to classical results from 1950's. As a result, mathematicians study some special classes of infinite finitely presented groups, and try to classify them up to weaker equivalence relations than isomorphism.

Geometric group theory studies infinite groups by way of viewing them as certain geometric objects. In 1980's M.Gromov proposed to study a vast class of the so called word-hyperbolic groups, whose behaviour as geometric objects resembles that of the hyperbolic plane. Instead of classifying them up to isomorphism, one may try to classify their “behaviour at infinity”, encoded in an object called Gromov boundary. Despite more than 30 years of efforts, there are still many quite basic open questions concerning the topological classification of Gromov boundaries of hyperbolic groups.

During the talk I will describe some recent developments concerning this problem. The first consists of a satisfactory description of the topology of the Gromov boundary of a free product of hyperbolic groups with amalgamation along finite groups. The other consists of showing that Gromov boundary of a hyperbolic group is

a space belonging to some countable family of spaces called Markov compacta, which are describable in certain algorithmic way out of finite amount of initial data.

SPIRALLING AND OTHER SOLUTIONS IN LIMITING PROFILES OF COMPETITION-DIFFUSION SYSTEMS

Susanna Terracini Università di Torino

Reaction-diffusion systems with strong interaction terms appear in many multispecies physical problems as well as in population dynamics. The qualitative properties of the solutions and their limiting profiles in different regimes have been at the center of the community's attention in recent years. A prototypical example is the system of equations

$$\begin{cases} -\Delta u + a_1 u = b_1 |u|^{p+q-2} u + c p |u|^{p-2} |v|^q u, \\ -\Delta v + a_2 v = b_2 |v|^{p+q-2} v + c q |u|^p |v|^{q-2} v \end{cases}$$

in a domain $\Omega \subset \mathbb{R}^n$ which appears, for example, when looking for solitary wave solutions for Bose-Einstein condensates of two different hyperfine states which overlap in space. The sign of b_i reflects the interaction of the particles within each single state. If b_i is positive, the self interaction is attractive (focusing problems). The sign of c , on the other hand, reflects the interaction of particles in different states. This interaction is attractive if $c > 0$ and repulsive if $c < 0$. If the condensates repel, they eventually separate spatially giving rise to a free boundary. Similar phenomena occurs for many species systems. As a model problem, we consider the system of stationary equations:

$$\begin{cases} -\Delta u_i = f_i(u_i) - \beta u_i \sum_{j \neq i} g_{ij}(u_j) \\ u_i > 0. \end{cases}$$

The cases $g_{ij}(s) = \beta_{ij} s$ (Lotka-Volterra competitive interactions) and $g_{ij}(s) = \beta_{ij} s^2$ (gradient system for Gross-Pitaevskii energies) are of particular interest in the applications to population dynamics and theoretical physics respectively.

Phase separation has been described in the recent literature, both physical and mathematical. Relevant connections have been established with optimal partition problems involving spectral functionals. The classification of entire solutions and the geometric aspects of phase separation are of fundamental importance as well. We intend to focus on the most recent developments of the theory in connection with problems featuring anomalous diffusions, non-local and non symmetric interactions

Guest Plenary Speaker

FUNCTIONAL INEQUALITIES, FLOWS, SYMMETRY AND SPECTRAL ESTIMATES

Maria J. Esteban CNRS & Université Paris-Dauphine

In this talk I will review recent result about how the use of linear and nonlinear flows has been key to prove functional inequalities and qualitative properties for their extremal functions. I will also explain how from these inequalities and their best constants, optimal spectral estimates can be obtained for Schrödinger operators. This is a topic which is at the crossroads of nonlinear analysis and probability, with implications in differential geometry and potential applications in modelling in physics and biology.

Sessions

1

Model Theory

Organizers:

Piotr Kowalski (Uniwersytet Wrocławski)

Antongiulio Fornasiero (University of Florence)

MODEL THEORY OF STEINER TRIPLE SYSTEMS

Silvia Barbina Open University

A Steiner triple system is a set S together with a collection B of subsets of S of size 3 such that any two elements of S belong to exactly one element of B . It is well known that the class of finite Steiner triple systems has a Fraïssé limit M . In joint work with Enrique Casanovas we show that the theory of M is the model completion of the theory of Steiner triple systems. We also prove that T is not small, has quantifier elimination, TP2, NSOP1, elimination of hyperimaginaries and weak elimination of imaginaries.

INP-MINIMAL GROUPS, RINGS, AND BILINEAR FORMS

Jan Dobrowolski University of Leeds

In a recent work with Frank Wagner we prove that every inp-minimal omega-categorical group is virtually abelian-by-finite, answering in particular two questions of Kaplan, Levi, and Simon. After introducing the context, I will discuss some of the ideas of the proof, in particular, the use of bilinear forms. An analogous theorem for rings is also obtained.

DECIDABILITY OF THEORIES OF MODULES

Lorna Gregory L'Università degli Studi della Campania "Luigi Vanvitelli"

In 1955, Szmielew proved that the theory of abelian groups, equivalently the theory of \mathbb{Z} -modules, is decidable. On the other hand, in the mid70's Baur and independently Kokorin and Martýanov proved that the theory of $k\langle x, y \rangle$ -modules is undecidable for any field k . In this talk, I will give an overview of decidability and undecidability results for theories of modules. I will present modern techniques for proving decidability of theories of modules. I will then specialise to recent results for finite-dimensional algebras and commutative domains.

HOW CAN WE GET WELL-BEHAVING PAC STRUCTURES?

Daniel Hoffmann Uniwersytet Wrocławski

In my preprint "Model theoretic dynamics in Galois fashion", I considered substructures of a big stable structure, which are equipped with a group action of a fixed group G . The most interesting case was the case of existentially closed substructures with a group action. Such structures are pseudo-algebraically closed and there is a way to control their absolute Galois groups (at least in some situations). I will explain what do we know about the absolute Galois groups of the aforementioned PAC structures (referring to results of my another preprint "On Galois groups and PAC substructures") and give some remarks showing how the absolute Galois group of a PAC structure might control the theory of such a structure.

CONNECTED COMPONENTS OF RINGS

Grzegorz Jagiella Uniwersytet Wrocławski

I will talk about the notion of a type-definable ideal (or a subring) of bounded index in a ring and its relation to the model-theoretic connected components of the additive group of the ring. I will also explore the connections with definable topological dynamics. Joint work with Gismatullin and Krupiński.

THE QUANTUM LOGIC OF A RING WITH INVOLUTION

Sonia L'Innocente University of Camerino

This report is devoted at generalizing Olivier's construction of the universal commutative (von Neumann) regular ring over a commutative ring, in order to obtain the universal $*$ -regular ring R over a noncommutative ring R with an involution $*$. The construction of a universal $*$ -regular ring R proceeds by adjoining to every element in the ring R its Moore-Penrose inverse. The involution of R induces an involution on the modular lattice L of positive primitive formulae in the language of left R -modules. It is shown that R coordinatizes the universal quantum logic of the ring R given by the quotient lattice of L modulo the least congruence for which the involution designates an orthogonal complement. This congruence is generated by the Laws of Contradiction and Excluded Middle, so that the R -modules, that arise from the universal $*$ -regular ring, are axiomatized by these laws. A crucial example in the context of universal enveloping algebras will be analysing.

VALUED HYPERFIELDS, TRUNCATED DVRs, AND VALUED FIELDS

Junguk Lee Mathematical Institute, Wrocław University

M. Krasner introduced a notion of valued hyperfield analogous to a valued field with a multivalued addition operation, and used it to do a theory of limits of local fields. P. Deligne did the theory of limits of local fields in a different way by defining a notion of triple, which consists of truncated discrete valuation rings and some additional data. Typical examples of a valued hyperfields and truncated discrete valuation rings are the n -th valued hyper field, which is quotient of a valued field by a multiplicative subgroup of the form $+m^n$, where m is the maximal ideal of a valuation ring, and the n -th residue ring, which is a quotient of a valuation ring by the n -th power of the maximal ideal. J. Tolliver showed that discrete valued hyperfields and triples are essentially same, stated by P. Deligne without a proof. W. Lee and the author showed that given complete discrete valued fields of mixed characteristic with perfect residue fields, any homomorphism between the n -th residue rings of the valued fields is lifted to a homomorphism between the valued fields for large enough n . This lifting process is functorial.

Motivated by above results, we show that given complete discrete valued fields of mixed characteristic with perfect residue fields, any homomorphism between the n -th valued hyperfields of the valued fields can be lifted to a homomorphism between the valued fields for large enough n , which is functorial. We also compute an upper bound of such a minimal n effectively depending only on the ramification index. Most of all, any homomorphism between the first valued hyperfields of valued fields is uniquely lifted to a homomorphism between the valued fields in the case of tamely ramified valued fields. From this lifting result, we prove a relative completeness AKE-theorem via valued hyperfields for finitely ramified valued fields with perfect residue fields.

NONSTANDARD METHODS IN COMBINATORICS

Lorenzo Luperi Baglini University of Vienna

In certain areas of combinatorics of numbers and Ramsey theory, several non-elementary methods are applied, including ergodic theory, Fourier analysis, (discrete) topological dynamics, algebra in the space of ultrafilters. Recently, certain methods based on non-standard analysis have been applied to both density results and Ramsey results. Basically, the two main ideas of this approach are: (1) hyperfinite versions of infinitary results; (2) identifying Stone-Ćech compactifications with quotients of nonstandard extensions. We will give a brief presentation of the techniques that arise from these ideas.

TRANSSERIES AS SURREAL ANALYTIC FUNCTIONS

Vincenzo Mantova University of Leeds

Transseries, such as LE series, arise when dealing with certain asymptotic expansions of real analytic function. Most transseries, though, are not convergent, and cannot represent real analytic functions, if only just for cardinality reasons. On the other hand, we can show that LE series do induce germs of non-standard analytic functions on the surreal line. More generally, call “omega-series” the surreal numbers that can be generated from the real numbers and the ordinal omega by closing under exponentiation, logarithm and infinite sum. Then omega-series form a proper class of transseries including LE series. It turns out that all omega-series induce (germs of) surreal analytic functions. Moreover, they can be composed and differentiated in a way that is consistent with their interpretation as functions, extending the already known composition and derivation of LE series, and the derivation coincides with the simplest one on surreal numbers. This is joint work with A. Berarducci.

DEFINABLE SETS IN LINEAR ORDERS

Slavko Moconja Institute of Mathematics, University of Wrocław

In this talk we will distinguish and discuss several conditions holding in colored linear orders. We will try to describe linear orders which are characterized by these conditions, where we are especially interested in characterizations given by descriptions of definable sets. We present joint work with Predrag Tanović and Dejan Ilić.

FIRST-ORDER MODEL THEORY OF FREE PROJECTIVE PLANES

Gianluca Paolini Einstein Institute of Mathematics of the Hebrew University of Jerusalem

We prove that the theory of open projective planes is complete and strictly stable, and infer from this that Marshall Hall’s free projective planes $(\pi^n : 4 \leq n \leq \omega)$ are all elementary equivalent and that their common theory is strictly stable and decidable, being in fact the theory of open projective planes. We further characterise the elementary substructure relation on the class of open projective planes, and show that $(\pi^n : 4 \leq n \leq \omega)$ is an

elementary chain. We then prove that for every infinite cardinality κ there are κ non-isomorphic open projective planes of power κ , improving known results on the number of open projective planes. Finally, we prove that π^ω is type-homogeneous and characterise the forking independence relation in models of the theory.

LINEAR ORDERS, CYCLIC ORDERS AND HEREDITARY G -COMPACTNESS

Tomasz Rzepecki Uniwersytet Wrocławski

In my talk, I will recall the notion of the *Laszar distance*, which is a natural (invariant) metric (with values in $\mathbf{N} \cup \{\infty\}$) on the space of small tuples in a fixed monster model. A maximal set of elements which are pairwise at finite distance from one another is called a *Laszar strong type*. We say that a theory is *G-compact* if every Laszar strong type has finite diameter (with respect to the Laszar distance). In particular, one can show that every simple theory (and, by extension, every stable theory) is G-compact.

One of the first examples of a non-G-compact theory was given in [1]. I will roughly explain the example and show how it can be generalised to show that in every sufficiently saturated infinite linear order we can define a structure whose theory is not G-compact, by reducing the problem to the case of dense and discrete linear orders.

A long-standing conjecture says that an unstable NIP theory always interprets an infinite linear order. If the conjecture holds, it would imply that NIP+“hereditarily G-compact” implies stable. If there is enough time, I will briefly discuss the notion of hereditary G-compactness, and how the method from [1] can be used for non-linear posets with certain “additivity” properties.

- [1] Enrique Casanovas, Daniel Laszar, Anand Pillay and Martin Ziegler. ‘Galois Groups of First Order Theories’. In: *J. Math. Log.* 1.2 (2001), pp. 305–319.

THE TORSION FREE PART OF THE ZIEGLER SPECTRUM OF ORDERS OVER DEDEKIND DOMAINS

Carlo Toffalori University of Camerino

I present some recent results obtained with Lorna Gregory and Sonia L’Innocente on the model theory of modules over orders O over Dedekind domains R , in particular on the torsion free part of their Ziegler spectrum. I discuss the role of lattices. Here orders are meant as certain widely considered finitely generated ring extensions of Dedekind domains R , and lattices over an order O are finitely generated R -torsionfree modules over O . These results follow some suggestions of Gena Puninski and enlarge a past analysis with Mike Prest and Annalisa Marcja over group rings. In particular I propose some adaptations to pure injective modules over orders of the Maranda Theorem on lattices.

2

Number Theory

Organizers:

Łukasz Pańkowski (Adam Mickiewicz University in Poznań)

Sandro Bettin (University of Genova)

Ilaria Del Corso (Universita' di Pisa)

Maciej Radziejewski (Adam Mickiewicz University in Poznań)

THE ATKIN-LEHNER OPERATOR ON DRINFELD MODULAR FORMS

Andrea Bandini University of Pisa

Let $F := \mathbb{F}_q(t)$ with q a power of a prime p , fix $\frac{1}{t}$ as the prime at ∞ and let $A := \mathbb{F}_q[t]$ be the “ring of integers” of F . Let F_∞ be the completion of F at $\frac{1}{t}$ with ring of integers A_∞ and denote by \mathbb{C}_∞ the completion of an algebraic closure of F_∞ . The Drinfeld upper half-plane is the set $\Omega := \mathbb{P}^1(\mathbb{C}_\infty) - \mathbb{P}^1(F_\infty)$ together with a structure of rigid analytic space.

Let $\Gamma_0(\mathfrak{n})$ be an arithmetic subgroup of $GL_2(A)$ of level \mathfrak{n} (where \mathfrak{n} is an ideal in A). The Drinfeld modular forms of weight k , type m and level \mathfrak{n} are meromorphic functions $\varphi : \Omega \rightarrow \mathbb{C}_\infty$ which are invariant for the action of $\Gamma_0(\mathfrak{n})$ and enjoy properties analogous to the classical modular forms (in characteristic 0) for arithmetic subgroups of $GL_2(\mathbb{Z})$ (for example they are required to be holomorphic at cusps). In particular they have a Fourier expansion and they admit an action by Hecke operators. We denote cusp forms (i.e. Drinfeld modular forms which vanish at all cusps) by $S_{k,m}^1(\mathfrak{n})$: their structure is still quite mysterious due to the absence of an analog of Petersson inner product and to a Fourier expansion on which the action of Hecke operators is provided by rather complicated formulas.

In an attempt to provide the foundations of Hida-Coleman theory for families of Drinfeld modular forms we study the degeneracy maps $S_{k,m}^1(1) \rightarrow S_{k,m}^1(t)$ and the trace maps (the other way around) and use them to define oldforms and newforms in $S_{k,m}^1(t)$. Via an explicit description of the action of U_t (the Atkin-Lehner operator, part of the Hecke algebra) on $S_{k,m}^1(t)$ we are able to compute slopes (i.e. t -adic valuations of eigenvalues) for eigenforms and, building on that, we shall describe various results and conjectures on the structure of $S_{k,m}^1(t)$, on the diagonalizability of U_t and on the distribution of slopes as the weight k varies. This is all joint work with Maria Valentino.

AN EFFECTIVE CRITERION FOR PERIODICITY OF p -ADIC CONTINUED FRACTIONS

Laura Capuano University of Oxford

It goes back to Lagrange that a real quadratic irrational always has a periodic continued fraction. Starting from decades ago, several authors proposed different definitions of a p -adic continued fraction, and the definition depends on the chosen system of residues mod p . It turns out that the theory of p -adic continued fractions has many differences with respect to the real case; in particular, no analogue of Lagrange’s theorem holds, and the problem of deciding whether the continued fraction is periodic or not seemed to be not known. In recent work with F. Veneziano and U. Zannier we investigated the expansion of quadratic irrationals, for the p -adic continued fractions introduced by Ruban, giving an effective criterion to establish the possible periodicity of the expansion. This criterion, somewhat surprisingly, depends on the real value of the p -adic continued fraction.

THE EQUIVARIANT LOCAL ε -CONSTANT CONJECTURE FOR UNRAMIFIED TWISTS OF $\mathbb{Z}_p(1)$

Alessandro Cobbe Universität der Bundeswehr München

Let N/K be a finite Galois extension of p -adic number fields. We study the equivariant local ε -constant conjecture, denoted by $C_{EP}^{na}(N/K, V)$, as formulated in various forms by Kato, Benois and Berger, Fukaya and Kato and others, for certain 1-dimensional twists $T = \mathbb{Z}_p(\chi^{\text{nr}})(1)$ of $\mathbb{Z}_p(1)$ and $V = T \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$. We show the

validity of $C_{EP}^{na}(N/K, V)$ for certain wildly and weakly ramified abelian extensions N/K . This is a joint work with Werner Bley.

COMPOSITE FACTORS OF BINOMIALS AND LINEAR SYSTEMS IN ROOTS OF UNITY

Roberto Dvornicich Dipartimento di Matematica, Università di Pisa

This is a joint paper with Umberto Zannier. We completely classify binomials in one variable which have a nontrivial factor which is *composite*, i.e. of the shape $g(h(x))$ for polynomials g, h both of degree > 1 . In particular, we prove that, if a binomial has such a composite factor, then $\deg g \leq 2$ (under natural necessary conditions). This is best-possible and improves on a previous bound $\deg g \leq 24$. This result provides evidence toward a conjecture predicting a similar bound when binomials are replaced by polynomials with any given number of terms. As an auxiliary result, which could have other applications, we completely classify the solutions in roots of unity of certain systems of linear equations.

HYBRID CONTINUED FRACTIONS AND OTHER APPLICATIONS OF p -ADIC HERON'S ALGORITHM

Antonino Leonardis

This work will continue the author's previous works on continued fractions and Heron's algorithm. Extending the notion of continued fraction to the p -adic fields, one can find continued fractions which converge in both real and p -adic topologies to the "same" algebraic number, some of which are given by the Heron's algorithm. The definition can be possibly generalized to other global fields, as left as an open question. We will end the part on hybrid convergence with many numerical examples. After that, we will recall the basic algorithms on the p -adic fields studied by the author and see some applications of them to computer science, where Heron's algorithm gives a fast calculation convergence, and cryptography, showing some methods to get pseudo-random numbers.

A NOTE ON p -ADIC VALUATIONS OF STIRLING NUMBERS OF THE SECOND KIND

Piotr Miska Jagiellonian University in Kraków, Institute of Mathematics

The Stirling number of the second kind $S(n, k)$, where $n, k \in \mathbb{N}$, counts the number of partitions of a set with n elements into exactly k nonempty subsets. The problem of p -adic valuations (with emphasize on 2-adic valuations) of Stirling numbers of the second kind and their relatives generated a lot of literature. It was considered by e.g. Lengyel, Clarke, De Wannemaker, Bennet and Mosteig. In 2008 Amdeberhan, Manna and Moll stated a conjecture on general description of 2-adic valuations of Stirling numbers of the second kind. Later, in 2010, Berrizbeitia, Medina, Moll, Moll and Noble generalized this conjecture on p -adic valuations of numbers $S(n, k)$ for arbitrary prime number p . In this talk we will show results on p -adic valuations of numbers $S(n, k)$ inspired by these conjectures.

SIGNED HARMONIC SUMS

Giuseppe Molteni Università degli Studi di Milano

The set of numbers $\sum_{n=1}^N s_n/n$ with $s_n \in \{\pm 1\}$ for every n and $N \in \mathbb{N}$ is dense in \mathbb{R} . We discuss some properties of the part of this set which is close to a given number τ . In particular we discuss some special features of the sequence converging to τ generated by a greedy algorithm, and the upper/lower bounds for the best approximation to τ which is possible to obtain with sums of a given length N . This is an account of a joint work with Sandro Bettin and Carlo Sanna [1,2].

BIBLIOGRAPHY: [1] S. Bettin, G. Molteni, and C. Sanna, “Small values of signed harmonic sums”, preprint arXiv:1806.05402, 2018.

[2] S. Bettin, G. Molteni, and C. Sanna, “Greedy approximations by signed harmonic sums and the Thue-Morse sequence”, preprint arXiv:1805.00075, 2018.

LOCAL-GLOBAL DIVISIBILITY QUESTIONS IN COMMUTATIVE ALGEBRAIC GROUPS

Laura Paladino University of Calabria

Let k be a number field and let \mathcal{A} be a commutative algebraic group defined over k . We denote by M_k the set of places $v \in k$ and by k_v the completion of k at the valuation v . Suppose that for all but finitely many $v \in M_k$, there exists $D_v \in \mathcal{A}(k_v)$ such that $P = qD_v$, where P is a fixed k -rational point of \mathcal{A} and q is a fixed positive integer. Is it possible to conclude that there exists $D \in \mathcal{A}(k)$ such that $P = qD$? The answer to this question, known as Local-Global Divisibility Problem, is linked to the behaviour of a cohomological group, whose definition is similar to the one of the Tate-Shafarevich group $Sha(k, \mathcal{A}[q])$, where $\mathcal{A}[q]$ denotes the q -torsion subgroup of \mathcal{A} . We show some sufficient condition to have both an affirmative answer for the local-global divisibility by q in \mathcal{A} and the triviality of $Sha(k, \mathcal{A}[q])$, when $q = p$ is a prime number. If \mathcal{A} is an abelian variety principally polarized, the vanishing of $Sha(k, \mathcal{A}[p])$ implies a local-global principle for divisibility by p for the elements of $H^r(k, \mathcal{A})$, for all $r \geq 0$, giving an answer to a question posed by Cassels.

FUNCTIONAL EQUATION OF THE STANDARD TWIST: SPECIAL CASES

Alberto Perelli Università di Genova

The standard twist $F(s, \alpha)$ of L -functions $F(s)$ in the Selberg class has several interesting properties and plays a central role in the Selberg class theory. It is therefore natural to study its finer analytic properties, for example the functional equation. In this talk we deal with a special case, where the functional equation of $F(s)$ has the gamma factor $\Gamma(s + (\kappa - 1)/2)$, κ being a half-integer. We show that the standard twist $F(s, \alpha)$ satisfies a functional equation reflecting s to $1 - s$, whose shape is not far from a Riemann-type functional equation of degree 2 and may be regarded as a degree 2 analog of the Hurwitz-Lerch functional equation. We also deduce some result on the growth on vertical strips and on the distribution of zeros of $F(s, \alpha)$. (joint work with J.Kaczorowski)

GALOIS PROPERTIES OF RINGS OF INTEGER-VALUED POLYNOMIALS

Giulio Peruginelli Dipartimento di Matematica, University of Padova

Let K be a number field with ring of integers O_K . Recently, Loper and Werner introduced the following ring which generalizes the classical definition of ring of integer-valued polynomials: $\text{Int}_{\mathbb{Q}}(O_K) = \{f \in \mathbb{Q}[X] \mid f(O_K) \subseteq O_K\}$. If $K = \mathbb{Q}$ then we get the classical ring $\text{Int}(\mathbb{Z})$ of polynomials with rational coefficients mapping \mathbb{Z} into \mathbb{Z} . Loper and Werner prove that $\text{Int}_{\mathbb{Q}}(O_K)$ is a Prüfer domain, which is strictly contained in $\text{Int}(\mathbb{Z})$ if K is a proper extension of \mathbb{Q} . Here, we show that in case K, F are Galois extensions of \mathbb{Q} , then $\text{Int}_{\mathbb{Q}}(O_K) = \text{Int}_{\mathbb{Q}}(O_F)$ if and only if $K = F$. We also characterize a basis for $\text{Int}_{\mathbb{Q}}(O_K)$ as a \mathbb{Z} -module when K/\mathbb{Q} is a tamely ramified Galois extension. This is a joint work with Bahar Heidaryan and Matteo Longo.

More generally, we also give the following new generalization: for any number fields K, F , if $\text{Int}_{\mathbb{Q}}(O_K) = \text{Int}_{\mathbb{Q}}(O_F)$, then K, F are conjugated over \mathbb{Q} .

ON THE LAST NONZERO DIGITS OF $n!$

Bartosz Sobolewski Jagiellonian University

Let $\ell_b(n!)$ be the last nonzero digit in the base- b expansion of $n!$. We will determine for which bases b the sequence $\{\ell_b(n!)\}_{n \geq 0}$ is automatic, i.e., its terms can be computed by a finite-state machine reading the digits of n . Using this description we will compute how often (in the asymptotic sense) $\ell_b(n!)$ takes on each value $1, 2, \dots, b-1$. Similar results have already been obtained in particular for $b = 4$ (Deshouillers, Luca) and $b = 12$ (Deshouillers, Ruzsa).

ON THE SETS WITH SMALL k -FOLD SUMSETS AND FOX-KLEITMAN CONJECTURE

Katarzyna Taczala Adam Mickiewicz University in Poznań

In 2014 Eberhard, Green and Manners proved a structural theorem about the sets of doubling less than 4. We will discuss the generalization of this theorem to the sets with small k -fold sumsets. We will also show how this result can be applied to confirm a conjecture of Fox and Kleitman on the maximal degree of regularity of the equation $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i + b$ for $b \in \mathbb{N}$. This is joint work with Tomasz Schoen.

ADDITIVE PROBLEMS WITH PRIME VARIABLES

Alessandro Zaccagnini Dipartimento di Scienze Matematiche, Fisiche e Informatiche, Università di Parma

I will review several recent results, in collaboration with Marco Cantarini, Alessandro Gambini and Alessandro Languasco, concerning averages of the number of representations of an integer as a sum of powers of primes with a given set of exponents. The proof needs the original version of Hardy and Littlewood's circle method.

3

Projective Varieties and their Arrangements

Organizers:

Tomasz Szemberg (Pedagogical University of Cracow)

Maria Evelina Rossi (University of Genoa)

Alessandra Sarti (University of Poitiers)

Sławomir Rams (Jagiellonian University)

HYPERELLIPTIC CURVES ON ABELIAN SURFACES

Paweł Borówka Jagiellonian University in Kraków

There is a one to one correspondence between symmetric hyperelliptic curves on an abelian surface and rational curves on its Kummer. The aim of the talk is to show that there are finitely many smooth hyperelliptic curves on a general polarised abelian surface, compute the number of these and show that it is non-zero only for polarisations of type $(1, d)$, $d < 5$. Moreover, since the construction is explicit, one can consider their configuration and recover famous 16_6 configuration or 32_{10} .

CONTACT FANO MANIFOLDS

Jarosław Buczyński IMPAN and MIMUW

A complex manifold is a contact manifold if there is a distribution in the tangent bundle which is as non-integrable as possible. I will report on recent progress in classification of projective contact manifolds focusing on the case of (complex) dimension 7 and 9. Our work implies the classification of quaternion-Kähler manifolds of (real) dimensions 12 and 16, a famous problem from Riemannian geometry. The tools we use include representation theory and actions of (complex) reductive groups on manifolds, symplectic geometry, characteristic classes, and equivariant localisation theorems. The talk is based mainly on: "Algebraic torus actions on contact manifolds" Jarosław Buczyński, Jarosław A. Wiśniewski, with an appendix by Andrzej Weber, arXiv:1802.05002.

HADAMARD PRODUCT OF DEGENERATE SUBVARIETIES

Enrico Carlini Politecnico di Torino

The Hadamard product of matrices and tensors is widely used in matrices and tensors theory. The Hadamard product of varieties is a relatively new topic the study of which arose in connection with Algebraic Statistics. In this talk we will recall some basic facts about the Hadamard product of varieties. Then we will focus on degenerate subvarieties. In particular, we will discuss dimension, degree, and singularities.

ON SHGH CONJECTURE AND UNEXPECTED CURVES OF LOW DEGREE

Łucja Farnik Institute of Mathematics, Polish Academy of Sciences

The problem of computing the dimension of the linear systems of plane curves passing through a given set of multiple points is one of the most classical issues in Algebraic Geometry. However, it is still an open problem. Despite many partial results, a complete solution is not known even if the fixed points are in general position. The famous Segre-Harbourne-Gimigliano-Hirschowitz conjecture, which predicts the answer, stands unsolved for more than 50 years.

In case when fixed points are in special position, unexpected situations may occur. Recently Di Gennaro, Ilardi and Vallès described a special configuration Z of nine points with a remarkable property: a general triple point always fails to impose independent conditions on the ideal of Z in degree 4. The peculiar structure and properties of such unexpected curves were studied by Cook II, Harbourne, Migliore and Nagel. In this talk I will

present the classification of low degree unexpected curves. In particular, I will sketch a proof that the above mentioned configuration Z is the unique one giving rise to an unexpected quartic. This is joint work with F. Galuppi, L. Sodomaco and B. Trok.

THE ACM PROPERTY IN MULTIPROJECTIVE SPACES

Giuseppe Favacchio Università di Catania

If X is a finite set of points in a multiprojective space $\mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_r}$ with $r \geq 2$, then X is not necessarily arithmetically Cohen-Macaulay (ACM). Several characterizations of ACM sets of points in $\mathbb{P}^1 \times \mathbb{P}^1$ are known. More recently, a characterization of the ACM property was given for sets of (reduced) points in $(\mathbb{P}^1)^r = \mathbb{P}^1 \times \cdots \times \mathbb{P}^1$ and partial results were found in $\mathbb{P}^1 \times \mathbb{P}^n$. In this talk I will discuss recent progress on the ACM property for multiprojective varieties. This talk is based upon joint works with Elena Guardo and with Juan Migliore.

K3 SURFACES WITH A LEVEL N STRUCTURE

Alice Garbagnati Università degli Studi di Milano-Dipartimento di Matematica

A K3 surface is said to have a level n structure if it is birational to the quotient of another K3 surface by an automorphism of order n . During this talk I will discuss the geometry of K3 surfaces with level n structure, for an odd prime number n , and some properties of their moduli spaces. In particular, I will compare some of the moduli spaces of these K3 surfaces with the moduli spaces of curves with level n structure and low genus.

ON THE MORIN PROBLEM

Grzegorz Kapustka Uniwersytet Jagielloński

We will study the Morin problem and present a method of classification of finite complete families of incident planes in P^5 as a result we prove that there is exactly one, up to $\text{Aut}(P^5)$, configuration of maximal cardinality 20 and a unique one parameter family containing all the configurations of 19 planes. The method is to study projective models of appropriated moduli spaces of twisted sheaves on K3 surfaces. This is a joint work with A. Verra.

CONTAINMENT AND COMBINATORICS

Halszka Tutaj-Gasińska Jagiellonian University

We consider two configurations of twelve lines with nineteen triple points. Both of them have the same arrangemental features, i.e. both configurations nine of twelve lines have five triple points and one double point, and the remaining three lines have four triple points and three double points. Taking the ideal of the triple points of these configurations it turns out that for one of the configurations the containment of the third symbolic power of the ideal in the second algebraic power holds, while for the other it does not.

THE NUMBER OF ENRIQUES QUOTIENTS OF A SINGULAR $K3$ SURFACE

Daide Cesare Veniani Johannes Gutenberg-Universität Mainz

TBA

4

Algebraic Geometry

Organizers:

Joachim Jelisiejew (Polish Academy of Sciences)

Cinzia Casagrande (University of Torino)

MATRIX SCHUBERT VARIETIES AND TORUS ACTIONS

Maria Donten-Bury University of Warsaw

We investigate certain torus actions on matrix Schubert varieties and similar varieties which admit a combinatorial construction. We attempt to classify the cases where the action has low complexity, study their properties related to deformation theory and their description in terms of polyhedral divisors. This is an ongoing joint project with Laura Escobar and Irem Portakal.

EQUIVALENCE OF CALABI-YAU TYPE MANIFOLDS

Michał Kapuska Jagiellonian University and Polish Academy of Sciences

I will discuss several notions of equivalence between algebraic manifolds and relations between them. I will focus mainly on Derived equivalence and L-equivalence. It appears that these notions are particularly interesting in the context of Calabi–Yau type manifolds. The discussion will be performed through presenting several examples of such non-trivial equivalences. The talk will include results from joint work with G. Kapuska, R. Moschetti, M. Rampazzo.

FROM TOPOLOGY TO ALGEBRAIC GEOMETRY AND BACK AGAIN

Mateusz Michałek IM PAN / MPI MiS

I will present applications of secants in topology through k -regular embeddings. An embedding of a variety in an affine space is called k -regular if any k points are mapped to linearly independent points. Numeric conditions for the existence of such maps are an object of intensive studies of algebraic topologists dating back to the problem posed by Borsuk in the fifties. Current world record results were obtained by Pavle Blagojevic, Wolfgang Lueck and Guenter Ziegler. Our results relate k -regular maps to punctual versions of secant varieties. This allows us to prove existence of such maps in special cases. The main new ingredient is providing relations to the geometry of the punctual Hilbert scheme and its Gorenstein locus. The talk is based on two joint works: with Jarosław Buczyński, Tadeusz Januszkiewicz and Joachim Jelisiejew and with Christopher Miller

A GEOMETRIC CHARACTERIZATION OF COMPLETE FLAG MANIFOLDS

Gianluca Occhetta Università di Trento

Let G be a semisimple algebraic group and B a Borel subgroup; the complete flag manifold G/B is a Fano manifold whose elementary contractions are smooth \mathbb{P}^1 -fibrations. In particular the number of such fibrations is equal to the Picard number of the manifold. I will show how these manifolds can be characterized by this property, namely that a smooth complex projective manifold X of Picard number n which admits n contractions which are smooth \mathbb{P}^1 -fibrations is isomorphic to a complete flag manifold G/B . I will also discuss related results, concerning Bott-Samelson varieties and their deformations. The results presented in this talk have been obtained in collaboration with Roberto Muñoz, Luis E. Sola Conde, Kiwamu Watanabe and Jarosław A. Wisniewski.

DENSITY OF NOETHER-LEFSCHETZ LOCI OF POLARIZED IRREDUCIBLE HOLOMORPHIC SYMPLECTIC VARIETIES AND APPLICATIONS

Gianluca Pacienza IECL - Université de Lorraine & CNRS

In the talk we will explain how to derive from deep results due to Clozel-Ullmo the density of Noether-Lefschetz loci inside the moduli space of marked (polarized) irreducible holomorphic symplectic (IHS) varieties. In particular we obtain the density of Hilbert schemes of points on projective $K3$ surfaces and of projective generalized Kummer varieties in their moduli spaces. We will present applications to the existence of rational curves on projective deformations of such varieties, to the study of relevant cones of divisors, and a refinement of Hassett's result on cubic fourfolds whose Fano variety of lines is isomorphic to a Hilbert scheme of 2 points on a $K3$ surface. We also discuss Voisin's conjecture on the existence of coisotropic subvarieties on IHS varieties and relate it to a stronger statement on Noether-Lefschetz loci in their moduli spaces. This is based on a joint work with Giovanni Mongardi.

ON SOME RECENT DEVELOPMENTS ON THE BOUNDED NEGATIVITY CONJECTURE

Piotr Pokora Institute of Mathematics, Polish Academy of Sciences

The main aim of my talk is to present some recent developments on the bounded negativity conjecture. The conjecture states that for every smooth complex projective surface X there exists an integer $b(X)$ such that for all irreducible and reduced curves $C \subset X$ one has $C^2 \geq -b(X)$. There are some surfaces for which the BNC holds, for instance surfaces with the canonical divisor $-K_X$ being \mathbb{Q} -effective, but in general the question is open. It is notoriously difficult to predict where a certain blowing up of a given surface has bounded negativity. In my talk, I will focus on an interesting relation between the BNC and Zariski decompositions of integral pseudo-effective divisors, which is the content of a joint paper with Thomas Bauer and David Schmitz. Time permitting, I will present effective results providing bounds on the self-intersection numbers of reduced curves on blow-ups of the complex projective plane.

BRIDGELAND STABILITY FOR SEMIORTHOGONAL DECOMPOSITIONS, HYPERKAEHLER MANIFOLDS AND CUBIC FOURFOLDS

Paolo Stellari Università degli Studi di Milano

We illustrate a new method to induce stability conditions on semiorthogonal decompositions and apply it to the Kuznetsov component of the derived category of cubic fourfolds. We use this to generalize results of Addington-Thomas about cubic fourfolds and to study the rich hyperkaehler geometry associated to these hypersurfaces. This is the content of joint works with Arend Bayer, Howard Nuer, Marti Lahoz, Emanuele Macri and Alex Perry.

5

Algebraic Geometry and Interdisciplinary Applications

Organizers:

Weronika Buczyńska (MIMUW)

Alessandra Bernardi (Università Cattolica del Sacro Cuore)

NONSATURATED APOLARITY AND BORDER RANK OF MONOMIALS

Jarosław Buczyński IMPAN and MIMUW

Classical apolarity lemma is useful to calculate the Waring rank of polynomials and their decompositions and exploits saturated (in fact, even radical) homogeneous ideals. We introduce the limit version of apolarity which exploits also nonsaturated homogeneous ideals and calculates the border rank instead. We also present a couple of applications of this apolarity, including a calculation of the border rank of some monomials not covered by the earlier results of Oeding. The talk is based on a joint work with Weronika Buczyńska.

ON THE GEOMETRY OF SYMMETRIC TENSORS.

Luca Chiantini Università di Siena

Using tools of algebraic geometry for the analysis of finite subsets of projective spaces, it is possible to carry on a detailed analysis of symmetric tensors (i.e. polynomial forms). Many properties of tensors which are relevant for computational purposes follow from an application of advanced techniques for the study of Hilbert functions. I will survey on some recent results on the subject.

BOUNDS FOR WALDSCHMIDT CONSTANTS

Marcin Dumnicki Jagiellonian University

For an ideal I of points in P^n we define its symbolic power $I^{(m)}$ to be the collection of all forms vanishing to order $m - 1$ at all points in $V(I)$. Computing the least degree $\alpha = \alpha(I^{(m)})$ of a non-zero form in $I^{(m)}$ is highly non-trivial, but in many applications we are interested in an asymptotic invariant, $a(I) = \lim_{m \rightarrow \infty} \alpha(I^{(m)})/m$. This invariant appeared in papers of Skoda and Nagata and has some impact on problems away algebraic geometry. Since computation of $a(I)$ is beyond our knowledge, I will discuss some methods of looking for good bounds for $a(I)$.

VARIETIES OF APOLAR SUBSCHEMES OF $P^1 \times P^1$

Matteo Gallet Research Institute for Symbolic Computation (RISC) - Linz

I will report about a joint work with Kristian Ranestad and Nelly Villamizar, in which we investigate the possible ways in which a general bihomogeneous form of bidegree $(2,2)$ can be written as the sum of the squares of four bihomogeneous forms of degree $(1,1)$ that are the product of two linear forms. The key tool we use is the extension to our context of the classical notion of apolarity. In this way, the set of possible decompositions of the $(2,2)$ form is identified with the variety of apolar schemes (VPS) to the form. We show that the VPS is isomorphic to the blow-up along a rational normal quartic curve of a linear section of the Grassmannian of lines in \mathbb{P}^3 .

SOME BARRIERS FOR GEOMETRIC METHODS IN COMPLEXITY THEORY

Fulvio Gesmundo University of Copenhagen

The flagship conjecture in Algebraic Complexity Theory states that there is no polynomially bounded function $n(m)$ such that the permanent of an m -by- m matrix of variables can be expressed as the determinant of an n -by- n matrix whose entries are affine linear forms. Throughout the years, several approaches to this problem have been proposed, based on methods coming from algebraic geometry and representation theory. Recently, no-go results for some of these approaches have been proved. We discuss some of these results with particular focus on barriers for geometric complexity theory and for the method of shifted partials.

INJECTIVITY FOR PROJECTIONS OF SEGRE-VERONESE VARIETIES

Paul Goerlach Max Planck Institute MiS Leipzig

Segre-Veronese varieties are embeddings of products of several projective spaces by a complete linear system. They parameterize decomposable partially symmetric tensors. We investigate the existence of a linear projection mapping an n -dimensional Segre-Veronese variety injectively into n -dimensional projective space. Equivalently, we study whether an n -dimensional product of projective spaces be mapped injectively into \mathbb{P}^{2n} by polynomials with a prescribed multidegree. This is closely related to considering partially symmetric tensors modulo suitable linear subspaces and studying whether the corresponding notion of rank ≤ 2 is a closed condition.

MATRIX PRODUCT STATES AND ALGEBRAIC GEOMETRY

Mateusz Michałek IM PAN / MPI MiS

Matrix product states (also their uniform versions and Projected Entangled Pair States) describe the locus of special tensors in tensor spaces. They play a prominent role in quantum physics and numerical computing. In flavour, they are similar to secant varieties, however have been much less studied by algebraic geometers. I will present an algebraic approach to matrix product states, focusing on work in progress with Seynnaeve, Shitov and Verstraete. A few purely algebraic open problems will be presented. The talk is based on work in progress with Seynnaeve, Shitov and Verstraete.

ON THE ALGEBRAIC BOUNDARIES AMONG TYPICAL RANKS FOR REAL BINARY FORMS.

Giovanni Staglianò Università Politecnica delle Marche

We describe the algebraic boundaries of the regions of real binary forms with fixed typical rank and of degree at most eight, showing that they are dual varieties of suitable coincident root loci. This is a joint work with M. C. Brambilla. (Preprint: <https://arxiv.org/abs/1804.08309v1>)

ON ABSOLUTE LINEAR HARBOURNE CONSTANTS

Justyna Szpond Pedagogical University of Cracow

Arrangements of lines were introduced to algebraic geometry by Hirzebruch in his papers concerning the geography of surfaces (i.e. construction of surfaces X with prefixed invariants $c_1^2(X)$ and $c_2(X)$). Recently arrangements of lines appeared in the ideas revolving around the Bounded Negativity Conjecture (BNC for short). In Bounded Negativity and Arrangements of Lines, (International Mathematics Research Notices 2015, 9456 - 9471) the authors introduced and began to study linear Harbourne constants. Even though the Bounded Negativity fails in positive characteristic, it is clear from definition that for a fixed d , the linear Harbourne constant $H(d)$ is a finite number. It is natural to wonder about its value or at least some estimates. A configuration \mathcal{L} is a finite set of mutually distinct lines $\mathcal{L} = \{L_1, \dots, L_d\}$. Given a configuration \mathcal{L} , we define its singular set $\mathcal{P}(\mathcal{L}) = \{P_1, \dots, P_s\}$ as a set of points where two or more lines intersect. This is the same as the singular locus of the divisor $L_1 + \dots + L_d$. For a point $P \in \mathcal{P}(\mathcal{L})$, we denote by $m_{\mathcal{L}}(P)$ its multiplicity, i.e. the number of lines which pass through P . The linear Harbourne constant of a configuration of lines \mathcal{L} in the projective plane $\mathbb{P}^2(\mathbb{K})$ is the rational number

$$H(\mathbb{K}, \mathcal{L}) = \frac{d^2 - \sum_{k=1}^s m_{\mathcal{L}}(P_k)^2}{s}.$$

The linear Harbourne constant of d lines over \mathbb{K} is defined as the minimum

$$H(\mathbb{K}, d) := \min H(\mathbb{K}, \mathcal{L})$$

taken over all configurations \mathcal{L} of d lines. Finally the absolute linear Harbourne constant of d lines is the minimum

$$H(d) := \min_{\mathbb{K}} H(\mathbb{K}, d)$$

taken over all fields \mathbb{K} . In my talk I will present the exact values of absolute linear Harbourne constants of d lines for all $d \leq 31$ and $d = q^2 + q + 1$, where q is a power of prime number, and give a conjecture for other d . This is based on the joint work with Marcin Dumnicki and Daniel Harrer.

6

Arithmetic Algebraic Geometry

Organizers:

Bartosz Naskręcki (Adam Mickiewicz University)

Davide Lombardo (Università di Pisa)

Maciej Zdanowicz

INDEPENDENCE OF POINTS ON ELLIPTIC CURVES COMING FROM MODULAR CURVES

Gregorio Baldi UCL

Modular curves naturally parametrise elliptic curves, in particular it makes sense to consider isogeny classes inside such curves. Given a correspondence between a modular curve S and an elliptic curve E , we prove that the intersection of any finite rank subgroup of E with the set of points on E coming from an isogeny class on S is finite. This is indeed predicted by the Zilber-Pink conjecture. The proof relies on Serre's open image theorem and various equidistribution results.

GEOMETRIC LANG-VOJTA CONJECTURE FOR SURFACES DOMINATING \mathbb{G}_m^2

Laura Capuano University of Oxford

The celebrated Lang-Vojta Conjecture predicts degeneracy of S -integral points on varieties of log general type defined over number fields. It admits a geometric analogue over function fields, where stronger results have been obtained applying a method developed by Corvaja and Zannier. In this talk, we present a recent result for non-isotrivial surfaces over function fields dominating \mathbb{G}_m^2 . This extends Corvaja and Zannier's result in the isotrivial case and it is based on a refinement of gcd estimates for polynomials evaluated at S -units. This is a joint work with A. Turchet.

ARAKELOV GEOMETRY AND GEOMETRY OF NUMBERS IN INFINITE RANK

François Charles Université Paris-Sud

We will describe basic aspects of geometry of numbers for lattices of infinite rank. We will show how cohomological invariants for these appear naturally in Arakelov geometry, especially in the study of arithmetic positivity, where the usual basic vanishing results have an arithmetic counterpart. This is joint work with Jean-Benoît Bost.

A METHOD TO COMPUTE THE GEOMETRIC PICARD LATTICE OF A K3 SURFACE OF DEGREE 2

Dino Festi Johannes Gutenberg-Universität Mainz

K3 surfaces are surfaces of intermediate type, i.e., they are in between surfaces whose arithmetic and geometry is fairly well understood (rational and ruled surfaces) and surfaces that are still largely mysterious (surfaces of general type). The Picard lattice of a K3 surface contains much information about the surface, both from a geometric and an arithmetic point of view. For example, it tells about the existence of elliptic fibrations on the surface; if the surface is over a number field, then by looking at the Picard lattice one can have information about the Brauer group, and the potential density of rational points. Although much effort, there is not yet a practical algorithm that, given an explicit K3 surface, returns the Picard lattice of the K3 surface. In this talk we are going to give an overview on how practically compute the geometric Picard lattice of a K3 surface of degree two over a field of characteristic zero.

CLASS NUMBERS OF DIVISION FIELDS OF ABELIAN VARIETIES

Jędrzej Garnek Adam Mickiewicz University, Poznań

Let A be an abelian variety defined over a number field K . Fix a prime p and a natural number n and consider the field K_n , obtained by adjoining to K all the coordinates of the p^n -torsion points of A . We give an effective lower bound on the p -part of the class group of K_n for large n , depending mostly on dimension and Mordell-Weil rank of A . The proof uses Galois representations attached to abelian varieties, in particular the theory of Kummer extensions of an abelian variety of Ribet and Bashmakov.

INTEGRAL POINTS ON ELLIPTIC CURVES ASSOCIATED WITH GENERALIZED TWIN PRIMES.

Tomasz Jędrzejak Uniwersytet Szczeciński

We provide a description of the integral points on elliptic curves of the shape $y^2 = x(x - 2^m)(x + p)$, where p and $p + 2^m$ are primes. In particular, we show that for $m = 2$ such a curve has no nontorsion integral point, and for $m = 1$ it has at most one such a point (with $y > 0$). Our proofs rely upon the numerical computations and a variety of results on quartic and other Diophantine equations (e.g. simultaneous Pell type equations) combining with elementary analysis.

FROBENIUS STRUCTURES ON GKZ HYPERGEOMETRIC SYSTEM AND APPLICATIONS

Kiran Kedlaya University of California, San Diego

The notion of an A -hypergeometric system, or a GKZ (Gelfand-Kapranov-Zelevinsky) hypergeometric system, provides a common framework including various generalizations of the Gaussian hypergeometric equation. It was discovered by Dwork that there is a natural (and technically rather easy) way to construct Frobenius structures on such systems. We sketch the construction and indicate how it is useful for computing zeta functions of certain motives.

TORSION AND ISOGENIES OF BASE CHANGES OF ELLIPTIC CURVES

Filip Najman University of Zagreb

We study how elliptic curves defined over \mathbb{Q} gain torsion points and isogenies upon base change. In particular, we determine for which integers d can an elliptic curve defined over \mathbb{Q} gain a p -isogeny or a p -torsion point over a degree d number field. In most cases our results are unconditionally best possible and are always best possible if one assumes a weaker version of Serre's uniformity conjecture. This is partially joint work with Enrique Gonzalez-Jimenez.

REGULATORS OF ELLIPTIC CURVES.

Fabien Pazuki University of Copenhagen

In a recent collaboration with Pascal Autissier and Marc Hindry, we prove that up to isomorphisms, there are only finitely many elliptic curves defined over a fixed number field, with bounded rank $r \geq 4$ and bounded Mordell-Weil regulator.

P-ADIC DIFFERENTIAL EQUATIONS ON BERKOVICH CURVES

Andrea Pulita Université Grenoble Alpes

I will give an expository talk about some recent developments by F.Baldassarri, K.S.Kedlaya, J.Poineau and myself in the theory of differential equations (i.e. vector bundles with a connection) on smooth analytic curves defined over a p-adic field. The language that is particularly well adapted in this topic is that of Berkovich which I will introduce carefully in the talk. Time permitting I will mention some more recent developments.

7

Group Theory

Organizers:

Waldemar Holubowski (Silesian University of Technology)

Patrizia Longobardi (University of Salerno)

Mercede Maj (University of Salerno)

ON GROUPS WITH AUTOMORPHISMS WHOSE FIXED POINTS SATISFY ENGEL-LIKE CONDITIONS

Cristina Acciarri Department of Mathematics, University of Brasilia

Many well-known results in the literature show that, if A is a finite group acting on a finite group G , then the structure of the centralizer $C_G(A)$ of A has influence over the structure of G . The influence is especially strong if the action of A on G is coprime, that is if $(|A|, |G|) = 1$.

In this talk we will discuss results that show this phenomenon in the particular situation where, for any nontrivial element a in A , the centralizers $C_G(a)$ consist of elements satisfying Engel-like conditions.

A GENERALIZED TRUNCATED LOGARITHM

Marina Avitabile Università di Milano Bicocca

In the talk I will introduce a generalization $G^{(\alpha)}(X)$ of the truncated logarithm $\mathfrak{L}_1(X) = \sum_{k=1}^{p-1} X^k/k$ in prime characteristic p , which depends on a parameter α . The main motivation of this study is $G^{(\alpha)}(X)$ being an inverse, in an appropriate sense, of a parametrized generalization of the truncated exponential given by certain Laguerre polynomials. Such Laguerre polynomials play a role in a *grading switching* technique for non-associative algebras - whose aim is to produce a new grading of an algebra from a given one - because they satisfy a weak analogue of the functional equation $\exp(X)\exp(Y) = \exp(X+Y)$ of the exponential series. I will also present some functional equations satisfied by $G^{(\alpha)}(X)$ motivated by known functional equations for $\mathfrak{L}_1(X) = -G^{(0)}(X)$. All the results mentioned are a joint work with Sandro Mattarei.

THE MODULAR GROUP ALGEBRAS OF p -GROUPS OF MAXIMAL CLASS II

Czesław Bagiński Białystok University of Technology

Let p be a prime number, G and H finite p -groups and F the prime field of characteristic p . The long-standing conjecture, due to R. Brauer and called the Modular Isomorphism Problem (MIP), states that if $F[G]$ and $F[H]$ are isomorphic algebras then G and H are isomorphic groups. The problem is settled in the positive in many cases for instance for 2-groups of coclass 1 and 2 but two groups, but a general positive solution seems far away at present and for last several years there was no essential progress in this direction. It's not settled even for groups which, in some sense, are close to abelian p -groups, for instance for groups having an abelian maximal subgroup with certain additional strict assumptions concerning the internal structure. In this paper we study 3-groups of coclass 1 and settle MIP but two families of groups. Moreover, the conjecture that the ideals belonging to the lower central series of a group base are determined by the structure of the group algebra is established for 3-groups of coclass 1 of order exceeding 81, and it is proved that the nilpotency class is determined by the structure of the group algebra for p -groups of maximal class.

This work was done in collaboration with J. Kurdics (University of Nyiregyhaza, Hungary) during his visit at Białystok University of Technology and the results will appear in Communications in Algebra. The research was supported by the Białystok University of Technology grant S/WI/1/2014 and funded by resources for research by the Ministry of Science and Higher Education of Poland.

VANISHING CONJUGACY CLASS SIZES IN FINITE GROUPS

Mariagrazia Bianchi University of Milano Statale

Many authors have investigated the relationship between the structure of a finite group G and arithmetical data connected to G . The arithmetical data can take various forms, for example authors have considered the set of conjugacy class sizes, or the set of character degrees. The link between these different sets is also of interest as demonstrated by the following result of Casolo and Dolfi. Suppose p and q are distinct primes and pq divides the degree of some irreducible character, then pq also divides the size of some conjugacy class. To prove this result the authors considered groups for which p and q both divide a conjugacy class size but pq does not, they prove that such a group must be $\{p, q\}$ -solvable. Recently, instead of considering all conjugacy class sizes, authors have been considering a subset of conjugacy class sizes “filtered” by the irreducible characters, namely the vanishing conjugacy class sizes. An element g of G is called a vanishing element if there exists an irreducible character χ of G such that $\chi(g) = 0$. A conjugacy class of such an element is called a vanishing conjugacy class of G . Motivated by Casolo and Dolfi’s result we consider the case when pq does not divide a vanishing class size in G .

ON GROUPS CONTAINING SUBSETS WITH REMOVABLE POINTS

Agnieszka Bier Institute of Mathematics, Silesian University of Technology

In the talk we discuss the problem of existence of so called sets with removable points in groups, i.e. sets which are congruent (modulo the group action on the left) to their proper subsets. It was shown by Straus in [2] that group G has a set E congruent to any proper subset $E \setminus \{p\}$ ($p \in E$), if and only if G has a free subgroup of rank two. Later, it was conjectured by Tomkowicz and Wagon in [3] that the statement holds also for so groups containing subsets with only two removable points. In [1] it was shown that if a group contains a set with points a and b which are removable by elements g and h respectively, then g and h generate a free subsemigroup in the group. However, the general question remained open. We derive a number of properties of the considered sets in groups and give the affirmative answer to the question whether a group containing a subset with two removable points necessarily contains a nonabelian free subgroup. This is a joint work with Piotr Ślanina.

- [1] Mycielski J., Tomkowicz G., „Paradoxical sets and sets with two removable points”, J. Geom. (2018) 109:28
 - [2] Straus E., „On Sierpiński sets in groups”, Fundam. Math. 45 (1958), p. 332–333
 - [3] Tomkowicz G., Wagon S., „The Banach-Tarski Paradox”, Second edn. Cambridge University Press, Cambridge (2016)
-

INVOLUTIONS, AND MORE, IN THE MULTIPLE HOLOMORPH OF A FINITE p -GROUP OF NILPOTENCE CLASS TWO

Andrea Caranti Università degli Studi di Trento

Let G be a group, and $S(G)$ be the group of permutations on the set G . The (abstract) holomorph of G is the natural semidirect product $\text{Aut}(G)G$. Write $\text{Hol}(G)$ for the normalizer of the image in $S(G)$ of the right regular representation of G ,

$$\text{Hol}(G) = N_{S(G)}(\rho(G)) = \text{Aut}(G)\rho(G) \cong \text{Aut}(G)G,$$

If N is any regular subgroup of $S(G)$, then $N_{S(G)}(N)$ is isomorphic to the holomorph of N .

G.A. Miller has shown that the group

$$T(G) = N_{S(G)}(\text{Hol}(G))/\text{Hol}(G)$$

acts regularly on the set of the regular subgroups N of $S(G)$ which are isomorphic to G , and have the same holomorph as G , in the sense that $N_{S(G)}(N) = \text{Hol}(G)$.

If G is non-abelian, inversion on G yields an involution in $T(G)$. Other non-abelian regular subgroups N of $S(G)$ having the same holomorph as G yield (other) involutions in $T(G)$. In the cases studied in the literature, $T(G)$ turns out to be a finite 2-group, which is often elementary abelian.

In this talk we will discuss examples of finite p -groups G of nilpotence class two for which $T(G)$ is non-abelian, and not a 2-group. Linear techniques will play an important role.

FROM SURJUNCTIVE GROUPS TO SURJUNCTIVE DYNAMICAL SYSTEMS

Tullio Ceccherini-Silberstein Università del Sannio

Surjunctive groups were introduced by W.Gottschalk in a symbolic dynamical setting: a group G is termed SURJUNCTIVE if, given any finite alphabet set A , one has that every INJECTIVE cellular automaton (=continuous and G -equivariant map) $\tau: A^G \rightarrow A^G$ is SURJECTIVE. Lawton showed that all residually finite groups are surjunctive. It is a consequence of the Garden of Eden theorem (TCS-A.Machi-F.Scarabotti) that all amenable groups are surjunctive. These results were generalized by M. Gromov and B. Weiss who showed that all sofic groups are surjunctive. We shall then discuss the notion of surjunctivity in the framework of general dynamical systems focusing on the so-called algebraic dynamical systems (in the sense of K.Schmidt) and present some recent results obtained in collaboration with S. Bhattacharya and M. Coornaert.

ALGEBRA OF SETS, PERMUTATION GROUPS AND INVARIANT FACTORS

Francesca Dalla Volta Università Milano-Bicocca

Let Ω be a finite set of size n and consider the incidence structure \mathcal{I} given by natural inclusion in the poset $(\mathcal{P}(\Omega) \subseteq)$. If G is a permutation group acting on Ω , G acts naturally on $\mathcal{P}(\Omega)$; denoted by L_k^n the set of k -subsets of Ω , this action extends in natural way to the vector space \mathbf{QL}_k^n :

$$g: r = \sum_{r_x \in \mathbf{Q}, x \in L_k^n} r_x x \mapsto r^g = \sum_{r_x \in \mathbf{Q}, x \in L_k^n} r_x x^g \text{ for } g \in G.$$

The orbit modules of G are the centralizers

$$T_k^G := \{ r \in \mathbf{QL}_k^n : r^g = r \text{ for all } g \in G \}$$

and T_k^G is spanned by the G -orbits on k -sets. Some aspects related to tactical decomposition arising from \mathcal{I} , and to the structure of T_k^G are considered

ON GROUPS WITH MANY SUBGROUPS WHICH ARE UNIFORMLY INERT

Ulderico Dardano Università di Napoli "Federico II"

A subgroup H of a group is called *uniformly inert* if there is n such that $|H : H^g \cap H| \leq n$ for all $g \in G$. According to a result by G. M. Bergman and H. W. Lenstra Jr., this is equivalent to the fact that there exists a normal subgroup N of G such that $|HN : (H \cap N)| < \infty$, that is H is commensurable with a normal subgroup N , say H is cn -subgroup of G .

A group in which all subgroups are cn is called a CN -group. Recently, it has been shown by C. Casolo, U. Dardano and S. Rinauro that a CN -group G such that every periodic image of G is locally finite is finite-abelian-by-finite, that is G has a subgroup G_0 with finite index in G and such that G'_0 is finite. Moreover, there are soluble CN -groups which are neither abelian-by-finite nor finite-by-abelian.

I will discuss the following results from a submitted joint work with S. Rinauro:

Theorem *Let G be a group such that every periodic image of G is locally finite. If G has finitely many conjugacy classes of subgroups which are not cn , then all subgroups of G are cn .*

Theorem *Let G be a generalized radical group with infinite Prüfer rank. If each subgroup of infinite rank is cn , then all subgroups are cn .*

Theorem *Let G be a group such that every periodic image of G is locally finite. If G satisfies the minimal condition on subgroups which are not cn , then either G satisfies the minimal condition on all subgroups or all subgroups of G are cn .*

Moreover, generalized soluble groups with the weak minimal condition on subgroups which are not cn will be featured.

COMMUTATORS IN FINITE p -GROUPS

Iker De las Heras University of the Basque Country

In a group G , the set $K(G)$ of commutators need not be a subgroup. In other words, the derived subgroup $G' = \langle K(G) \rangle$ may be strictly larger than $K(G)$. There exist, however, several families of groups in which the equality $G' = K(G)$ is satisfied. In this talk we will discuss this equality and provide different conditions for it to hold.

In particular, for finite nilpotent groups, the study of this property is obviously reduced to finite p -groups, where p is a prime. In this case, Guralnick showed that $G' = K(G)$ whenever G' is abelian and can be generated by 2 elements. We will see that the requirement that G' should be abelian is not necessary. Actually, we will show that all elements of G' arise as commutators from a single suitable element.

ON FUCHS' PROBLEM ABOUT THE GROUP OF UNITS OF A RING

Ilaria Del Corso Università di Pisa

On Fuchs' problem about the group of units of a ring June 28, 2018 Ilaria Del Corso Università di PISA. Fuchs in [Abelian groups, 3rd edn (Pergamon, Oxford, 1960); Problem 72] posed the following problem: Characterize the groups which are the groups of all units in a commutative and associative ring with identity. A partial approach to this problem was suggested by Ditor in 1971, with the following less general question: Which whole numbers can be the number of units of a ring? In the following years, these questions inspired the work of many authors,

and some partial answer to them and to related problems has been given. Recently, in two joint papers with R. Dvornicich, we studied the original Fuchs' question for both positive characteristic and zero characteristic rings, and we "almost" answered it. In fact, we have been able to obtain a pretty good description of the possible groups of units equipped with families of examples of both realizable and non-realizable groups. We also examined the interesting case of torsion-free rings and we completely classified the possible finite abelian groups of units which arise in this case. As a consequence of our results we completely answered Ditor's question on the possible cardinalities of the group of units of a ring. The study of the case of finitely generated abelian groups is a work in progress and I will also present some results on the group of units of torsion-free rings in this case. 11 The activities mentioned herein are performed in the framework of the PRIN 2015, title "Number Theory"

A FUBINI THEOREM FOR AMENABLE GROUPS

Antongiulio Fornasiero University of Florence

Let G be an amenable group. A subadditive monotone left-invariant function on G is a function f from the family of finite subsets of G to the nonnegative real numbers such that, for every A, B finite subsets of G , and every $g \in G$, $f(gA) = f(A)$ and $f(A) \leq f(A \cup B) \leq f(A) + f(B)$. Ornstein-Weiss Lemma gives a way to define an average value $A(f)$ for f (using Folner sequences): this functional A behaves like an integral of f . We show an analogue of Fubini's theorem: if H is a normal subgroup of G , there is a formula connecting the average on G with the averages on H and G/H . Similar results hold also for cancellative amenable monoids.

A REDUCTION THEOREM FOR NONSOLVABLE FINITE GROUPS

Francesco Fumagalli Università degli Studi di Firenze

Every finite group G has a normal series each of whose factors is either a solvable group or a direct product of nonabelian simple groups. The minimum number of nonsolvable factors attained on all possible such series is called the nonsolvable length of the group and denoted by $\lambda(G)$. For every integer n , we define a particular class of groups of nonsolvable length n , called n -rarefied, and we show that every finite group of nonsolvable length n contains an n -rarefied subgroup. As applications of this result, we improve the known upper bounds on $\lambda(G)$ and determine the maximum possible nonsolvable length for permutation groups and linear groups of fixed degree resp. dimension.

THE BREADTH-DEGREE TYPE OF A FINITE p -GROUP

Norberto Gavioli University of L'Aquila

The breadth b and the maximum degree p^d of the irreducible characters of a finite p -group G were already known to Philip Hall to be invariant under isoclinism. We define the pair (b, d) to be the breadth-degree type of G . Given b there exist stem p -groups of breadth b and arbitrarily large order. Similarly given d there exist stem p -groups with irreducible character maximum degree p^d and arbitrarily large order. We prove that the order of a stem p -group in the isoclinism class of a finite p -group G can be bounded by a function depending only on p and on the breadth-degree type (b, d) .

MILNOR-WOLF THEOREM FOR THE GROWTH OF GROUP ENDOMORPHISMS

Anna Giordano Bruno Università di Udine

We are interested in the growth of group endomorphisms and in an analogue of the Milnor-Wolf Theorem for the growth of finitely generated soluble groups. Indeed, we prove that if G is a locally virtually soluble group and if $\phi : G \rightarrow G$ is an endomorphism, then ϕ has either polynomial or exponential growth.

ON THE REGULAR ACTION OF THE UNIT GROUP ON RINGS

Małgorzata Hryniewicka Institute of Mathematics, University of Białystok

Let R be an associative unital ring with the unit group $U(R)$. Let R^+ be the additive group of the ring R . In the talk we will consider a natural group action of $U(R) \times U(R)$ on R^+ defined by $(a, b) \rightarrow x = axb^{-1}$ where $a, b \in U(R)$ and $x \in R$. We will call this action *the regular action* of the unit group $U(R)$ on the ring R . We will concentrate on rings with finite number of orbits under the regular action of $U(R)$ on R .

A NILPOTENCY CRITERION FOR SOME VERBAL SUBGROUPS

Carmine Monetta University of Salerno

A group-word is a non trivial element $w = w(x_1, \dots, x_s)$ of the free group F on free generators x_1, \dots, x_s . Given a group G , a group-word $w = w(x_1, \dots, x_s)$ can be seen as a function defined in the cartesian product G^s taking values in G , and the subgroup of G generated by all w -values is denoted by $w(G)$ and it is called the verbal subgroup of G corresponding to w . It is quite natural to ask if one could grasp any information about $w(G)$ imposing some condition on the w -values. Indeed, we are concerned with the following property when G is a finite group: “if a and b are w -values of coprime orders $|a|$ and $|b|$, respectively, then the order of ab is the product of $|a|$ and $|b|$ ”. The aim of this talk is to discuss some cases in which this property characterizes the nilpotency of the verbal subgroup $w(G)$.

NORMALITY CONDITIONS IN GROUPS OF LARGE CARDINALITY

Carmela Musella Dipartimento di Matematica e applicazioni “R. Caccioppoli”,
Università di Napoli “Federico II”

In the frame of a project on “large” groups (groups which are far from finiteness in some sense), some recent results concerning uncountable groups of regular cardinality k in which all subgroups of cardinality k satisfy certain embedding properties are presented. In particular, uncountable groups with conditions of normality, subnormality, permutability, Neumann’s properties (i.e. properties close to normality up to a finite section) or in which the normality is a transitive relation are considered.

ENGEL ELEMENTS IN SOME FRACTAL GROUPS

Marialaura Noce University of Salerno - University of the Basque Country

An element g of a given group G is a right Engel element if for every $x \in G$ there exists an integer $n = n(g, x) \geq 1$ such that $[g, x, \dots, x] = 1$. Moreover, an element of G is said to be a bounded Engel element if the choice of n is independent of x . Similarly, g is a left (or left bounded) Engel element if the variable x appears on the left. These sets are denoted by $R(G)$ and $L(G)$ ($\bar{L}(G)$), respectively. It is a long-standing problem, raised by Plotkin, whether $\bar{L}(G)$, $L(G)$ and $R(G)$ are subgroups or not. In this talk, we first introduce the notion of fractal and (weakly) branch group, and then we show that: 1) $\bar{L}(G) = 1$ and $R(G) = 1$ for certain classes of regular branch and weakly branch groups, respectively. Both include the first Grigorchuk group, and some GGS groups; 2) $L(G) = 1$ for some fractal nonabelian groups with torsion-free abelianization. This result applies, for example, to the Basilica group, the Brunner-Sidki-Vieira group, and also to the GGS-group with constant defining vector. This is a joint work with G. Fernández-Alcober and A. Garreta.

ON THE CHARACTER DEGREE GRAPH OF FINITE GROUPS

Emanuele Pacifici Università di Milano

Character Theory is one of the fundamental tools in the theory of finite groups, and, given a finite group G , the study of the set $\text{cd}(G) = \{\chi(1) \mid \chi \in \text{Irr}(G)\}$, of all degrees of the irreducible complex characters of G , is a particularly intriguing aspect of this theory. One of the methods that have been devised to approach such degree-set is to consider the prime graph $\Delta(G)$ attached to it. The character degree graph $\Delta(G)$ is thus defined as the (simple undirected) graph whose vertex set is the set of all the prime numbers that divide some $\chi(1) \in \text{cd}(G)$, while a pair $\{p, q\}$ of distinct vertices p and q belongs to the edge set if and only if pq divides an element in $\text{cd}(G)$. There is a well-developed literature on character degree graphs, and a large part of it is focused on studying to which extent specific properties of a group are reflected by graph theoretical features of its graph, or aimed at describing in detail the degree graph of interesting classes of groups. In this talk, we will discuss some recent developments in this research area (joint works with Z. Akhlaghi, C. Casolo, S. Dolfi and L. Sanus).

A NEW REPLACEMENT THEOREM

Gemma Parmeggiani Dipartimento di Matematica "Tullio Levi-Civita" Università di Padova

One of the important steps in the local analysis of finite groups is to detect, under certain conditions, non-trivial quadratic action on an abelian normal subgroup. This is usually done by applying Replacement Theorems. That is, results that replace a given group by a subgroup that acts quadratically. Thompson gave the first example for such a theorem using what was then called the Thompson subgroup. Later his idea was generalized and formulated in terms of abelian offenders and modules. In this talk I will present a Replacement Theorem which neither needs that the offender nor the group it acts on is abelian. This is a joint work with Ulrich Meierfrankenfeld and Bernd Stellmacher.

SOME NEW RESULTS ON THE $(2, 3)$ -GENERATION OF FINITE CLASSICAL GROUPS

Marco Antonio Pellegrini Università Cattolica del Sacro Cuore

In this talk I will describe some recent results concerning the $(2, 3)$ -generation of finite simple groups. Recall that a finite group is said to be $(2, 3)$ -generated if it can be generated by a pair x, y of elements having order 2 and 3, respectively. Although it is known that, apart from the infinite families of symplectic groups $PSp(4, 2^a)$ and $PSp(4, 3^a)$, there are only finitely many finite simple classical groups which are not $(2, 3)$ -generated, the problem of determining such exceptions is still open. In collaboration with M.C. Tamburini Bellani, we considered the linear and unitary groups: we proved that the exact list of exceptions is, for linear groups,

$$PSL(2, 9), PSL(3, 4), PSL(4, 2)$$

and, for unitary groups,

$$PSU(3, 3), PSU(3, 5), PSU(4, 2), PSU(4, 3), PSU(5, 2).$$

PYTHAGOREAN SCHOOL, FIBONACCI NUMBERS, INDUCTION PRINCIPLE AND MINIMUM PRINCIPLE

Giuseppe Pirillo INPS

First of all, we recall the results of some our recent articles: *Some recent results of Fibonacci numbers, Fibonacci words and Sturmian words* (Southeast Asian Bull. of Math., 2017), *La scuola pitagorica ed i numeri di Fibonacci* (Archimede, 2017), *L'origine pitagorica dei numeri di Fibonacci* (Periodico di Matematiche, 2017), *Figure geometriche su un portale del Duomo di Prato* (Prato Storia e Arte, 2017), *A characterization of Fibonacci numbers* (ArXiv, 2017). (We also recall the results of our paper *Numeri irrazionali e segmenti incommensurabili* (Nuova Secondaria, 2005) where there are some incommensurability results for *octagons, decagons* and *regular dodecagons*.)

We present our audacious thesis that the first mathematicians that discovered the “so called” Fibonacci numbers were some members of the Pythagorean School, well documented and active in Crotona in VI, V and IV century B.C. (the Indian discovery is later, VI century A.D., and the *Liber Abbaci* of Fibonacci dates back to XIII A.D.). Here we recall the following definition and proposition.

Definition. Let β a positive integer. When there exists a positive integer α such that, for some non-negative integer γ , the equality

$$\beta(\beta + \alpha) - \alpha^2 = (-1)^\gamma$$

holds, then we say that β is a Hippiasus number and that α is a Hippiasus successor of β .

Proposition. A positive integer is a Hippiasus number if, and only if, it is a Fibonacci number.

In our paper on ArXiv, where *Proposition* is proved using *induction principle* and also *minimum principle*, we “demonstrated” that *Fibonacci numbers* and *Cassini identity* are of Pythagorean origin and were “almost simultaneously” discovered with the incommensurability of side and diagonal of the regular pentagon.

We want to emphasize that Fibonacci numbers are of some importance in every mathematical discipline, also in group theory (see, for example, *On the structure of subsets of an orderable group with some small doubling properties* of G.A. Freiman, M. Herzog, P. Longobardi, M. Maj, A. Plagne, D.J.S. Robinson, Y.V. Stanchescu, where not only *Fibonacci numbers* but also *Cassini identity* are used).

In this context, a question has strongly come to our attention: are *induction principle* and *minimum principle* of Pythagorean origin? In the absence of historical documents (so rare for the Pythagorean period!) we try to give an answer based on solid mathematical arguments.

A GENERALIZATION OF THE BURNSIDE BASIS THEOREM

Agnieszka Stocka University of Białystok

Let G be a finite group and $\Phi(G)$ denotes the Frattini subgroup of G . A subset X of G is called g -independent if there is no proper subset Y of X such that $\langle Y, \Phi(G) \rangle = \langle X, \Phi(G) \rangle$. The group G has the embedding property if every g -independent subset of G can be embedded in a minimal generating set of G and G has property \mathcal{B} if all minimal generating sets of G have the same cardinality. If X is a set of prime power order elements, then we say that G has the pp-embedding property, respectively property \mathcal{B}_{pp} . From the Burnside basis theorem we now that all finite p -groups have such properties. During this talk we present some results about groups with the pp-embedding property and property \mathcal{B}_{pp} .

ON VARIETIES OF GROUPS IN WHICH EVERY FINITE GROUP IS NILPOTENT

Witold Tomaszewski Institute of Mathematics, Silesian University of Technology

The talk will be about varieties of groups in which every finite group is nilpotent. We will present examples of such varieties and properties of them. In the second part of the talk we will describe conditions for a binary word $w(x, y)$ to have the property that every finite group satisfying the law $w(x, y) \equiv 1$ is nilpotent.

ON CONCISENESS OF SOME COMMUTATOR WORDS

Maria Tota Università di Salerno

If $w = w(x_1, \dots, x_n)$ is a group-word in variables x_1, \dots, x_n , we denote by $w(G)$ the verbal subgroup of the group G generated by the set G_w of all values of w in G . A word w is called *concise* if for every group G whenever w is finite-valued in G the subgroup $w(G)$ is finite. Philip Hall had conjectured that every word is concise, and he proved this for every non-commutator word (i.e., a word such that the sum of the exponents of some variable involved in it is non-zero), and for lower central words. In [3] Turner-Smith showed that derived words are also concise, and J.C.R. Wilson [4] subsequently extended this result to all multilinear commutator words (which are words obtained by nesting commutators, but using always different variables). On the other hand, Hall's conjecture was eventually refuted in 1989 by Ivanov [2]. We will see that a commutator of two non-commutator words is a concise word [1].

- [1] C. Delizia, P. Shumyatsky, A. Tortora and M. Tota, *On conciseness of some commutator words*, Arch. Math., DOI: 10.1007/s00013-018-1215-8, to appear.
- [2] S. V. Ivanov, *P. Hall's conjecture on the finiteness of verbal subgroups*, Soviet Math. (Iz. VUZ) **33** (1989), no. 6, 59–70.
- [3] R. F. Turner-Smith, *Marginal subgroup properties for outer commutator words*, Proc. London Math. Soc. (3) **14** (1964), 321–341.
- [4] J.C.R. Wilson, *On outer-commutator words*, Canad. J. Math. **26** (1974), 608–620.



Geometric Function and Mapping Theory

Organizers:

Tomasz Adamowicz (Polish Academy of Sciences)

Valentino Magnani (University of Pisa)

Paweł Goldstein (University of Warsaw)

SOLUTIONS TO HESSIAN TYPE EQUATIONS

Śławomir Dinew Jagiellonian University

The class of Hessian type elliptic equations covers a wide range of model nonlinearities that are relevant to geometric analysis. During the talk I will introduce the main types of Hessian equations together with some basic regularity theory. Some geometric applications will be sketched.

FRACTIONAL POWERS OF HYPOELLIPTIC OPERATORS AND RELATIVE EXTENSION PROBLEM

Nicola Garofalo University of Padova

In this talk I define the fractional powers of sub-Laplaceans and prove that they are the Dirichlet-to-Neumann map of an extension problem inspired to the famous 2007 work of Caffarelli and Silvestre for the standard Laplacean. A key tool is an extension problem for the relevant fractional heat equation for which I compute the Poisson kernel. I then use the latter to: 1) find the Poisson kernel for the time-independent case; and 2) solve the extension problem.

A UNIQUENESS PROPERTY FOR ANALYTIC FUNCTIONS ON MMS

Grzegorz Łysik Jan Kochanowski University in Kielce

After recalling the Pizzetti formulas for real analytic functions we give a characterization of such functions only in terms of integral means over balls. The characterization justifies introduction of a definition of analytic functions on metric measure spaces. In the sequel shall prove that analytic functions on MMS possess the following uniqueness property: If a function vanishes on a nonempty open subset of a connected set Ω , then it vanishes on Ω .

CHARACTERIZATION OF MEAN VALUE HARMONIC FUNCTIONS

Antoni Kijowski Institute of Mathematics of the Polish Academy of Sciences

We study functions possessing the mean value property in metric measure spaces, [1, 4]. We treat them as a natural counterpart to harmonic functions in this setting and therefore examine their properties such as the maximum principle, the Harnack inequality, Lipschitz and Sobolev regularity. Finally, we state necessary and sufficient conditions for a function to attain the mean value property expressed via system of elliptic PDEs, [2, 3, 5, 6].

- [1] T. Adamowicz, M. Gaczkowski, P. Górka, *Harmonic functions on metric measure spaces*. arXiv:1601.03919 [math.MG].
- [2] A.K. Bose, *Functions satisfying a weighted average property*. Trans. Amer. Math. Soc., 118, (1965), 472-487.

- [3] A. Friedman, W. Littman, *Functions satisfying the mean value property*. Trans. Amer. Math. Soc., 102, (1962), 167-180.
 - [4] M. Gaczkowski, P. Głorka, *Harmonic Functions on Metric Measure Spaces: Convergence and Compactness*. Potential Anal. 31, 203-214, (2009).
 - [5] A. Kijowski, *Characterization of mean value harmonic functions on norm induced metric measure spaces with weighted Lebesgue measure*. arXiv:1804.10005 [math.AP].
 - [6] L. Zalcman, *Mean values and differential equations*. Israel J. Math., 14, (1973), 339-352.
-
-

EPSILON-REGULARITY FOR P-HARMONIC MAPS AT A FREE BOUNDARY ON A SPHERE

Katarzyna Mazowiecka Université catholique de Louvain

We prove an epsilon-regularity theorem for vector-valued p -harmonic maps, which are critical with respect to a partially free boundary condition, namely that they map the boundary into a round sphere. As a consequence we obtain partial regularity of stationary p -harmonic maps up to the boundary away from a set of $(n - p)$ -dimensional Hausdorff measure. Joint work with R. Rodiac and A. Schikorra.

PROPERTIES OF CONVEX SETS IN WIENER SPACES

Michele Miranda University of Ferrara

We show some recent results on convex sets in Wiener spaces. We characterize the essential and reduced boundary of open convex sets and investigate integration by parts formulae. Of particular interest is the investigation of trace theorems for functions of bounded variation on boundaries of subsets in Wiener spaces.

SINGULARITIES OF MINIMIZING HARMONIC MAPS

Michał Miśkiewicz University of Warsaw

Minimizing harmonic maps between manifolds are known to be smooth outside the so-called singular set. In general this is a rectifiable set of codimension 3, i.e., it can be covered by countably many Lipschitz pieces, but still may have many small gaps. In one special case of maps from a 4-dimensional domain into the 2-dimensional sphere Hardt and Lin proved that the singular set consists of topological curves. I will show a generalization to higher dimensional domains and discuss the topological obstruction responsible for preventing gaps in the singular set.

A C^m WHITNEY EXTENSION THEOREM FOR HORIZONTAL CURVES IN THE HEISENBERG GROUP

Andrea Pinamonti University of Trento

We characterize when a mapping from a compact subset of \mathbb{R} into the Heisenberg group can be extended to a C^m horizontal curve in \mathbb{H}^n . The talk is based on a joint work with Gareth Speight and Scott Zimmermann.

F. SERRA CASSANO-THE BERNSTEIN PROBLEM FOR AREA-MINIMIZING INTRINSIC GRAPHS IN THE SUB-RIEMANNIAN HEISENBERG GROUP

Francesco Serra Cassano University of Trento

We will deal with the so-called *Bernstein problem* for area-minimizing intrinsic graphs in the first Heisenberg group $\mathbb{H}^1 \equiv (\mathbb{R}^3, \cdot)$, understood as a Carnot group and equipped by the sub-Riemannian metric structure. More precisely, the problem reads as follows: if the intrinsic graph $\Gamma_f \subset \mathbb{H}^1$ of a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, that is

$$\Gamma_f := \{(0, y, t) \cdot (f(y, t), 0, 0) : (y, t) \in \mathbb{R}^2\},$$

is (locally) area minimizing in \mathbb{H}^1 , then must Γ_f be a plane, in the geometry of \mathbb{H}^1 ? We will positively and negatively answer to this problem, taking the regularity of f into account.

RECTIFIABILITY ISSUES IN SUB-RIEMANNIAN GEOMETRY

Davide Vittone Dipartimento di Matematica, Università di Padova

In this talk we will discuss two problems concerning “rectifiability” results in sub-Riemannian geometry and particularly in the model setting of Carnot groups. The first problem regards the rectifiability of boundaries of sets with finite perimeter in Carnot groups, while the second one concerns Rademacher-type results (existence of a tangent plane out of a negligible set) for (intrinsic) graphs with (intrinsic) Lipschitz regularity. We will introduce both problems and discuss the state-of-the-art. Eventually, we will present some recent results about the rectifiability of sets with finite perimeter in a certain class of Carnot groups (including the simplest open case, i.e., the Engel group) and about a Rademacher theorem for intrinsic Lipschitz graphs of any dimension in Heisenberg groups.

PUNCTURE REPAIR IN METRIC MEASURE SPACES

Ben Warhurst University of Warsaw

Motivated by recent interest concerning “puncture repair” in the conformal geometry of compact Riemannian manifolds, a brief exposition on generalisation to the setting of quasiconformal mappings on certain metric measure spaces will be discussed.

LOWER DIMENSIONAL RECTIFIABLE STRUCTURES IN \mathbb{R}^N AND ELLIPTIC EQUATIONS WITH NEUMANN BOUNDARY DATA

Anna Zatorska-Goldstein University of Warsaw

I will discuss an elementary linear elliptic equation on a lower dimensional rectifiable structure in \mathbb{R}^N with Neumann boundary data. The set may be described by means of a finite Borel measure μ supported on it. This allows us to reformulate the equation and the boundary condition and to establish existence and uniqueness of a weak solution via a variational method. The setting requires an appropriate definition of a Sobolev-type space dependent on the measure μ and an appropriate Poincaré-type inequality. I will present examples of structures that are not manifolds and which do not support a global Poincaré inequality, yet which are admissible for our setting.

9

Complex Analysis and Geometry

Organizers:

Sławomir Dinew (Jagiellonian University)

Adriano Tomassini (Università di Parma)

Giorgio Patrizio (Università degli Studi di Firenze)

Sławomir Kołodziej (Uniwersytet Jagielloński)

DETERMINANTAL POINT PROCESSES AND THE RECONSTRUCTION OF HOLOMORPHIC FUNCTIONS

Alexander Bufetov CNRS Marseille

Consider a Gaussian Analytic Function on the disk, that is, a random series whose coefficients are independent complex Gaussians. In joint work with Yanqi Qiu and Alexander Shamov, we show that the zero set of a Gaussian Analytic Function is a uniqueness set for the Bergman space on the disk: in other words, almost surely, there does not exist a nonzero square-integrable holomorphic function having these zeros. The key role in our argument is played by the determinantal structure of the zeros, and we prove, in general, that the family of reproducing kernels along a realization of a determinantal point process generates the whole ambient Hilbert space, thus settling a conjecture of Lyons and Peres. In a sequel paper, joint with Yanqi Qiu, we study how to recover a holomorphic function from its values on our set. The talk is based on the preprints arXiv:1806.02306 and arXiv:1612.06751

LOG-CONCAVITY OF THE VOLUME OF POSITIVE CURRENTS

Eleonora Di Nezza I.H.E.S.

In this talk we present a proof of the log-concavity property of total masses of positive currents on a given compact Kähler manifold, that was conjectured by Boucksom, Eyssidieux, Guedj and Zeriahi. The proof relies on the resolution of complex Monge-Ampère equations with prescribed singularities. As corollary we give an alternative proof of the Brunn-Minkowsky inequality for convex bodies. This is based on a joint work with Tamás Darvas and Chinh Lu.

A NEW APPROACH TO QUATERNIONIC MANIFOLDS

Graziano Gentili Università di Firenze

A recent definition of slice regular function of several quaternionic variables suggests a new notion of quaternionic manifold. This talk presents the definition of quaternionic regular manifold, as a space locally modeled on H^n , in a slice regular sense. Some significant classes of examples, including manifolds which carry a quaternionic affine structure, are presented.

THE ANALYTIC STRUCTURE OF THE SINGULAR SET OF A SLICE-REGULAR FUNCTION

Alessandro Perotti University of Trento, Italy

In real dimensions higher than two a recent function theory, the one of slice-regularity, has been able to reproduce many features of the theory of holomorphic functions of one complex variable. Here we consider the four dimensional case represented by slice-regular quaternionic functions. The Jacobian determinant of a quaternionic slice-regular function f is always non-negative. From this property and from a detailed study of the fibers of f , we get the analytic structure of the singular set of f . We also show that the sign property of the Jacobian implies some of the fundamental results of the theory of slice-regular functions. Some of them are partially known, others are new: quasi-open mapping theorem, maximum modulus principle, boundary injectivity theorem. (From joint work with Riccardo Ghiloni, Univ. of Trento)

10

Optimization, Microstructures, and Applications to Mechanics

Organizers:

Agnieszka Kałamajska (University of Warsaw)

Elvira Zappale (Università degli Studi di Salerno)

SELF-SIMILAR SOLUTION FOR THE FRACTIONAL LAPLACIAN IN CONES

Krzysztof Bogdan Wrocław University of Science and Technology

This is joint a work of Krzysztof Bogdan with Zbigniew Palmowski (Wrocław University of Science and Technology) and Longmin Wang (Nankai University). We fix $d \geq 1$ and $0 < \alpha < 2$. Let

$$v(y) = c|y|^{-d-\alpha}, \quad y \in \mathbf{R}^d,$$

where c is so chosen that

$$\int_{\mathbf{R}^d} [1 - \cos(\xi \cdot y)] v(y) dy = |\xi|^\alpha, \quad \xi \in \mathbf{R}^d.$$

Define

$$p_t(x) = (2\pi)^{-d} \int_{\mathbf{R}^d} e^{-ix \cdot \xi} e^{-t|\xi|^\alpha} d\xi, \quad x \in \mathbf{R}^d, t > 0.$$

This is a probability density for each t , due to the Lévy-Khintchine formula. We have scaling (aka self-similarity):

$$p_t(x) = t^{-d/\alpha} p_1(t^{-1/\alpha}x).$$

We define the semigroup of operators $P_t f = f * p_t$, and, e.g., for $f \in C_0^2(\mathbf{R}^d)$,

$$\lim_{t \rightarrow 0^+} \frac{P_t f(x) - f(x)}{t} = \lim_{\varepsilon \rightarrow 0^+} \int_{|y| > \varepsilon} [f(x+y) - f(x)] v(dy) =: \Delta^{\alpha/2} f(x),$$

Also, $p_t(y-x)$ is the heat kernel of the fractional Laplacian $\Delta^{\alpha/2}$ on \mathbf{R}^d .

Let $\emptyset \neq \Gamma \subset \mathbf{R}^d$ be an arbitrary open Lipschitz cone with vertex at $0 \in \mathbf{R}^d$. We consider the Dirichlet heat kernel $p_t^\Gamma(x, y)$ of $\Delta^{\alpha/2}$ for Γ . As the main result of [1] we give a nontrivial self-similar function

$$n_t(y) = t^{-(d+\beta)/\alpha} n_1(t^{-1/\alpha}y) \geq 0, \quad t > 0, \quad y \in \mathbf{R}^d, \quad (10.1)$$

such that for all $s, t > 0$,

$$n_{t+s}(y) = \int_{\Gamma} n_t(z) p_s^\Gamma(z, y) dz.$$

The latter means that n solves $\partial_t n = \Delta^{\alpha/2} n$ in $\Gamma \times \{t > 0\}$ with Dirichlet (zero) condition on Γ^c . Here β is a number between 0 and α , depending on the cone.

The existence of n is a consequence of the boundary Harnack principle for nonnegative harmonic functions of $\Delta^{\alpha/2}$ and sharp heat kernel estimates for the fractional Laplacian. In fact, we define

$$n_t(y) = \lim_{\Gamma \ni x \rightarrow 0} \frac{p_t^\Gamma(x, y)}{\int_{\Gamma} p_1^\Gamma(x, z) dz}, \quad t > 0, y \in \Gamma, \quad (10.2)$$

and we are able to give sharp estimates for $n_t(y)$.

The Analyst should be warned that the authors find it irresistible to use the language and methods of the probabilistic potential theory, in particular they use the isotropic α -stable Lévy process $X = \{X_t, t \geq 0\}$ in \mathbf{R}^d with the semigroup $\{P_t\}$. They also use \mathbf{P}_x , the law of the process starting from $x \in \mathbf{R}^d$, and, worst of all, they are addicted to the time of the first exit of X from Γ :

$$\tau_\Gamma := \inf\{t > 0 : X_t \notin \Gamma\}.$$

With this language, the above reformulates as the existence of the following Yaglom limit:

$$\lim_{t \rightarrow \infty} \mathbf{P}_x \left(\frac{X_t}{t^{1/\alpha}} \in A \mid \tau_\Gamma > t \right) = \int_A n_1(y) dy, \quad x \in \Gamma, \quad A \in \mathcal{B}(\mathbf{R}^d). \quad (10.3)$$

REFERENCES:

- [1] K. Bogdan, Z. Palmowski, and L. Wang, Yaglom limit for stable processes in cones, *Electron. J. Probab.* 23 (2018)
- [2] K. Bogdan, T. Grzywny, and M. Ryznar, Heat kernel estimates for the fractional Laplacian with Dirichlet conditions, *Ann. Probab.* 38 (2010)
- [3] K. Bogdan, The boundary Harnack principle for the fractional Laplacian, *Studia Math.* 123 (1997)
-
-

EVOLUTION PROBLEMS IN VISCOELASTIC MATERIALS**Sandra Carillo** SBAI Dept., SAPIENZA University of Rome, Italy

The name materials with memory is generally used to refer to materials whose mechanical and/or thermodynamical behaviour depends on time not only via the present time, but also through its past history. This is the case of linear viscoelastic bodies. The model of a viscoelastic body aims to describe the mechanical behaviour of a body whose stress-strain response depends on its mechanical history. Accordingly, under the analytical viewpoint, the model equation is a linear integro-differential one. The kernel of which is represented by the relaxation modulus. An overview on viscoelasticity problems is considered with special attention on various forms of the relaxation modulus. In dealing with classical viscoelastic materials the kernel is a regular function. New materials characterised by a relaxation modulus which is unbounded at the origin [1,2] or is not differentiable [3] are considered. Furthermore, the coupling between viscoelastic and magnetic effects are studied, in the regular one and three-dimensional cases, in [4,5] respectively. while the case of relaxation modulus unbounded at the origin is considered in [6]. An overview on non-classical memory kernels is provided in [7].

References

1. S. Carillo, M. Chipot, V. Valente, G. Vergara Caffarelli, A Singular Viscoelasticity Problem: an Existence and Uniqueness Result 26, n.ro 9/10 (2013), 1115-1125.
2. S. Carillo, A 3-dimensional singular kernel problem in viscoelasticity: an existence result, *Atti Acc. Peloritana dei Pericolanti*, (2018), in press.
3. S. Carillo, M. Chipot, V. Valente, G. Vergara Caffarelli, Gleanings some old and new result in linear viscoelasticity, preprint (2018).
4. S. Carillo, V. Valente, G. Vergara Caffarelli, A result of existence and uniqueness for an integro-differential system in magneto-viscoelasticity, *Applicable Analysis: An International Journal*, (90) n. 12, (2011) 1791–1802, ISSN: 0003-6811.
5. S. Carillo, V. Valente, G. Vergara Caffarelli, An existence theorem for the magneto- viscoelastic problem, *Discrete and Continuous Dynamical Systems Series S*, (5) n. 3, (2012), 435 - 447.
6. S. Carillo, M. Chipot, V. Valente, G. Vergara Caffarelli, A magneto-viscoelasticity problem with a singular memory kernel, *Nonlinear Analysis Series B*, 35C (2017) pp. 200-210.
7. S. Carillo, C. Giorgi, Non-classical memory kernels in linear viscoelasticity, Chapter 13 in *Viscoelastic and Viscoplastic Materials*, M.F. El-Amin Editor, ISBN 978-953-51-2602-7, Published: Sept 6, 2016, pp 295 – 331 (2016) InTech.

REGULARITY FOR A CLASS OF NONLINEAR ELLIPTIC EQUATIONS IN MINIMALLY REGULAR DOMAINS

Andrea Cianchi Università di Firenze

I will discuss a few aspects of the regularity of solutions to boundary value problems for nonlinear elliptic equations and systems of p-Laplacian type. In particular, second-order regularity properties of solutions, and the boundedness of their gradient will be focused. The results to be presented are optimal, in a sense, as far as the regularity of the right-hand sides of the equations and the boundary of the underlying domains are concerned. The talk is based on joint researches with V.Mazya.

OPTIMAL DESIGN FOR THIN STRUCTURES IN GENERALIZED SOBOLEV SPACES

Piotr Kozarzewski University of Warsaw, MIM

We deal with the problem

$$\begin{aligned} \inf_{\substack{v \in W^{1,\Psi}(\Omega(\varepsilon); \mathbb{R}^3) \\ \chi_{E(\varepsilon)} \in BV(\Omega(\varepsilon); \{0, 1\})}} & \left\{ \frac{1}{\varepsilon} \left(\int_{\Omega(\varepsilon)} (\chi_{E(\varepsilon)} W_1 + (1 - \chi_{E(\varepsilon)}) W_2) (\nabla v) dx - \int_{\Omega(\varepsilon)} \bar{f} \cdot v dx \right. \right. \\ & \left. \left. + \alpha P(E(\varepsilon); \Omega(\varepsilon)) \right) : v = 0 \text{ on } \partial \omega \times (-\varepsilon, \varepsilon), \frac{1}{\mathcal{L}^3(\Omega(\varepsilon))} \int_{\Omega(\varepsilon)} \chi_{E(\varepsilon)} dx = \lambda \right\}, \end{aligned} \quad (10.4)$$

where $\beta'(\Psi(|\xi| - 1) \leq W_i(|xi|) \leq \beta(1 + \Psi(|\xi|)) \forall \xi \in \mathbb{R}^{3 \times 3}$, $i = 1, 2$, and some $\beta \geq \beta' > 0$, while Ψ is an Orlicz convex function satisfying the ∇_2 and Δ_2 conditions. $E(\varepsilon) \subset \Omega(\varepsilon)$ is a measurable subset of $\Omega(\varepsilon)$ with finite perimeter. We assume that and the load $\hat{f} \in L^{\Psi^*}(\Omega(\varepsilon); \mathbb{R}^3)$, where Ψ^* is the conjugate Orlicz function of Ψ .

This problem, commonly appearing in mechanical engineering, like the study of thin structures, can be investigated via functionals defined as follows. For every $\varepsilon > 0$, let $J_\varepsilon : L^1(\Omega; \{0, 1\}) \times L^\Psi(\Omega; \mathbb{R}^3) \rightarrow [0, +\infty]$ we take

$$J_\varepsilon(\chi, u) := \begin{cases} \int_{\Omega} (\chi W_1 (\nabla_{1,2} u | \frac{1}{\varepsilon} \nabla_3 u) + (1 - \chi) W_2 (\nabla_{1,2} u | \frac{1}{\varepsilon} \nabla_3 u)) dx \\ - f \cdot u dx + \alpha | (D_{1,2} \chi | \frac{1}{\varepsilon} D_3 \chi) | (\Omega) & \text{in } BV(\Omega; \{0, 1\}) \times W^{1,\Psi}(\Omega; \mathbb{R}^3), \\ +\infty & \text{otherwise,} \end{cases} \quad (10.5)$$

where $M|v$ is a matrix whose first two columns come from matrix M and the third one is vector v .

Our main result is calculating the Γ -limit of the family (2) with respect to the strong topology of $L^1(\Omega; \{0, 1\}) \times L^\Psi(\Omega, \mathbb{R}^3)$, when $\varepsilon \rightarrow 0$.

RAYLEIGH-BENARD HEAT CONVECTION PROBLEM FOR THE MICROPOLAR FLUID AND NAVIER-STOKES MODELS.

Grzegorz Łukaszewicz Uniwersytet Warszawski

We compare the micropolar fluid model – describing fluids with microstructure, and the Navier-Stokes model – describing homogeneous fluids, in the context of the Rayleigh–Benard heat convection problem. Our aim is to show that the presence of microstructure makes the fluid flow more stable and also decreases heat convection.

The results are expressed in terms of upper estimates of the critical Rayleigh number and the Nusselt number, respectively, for both models. Talk based on the joint project with Piotr Kalita (Jagiellonian University) and Jose A. Langa (Universidad de Sevilla).

STARSHAPE OF THE SUPERLEVEL SETS OF SOLUTIONS TO EQUATIONS INVOLVING THE FRACTIONAL LAPLACIAN IN STARSHAPED RINGS.

Tadeusz Kulczycki Wrocław University of Science and Technology

We study solutions of the problem $-(-\Delta)^{\alpha/2}u = f(x, u)$ in $D_0 \setminus \overline{D_1}$, with exterior conditions $u = 0$ in $R^n \setminus D_0$ and $u = 1$ in $\overline{D_1}$, where $D_0, D_1 \subset R^n$ are open sets such that $\overline{D_1} \subset D_0$, $\alpha \in (0, 2)$, and f is a nonlinearity. Under different assumptions on f we prove that, if D_0 and D_1 are starshaped with respect to the same point $x_0 \in D_1$, then the same occurs for every superlevel set of u .

MECHANICS WITH STRUCTURED DEFORMATIONS

Marco Morandotti Technische Universität München

In this seminar I will present some results on mechanical problems in the framework of structured deformations. Structured deformations provide a mathematical framework to capture the effects at the macroscopic level of geometrical changes at submacroscopic levels: this leads naturally to the enrichment of the energies that underlie variational descriptions of important physical phenomena.

HISTORY-DEPENDENT HEMIVARIATIONAL INEQUALITIES AND THEIR APPLICATIONS TO CONTACT MECHANICS

Anna Ochal Jagiellonian University in Kraków

We present recent results on the existence and uniqueness of the solution to the hemivariational inequality of first order with the history-dependent operator. The proof is based on arguments of surjectivity for pseudomonotone operators and the Banach fixed point theorem. We study also the continuous dependence of the solution to the considered inequality w.r.t. the operators, functions and initial data involved in the problem. The interest in continuous dependence of the solution on the perturbed data is twofold. First, the associated regularized problems can be used in numerical methods. Second, it can be the first step in studying of optimal control and identification problems. Finally, we consider an example which shows how the abstract result is applicable to the model of the contact problem. This is a joint contribution with Stanislaw Migorski and Mircea Sofonea.

LEVEL CONVEXITY FOR SUPREMAL FUNCTIONALS.

Francesca Prinari Dipartimento di Matematica e Informatica di Ferrara

The existence of a level convex supremal functional is crucial in the problems involving supremal functionals as in the study of existence of absolute minimizers (the so called AML) and in the principles of comparison with distance functions for AML. In this talk we present the characterization of the effective strength set in the context of electrical resistivity. Moreover, since the question whether a supremal functional F admits a level convex supremal functional turns out to be relevant for applications, we give some results about the necessary conditions for the lower semicontinuity of a supremal functional. Under a mild assumption on the sublevel sets of a supremal functional F , we show that the lower semicontinuous envelopes of F with respect to the weak* topology and the uniform convergence are level convex (i.e. they have convex sub-level sets).

THE LEAST GRADIENT PROBLEM IN THE PLAIN

Piotr Rybka The University of Warsaw

The least gradient problem arises in many applications, e.g. in the free material design. We show existence of solutions in bounded, strictly convex planar regions, when the data are functions of bounded variation. Our main goal is to show existence of solutions in convex, but not necessarily strictly convex planar regions. In order to avoid technicalities we consider only continuous data, but BV data will do too. We formulate two sets of admissibility conditions. We show that they are sufficient for existence. This is a joint project with Wojciech Górny and Ahmad Sabra.

11

Variational and Set-valued Methods in Differential Problems

Organizers:

Wojciech Kryszewski (Nicolaus Copernicus University in Toruń)

Pasquale Candito (Università Mediterranea di Reggio Calabria)

Salvatore Marano (University of Catania)

Marek Galewski (Łódz University of Technology)

COLLECTIVE MOVEMENTS THROUGH DIFFERENTIAL INCLUSIONS

Rinaldo M. Colombo University of Brescia

Differential Inclusions provide a natural framework to model a variety of collective phenomena. Different sorts of control problems then arise, a key example being that of confinement strategies. The present talk overviews recent results and open problems in this direction.

UNILATERAL SOURCES AND SINKS OF ACTIVATOR IN REACTION-DIFFUSION SYSTEMS EXHIBITING TURING'S DIFFUSION-DRIVEN INSTABILITY

Martin Fencel University of West Bohemia in Pilsen, Faculty of Applied Sciences, Department of Mathematics

A reaction-diffusion system exhibiting Turing's diffusion-driven instability is considered. The equation for activator is supplemented by unilateral terms of the type $s_-(\mathbf{x})u^-$, $s_+(\mathbf{x})u^+$ describing sources and sinks active only if the concentration decreases below and increases above, respectively, the value of the basic spatially constant solution which is shifted to zero. We show that the domain of diffusion parameters in which spatially non-homogeneous stationary solutions can bifurcate from that constant solution is smaller than in the classical case without unilateral terms. It is a dual information to previous results stating that analogous terms in the equation for inhibitor imply the existence of bifurcation points even in diffusion parameters for which bifurcation is excluded without unilateral sources.

FILIPPOV LEMMA FOR MEASURE DIFFERENTIAL INCLUSIONS

Andrzej Fryszkowski Faculty of Mathematics and Information Sciences, Warsaw University of Technology

I am going to present a Filippov-type Lemma for differential inclusion

$$(1) \quad \frac{d}{d\mu}x(t) \in F(t, x(t)), \quad x(0) = x_0,$$

where $F : [0, T] \times \mathbb{R}^d \rightarrow \mathbb{R}^d$ is a given multifunction and μ is a finite Borel signed measure on $[0, T]$ (possibly atomic). By solution of (1) we mean a function $x : [0, T] \rightarrow \mathbb{R}^d$ such that $x(0) = x_0$ and

$$x(t) = x_0 + \int_{S(t)} v(s) d\mu(s)$$

for $t > 0$, where $v(\cdot)$ is a μ -integrable function such that $v(t) \in F(t, x(t))$ for μ -almost every $t \in [0, T]$ and $S(t)$ stands for either $(0, t]$ for each $t \in J$ or $[0, t)$. Such setting leads to at least two nonequivalent notions of a solution to (1) and therefore we formulate two different Filippov-type inequalities (Theorems 1 and 2). These two concepts coincide in case of the Lebesgue measure. The purpose of our considerations is to cover a class of impulsive control systems, a class of stochastic systems and differential systems on time scales.

ON LYAPUNOV PAIRS IN SEMILINEAR DIFFERENTIAL PROBLEMS WITH STATE-DEPENDENT IMPULSES

Grzegorz Gabor Nicolaus Copernicus University in Toruń

In the talk a problem of the existence of mild solutions to the following multivalued differential problem with state-dependent impulses in a Banach space E will be discussed:

$$(IP) \begin{cases} y'(t) \in Ay(t) + G(t, y(t)), & t \in [0, a], t \neq \tau_j(y(t)), j = 1, \dots, m \\ y(t^+) = y(t) + I_j(y(t)), & t \in [0, a], t = \tau_j(y(t)), j = 1, \dots, m \\ y(0) = y_0 \in E. \end{cases}$$

The basic and preliminary question is what function space is appropriate as a space including solutions. The answer will be given, and conditions guaranteeing that each mild solution to (IP) meets every barrier $\Gamma = Gr(\tau_i)$ exactly once will be presented. It is interesting that the idea of Lyapunov pairs, usually applied for stability problems, has become useful for this aim. The results presented in the talk have been recently published in: I. Benedetti, T. Cardinali, G. Gabor, P. Rubbioni, *Lyapunov pairs in semilinear differential problems with state-dependent impulses*, Set-Valued and Variational Analysis, DOI 10.1007/s11228-018-0490-7.

SOME DIFFERENTIAL MODELS RELATED TO THE DYNAMICS OF SUSPENSION BRIDGES WITH INTERNAL PIERS: LINEAR THEORY AND STABILITY

Maurizio Garrione Politecnico di Milano (Polytechnic University of Milan)

We discuss some beam and degenerate plate models with multi-point conditions, which have the final aim of describing the oscillations of a suspension bridge with four internal piers. First, the presence of the piers makes necessary to settle the linear theory for the stationary problem, which we briefly discuss by presenting a complete spectral result. Then, we investigate the optimal position of the piers in terms of suitable notions of (linear and nonlinear) stability, which are analyzed both by means of analytical tools and numerical simulations. Joint work with Filippo Gazzola (Polytechnic of Milan).

BREZIS-NIRENBERG TYPE THEOREMS, LOCAL MINIMIZERS, SOBOLEV SPACES

Leszek Gasiński Jagiellonian University

Since 1993, when the paper of H. Brezis and L. Nirenberg “H versus C local minimizers” was published, many versions, generalizations and competitive proofs of their theorem have appeared. A short survey of these results together with their applications will be presented.

TRAVELLING WAVE FIELDS IN NONLINEAR MEDIA

Jarosław Mederski Institute of Mathematics, Polish Academy of Sciences

We look for travelling wave fields

$$E(x, y, z, t) = U(x, y) \cos(kz + \omega t) + \tilde{U}(x, y) \sin(kz + \omega t), \quad (x, y, z) \in \mathbb{R}^3, t \in \mathbb{R}$$

in a nonlinear and not necessarily cylindrically symmetric medium. We derive a new semilinear elliptic problem for profiles $U, \tilde{U} : \mathbb{R}^2 \rightarrow \mathbb{R}^3$, which allows to find the exact propagation of travelling waves according to Maxwell equations. We are able to deal with super quadratic focusing effects, e.g. in the Kerr-like materials with the susceptibility of the form

$$\chi(x, y, \langle |E|^2 \rangle E) = \chi^{(3)}(x, y) \langle |E|^2 \rangle E.$$

A variational approach is presented for the semilinear problem. The energy functional associated with the equations is strongly indefinite. This is a joint work with Wolfgang Reichel.

SOME REMARKS ON EXISTENCE OF ENTIRE SOLUTIONS FOR A PROBLEM OF SEGREGATION AND INTERACTION OF SPECIES

Andrzej Szulkin Department of Mathematics, Stockholm University

We consider the elliptic system

$$\begin{cases} -\Delta u = \mu_1 |u|^{2^*-2} u + \lambda \alpha |u|^{\alpha-2} |v|^\beta u, \\ -\Delta v = \mu_2 |v|^{2^*-2} v + \lambda \beta |u|^\alpha |v|^{\beta-2} v, \\ u, v \in D^{1,2}(\mathbb{R}^N), \end{cases}$$

where $2^* := 2N/(N-2)$ ($N \geq 3$), $\alpha, \beta > 1$, $\alpha + \beta = 2^*$, $\mu_1, \mu_2 > 0$ and $\lambda < 0$. This system appears e.g. in two-species physical problems (Bose-Einstein condensates with two hyperfine states) and in population dynamics. The condition $\mu_1, \mu_2 > 0$ indicates that the interaction of species (or particles) of the same kind is attractive while $\lambda < 0$ indicates the repulsive interaction of species of different kind. We shall show that there exists a positive solution (u, v) (i.e., $u, v > 0$) having a prescribed symmetry. This generalizes a recent result by Clapp and Pistoia where it was additionally assumed that $\alpha, \beta \leq 2$ (hence $N \geq 4$ and if $N = 4$, then $\alpha = \beta = 2$). This is work in progress with Mónica Clapp.

REGULARITY OF THE OPTIMAL SETS FOR SPECTRAL FUNCTIONALS AND THE FREE BOUNDARY FOR THE VECTORIAL BERNOULLI PROBLEM

Susanna Terracini University of Turin

In this talk we deal with the regularity of optimal sets for a shape optimization problem involving a combination of eigenvalues, under a fixed volume constraints. As a model problem, consider

$$\min \left\{ \lambda_1(\Omega) + \dots + \lambda_k(\Omega) : \Omega \subset \mathbb{R}^d, \text{ open}, |\Omega| = 1 \right\},$$

where $\lambda_i(\cdot)$ denotes the eigenvalues of the Dirichlet Laplacian and $|\cdot|$ the d -dimensional Lebesgue measure. We prove that any minimizer Ω_{opt} has a regular part of the topological boundary which is relatively open and $C^{1,\alpha}$ regular and that the singular part has Hausdorff dimension smaller than $d - d^*$, where $d^* \geq 3$ is the minimal dimension allowing the existence of minimal conic solutions to the bow-up problem.

We examine the link between this and the problem of regularity of the free boundary for a vector-valued Bernoulli problem, with no sign assumptions on the boundary data. More precisely, given an open, smooth set of finite measure $D \subset \mathbb{R}^d$, $\Lambda > 0$ and $\varphi_i \in H^{1/2}(\partial D)$, we deal with

$$\min \left\{ \sum_{i=1}^k \int_D |\nabla v_i|^2 + \Lambda \left| \bigcup_{i=1}^k \{v_i \neq 0\} \right| : v_i = \varphi_i \text{ on } \partial D \right\}.$$

We prove that, for any optimal vector $U = (u_1, \dots, u_k)$, the free boundary $\partial(\bigcup_{i=1}^k \{u_i \neq 0\}) \cap D$ is made of a regular part, which is relatively open and locally the graph of a C^∞ function, a (one-phase) singular part, of Hausdorff dimension at most $d - d^*$, for a $d^* \in \{5, 6, 7\}$, and by a set of branching (two-phase) points, which is relatively closed and of finite \mathcal{H}^{d-1} measure having a stratified structure itself.

These are joint works with Dario Mazzoleni and Bozhidar Velichkov.

A PLANAR GEOMETRIC FLOW WITH LACK OF SCALE INVARIANCE

Enrico Valdinoci Università di Milano

We consider a geometric evolution in the plane in which the normal velocity is a nonlocal variant of the curvature. The flow is not scaling invariant and in fact has different behaviors at different spatial scales, thus producing phenomena that are different with respect to both the classical mean curvature flow and the fractional mean curvature flow. In particular, we give examples of neckpinch singularity formation, we show that sets with sufficiently small interior remain convex under the flow, but, on the other hand, in general the flow does not preserve convexity. We also take into account traveling waves for this geometric flow, showing that a new family of convex traveling sets arises in this setting. This type of flows has been used in image digitalization for denoising methods to preserve fine structures of the details (e.g. fingerprints). This is a joint project with Serena Dipierro and Matteo Novaga.

12

Topological Methods and Boundary Value Problems

Organizers:

Aleksander Ćwiszewski (Nicolaus Copernicus University)

Gennaro Infante (University of Calabria)

EXISTENCE AND MULTIPLICITY RESULTS FOR SOME SUPERCRITICAL QUASILINEAR ELLIPTIC PROBLEMS

Anna Maria Candela Università degli Studi di Bari Aldo Moro

In the last years we have investigated the existence of one or more weak solutions of the quasilinear elliptic model problem

$$\begin{cases} -\operatorname{div}(A(x,u)|\nabla u|^{p-2}\nabla u) + \frac{1}{p}A_t(x,u)|\nabla u|^p = g(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbf{R}^N$ is a bounded domain, $N \geq 3$, $p > 1$ and the real terms $A(x,t)$, $A_t(x,t) = \frac{\partial A}{\partial t}(x,t)$ and $g(x,t)$ are Carathéodory functions on $\Omega \times \mathbf{R}$. Even if the coefficient $A(x,t)$ makes the variational approach more difficult, taking $G(x,t) = \int_0^t g(x,s)ds$, suitable assumptions allow us to look for critical points of the functional

$$\mathcal{J}(u) = \frac{1}{p} \int_{\Omega} A(x,u)|\nabla u|^p dx - \int_{\Omega} G(x,u) dx$$

which is C^1 but not verifies the classical Palais–Smale condition on the Banach space $X = W_0^{1,p}(\Omega) \cap L^\infty(\Omega)$. Anyway, a variant of the Ambrosetti–Rabinowitz theorem, which is based on a weak version of the Cerami–Palais–Smale condition, applies and both an existence result and, if $A(x, \cdot)$ and $G(x, \cdot)$ are even, also a multiplicity one holds. We note that if $A(x,t)$ satisfies “good” growth hypotheses, then the nonlinear term $G(x,t)$ can have also a suitable supercritical growth. These are joint works with Giuliana Palmieri and Addolorata Salvatore.

TRAVELING-WAVE SOLUTIONS FOR MODELS OF COLLECTIVE MOVEMENTS WITH NEGATIVE DIFFUSIVITIES

Andrea Corli University of Ferrara

In this talk we consider the advection-diffusion equation

$$\rho_t + f(\rho)_x = (D(\rho)\rho_x)_x$$

in one space dimension, whose diffusivity can be negative. This equation arises in particular in the modeling of vehicular traffic flows or crowds dynamics, where a negative diffusivity simulates aggregation phenomena. We first study traveling-wave solutions connecting two states whose diffusivity has different signs; under some geometric conditions we prove the existence, uniqueness (in a suitable class of solutions) and sharpness of the corresponding profiles. The results are extended to the case of end states where the diffusivity is positive but becomes negative in some inner interval. The vanishing-viscosity limit is also considered. At last, we show that our conditions are satisfied by a large class of models in correspondence of real data. This is a joint work with Luisa Malaguti.

POSITIVE PERIODIC SOLUTIONS TO AN INDEFINITE MINKOWSKI-CURVATURE EQUATION

Guglielmo Feltrin Università degli Studi di Torino

We deal with the indefinite Minkowski-curvature equation

$$\left(\frac{u'}{\sqrt{1-(u')^2}} \right)' + \lambda a(t)g(u) = 0,$$

where $\lambda > 0$ is a parameter, $a(t)$ is a T -periodic sign-changing weight function and $g: [0, +\infty[\rightarrow [0, +\infty[$ is a continuous function having superlinear growth at zero. We prove that for both $g(u) = u^p$, with $p > 1$, and $g(u) = u^p/(1 + u^{p-q})$, with $0 \leq q \leq 1 < p$, the equation has no positive T -periodic solutions for λ close to zero and two positive T -periodic solutions (a “small” one and a “large” one) for λ large enough. Moreover, in both cases the “small” T -periodic solution is surrounded by a family of positive subharmonic solutions with arbitrarily large minimal period. The proof of the existence of T -periodic solutions relies on a recent extension of Mawhin’s coincidence degree theory for locally compact operators in product of Banach spaces, while subharmonic solutions are found by an application of the Poincaré–Birkhoff fixed point theorem, after a careful asymptotic analysis of the T -periodic solutions for $\lambda \rightarrow +\infty$.

This talk is based on a joint work with A. Boscaggin (University of Turin).

BIFURCATION OF EQUILIBRIUM FORMS OF AN ELASTIC ROD ON A TWO-PARAMETER WINKLER FOUNDATION

Joanna Janczewska Faculty of Applied Physics and Mathematics, Gdańsk University of Technology

We consider two-parameter bifurcation of equilibrium states of an elastic rod lying on a deformable foundation which is being compressed by forces at the ends. We consider mixed boundary conditions which are as follows. The beam is free at the left end, and so it may move. However, we require the shear force at the left end to vanish. At the right end, we assume the beam to be simply supported. Our main theorem shows that bifurcation occurs if and only if the linearization of our problem has nontrivial solutions. In fact our proof, based on the concept of the Brouwer degree, gives more, namely that from each bifurcation point there branches off a continuum of solutions. The lecture is based on the joint work with M. Izydorek, N. Waterstraat and A. Zgorzelska [Nonlinear Anal. Real World Appl. 39 (2018), 451-463].

ON NON-AUTONOMOUSLY FORCED BURGERS EQUATION

Piotr Kalita Jagiellonian University

We study the non-autonomously forced Burgers equation

$$u_t(x,t) + u(x,t)u_x(x,t) - u_{xx}(x,t) = f(x,t)$$

on the space interval $(0, 1)$ with the Dirichlet and periodic boundary conditions. For both situations we prove that there exists the unique H^1 bounded eternal trajectory of this equation which attracts all trajectories both in

pullback and forward sense. We also prove that for the Dirichlet case this attraction is exponential. Finally, we present computer assisted method based on Finite Element approach which allows to construct this trajectory, with some accuracy, for the problem with Dirichlet conditions. The results are obtained in a joint work with Piotr Zgliczyński.

THE RIEMANN-HILBERT PROBLEM AND SINGULARITY FORMATION IN THE LOCALIZED INDUCTION APPROXIMATION

Piotr Kokocki Nicolaus Copernicus University in Toruń

The Riemann-Hilbert problem and singularity formation in the localized induction approximation We are concern with the differential equation of the form

$$z_t = -k_s n - \frac{1}{2} k^2 T, \quad t, s \in \mathbb{R}, \quad (12.1)$$

where z is the flow of regular curves living in the complex plane, s is the arc-length parameter, T is the field of tangent vectors, $n := iT$ is the oriented normal vector field and k is the curvature defined by $T_s = kn$. The equation (12.1) is called the *localized induction approximation* and can be considered as the geometric flow, whose evolution is similar to the contour dynamics of a vortex patch subjected to the 2D Euler equation (see e.g. [3]). Our aim is to show the existence of a regular family of self-similar solutions for (12.1), which develops a spiral singularity at finite time. To be more precise, we prove that for any $a \in (-\frac{\pi}{2}, \frac{\pi}{2})$ and $\mu \in \mathbb{R}$, there are $\theta^+, \theta^- \in [0, 2\pi)$ and a self-similar solution $z^{\mu, a}$ of the equation (12.1), such that $\theta^+ - \theta^- = 2a$ and $\lim_{t \rightarrow 0} z^{\mu, a}(t) = z_0^{\mu, a}$ in the space of tempered distributions, where

$$z_0^{\mu, a}(s) := \begin{cases} \frac{s}{\sqrt{1+4\mu^2}} e^{i(\theta^- - 2\mu \log|s|)} \chi_{(-\infty, 0)}(s), & s > 0, \\ \frac{s}{\sqrt{1+4\mu^2}} e^{i(\theta^+ - 2\mu \log|s|)} \chi_{(0, +\infty)}(s), & s < 0. \end{cases} \quad (12.2)$$

The function (12.2) is called a *logarithmic spiral* and plays a crucial role in the fluid mechanics and turbulence modeling. The method of the proof is searching for the solutions $z^{\mu, a}$ in a class of self-similar functions, whose profiles are purely imaginary solutions of the second Painlevé (PII) equation

$$u''(x) = xu(x) + 2u^3(x) - \alpha \quad (12.3)$$

with $\alpha := -i\mu$. The obtained results improves those from [5], where the authors use the perturbation argument together with the methods of ODEs to prove the existence of solutions $z^{\mu, a}$, when $|\mu| + |a|$ is sufficiently close to zero.

- [1] J. Baik, R. Buckingham, J. DiFranco, A. Its, *Total integrals of global solutions to Painlevé II*, Nonlinearity **22** (2009), no. 5, 1021–1061.
- [2] A.S. Fokas, A.R. Its, A. Kapaev, V. Novokshenov, *Painlevé transcendents. The Riemann-Hilbert approach*, Mathematical Surveys and Monographs, 128. American Mathematical Society, RI, 2006.
- [3] R.E. Goldstein, D.M. Petrich, *Solitons, Euler's equation, and vortex patch dynamics*, Phys. Rev. Lett. **69** (1992), no. 4, 555–558.
- [4] P. Kokocki, *The Riemann-Hilbert problem and singularity formation in the contour dynamics for the 2D Euler equation*, in preparation
- [5] G. Perelman, L. Vega, *Self-similar planar curves related to modified Korteweg-de Vries equation*, J. Differential Equations **235** (2007), 56–73.

- [6] N.J. Zabusky, M.H. Hughes, K.V. Roberts, *Contour dynamics for the Euler equations in two dimensions*, J. Comput. Phys. 30 (1979), no. 1, 96–106.

NONLOCAL SOLUTIONS OF PARABOLIC EQUATIONS WITH STRONGLY ELLIPTIC DIFFERENTIAL OPERATORS

Luisa Malaguti University of Modena and Reggio Emilia

The talk deals with second order parabolic equations on bounded domains in arbitrary Euclidean spaces. Their interest comes from being models for describing reaction-diffusion processes in several frameworks. A linear diffusion term in divergence form is included which generates a strongly elliptic differential operator. A further linear part, of integral type, is present which accounts of nonlocal diffusion behaviours. The main result provides a unifying method for studying the existence and localization of solutions satisfying nonlocal associated boundary conditions. The Cauchy multipoint and the mean value conditions are included in this discussion. The problem is transformed into its abstract setting and the proofs are based on the homotopic invariance of the Leray-Schauder topological degree. A bounding function theory is developed, which is new in this infinite dimensional context. It allows that the associated vector fields have no fixed points on the boundary of their domains and then it makes possible the use of a degree argument. The result come from a joint paper with Irene Benedetti (University of Perugia) and Valentina Taddei (University of Modena and Reggio Emilia).

EXISTENCE AND MULTIPLICITY OF EVANESCENT POSITIVE SOLUTIONS OF ELLIPTIC SYSTEMS

Aleksandra Orpel University of Łódź

We investigate positive evanescent solutions for the following elliptic system

$$\begin{cases} \Delta u(x) + f_1(x, u(x), v(x)) + g_1(x)x \cdot \nabla u(x) = 0 \\ \Delta v(x) + f_2(x, u(x), v(x)) + g_2(x)x \cdot \nabla v(x) = 0, \end{cases} \quad (12.4)$$

considered in $\Omega_R = \{x \in \mathbb{R}^n, |x| > R\}$, with $R > 0$ and $n > 2$. We formulate sufficient conditions for the existence of infinitely many solutions with the asymptotic decay $u(x) = O(|x|^{2-n})$ and $v(x) = O(|x|^{2-n})$ as $|x| \rightarrow \infty$. Moreover, the asymptotic behavior of the energy of solutions will be described. Our results are based on a certain iteration approach in which we apply the sub- and super-solution method.

INFINITELY MANY SOLUTIONS FOR QUASILINEAR ELLIPTIC EQUATIONS WITH LACK OF SYMMETRY

Addolorata Salvatore Università degli Studi di Bari Aldo Moro

We investigate the existence of weak bounded solutions of the quasilinear elliptic model problem

$$\begin{cases} -\operatorname{div}(A(x, u)\nabla u) + \frac{1}{2}A_t(x, u)|\nabla u|^2 = g(x, u) + h(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbf{R}^N$ is an open bounded domain, $N \geq 3$, the real terms $A(x,t)$, $A_t(x,t) = \frac{\partial A}{\partial t}(x,t)$, $g(x,t)$ are Caratheodory functions on $\Omega \times \mathbf{R}$ and $h : \Omega \rightarrow \mathbf{R}$ is a given measurable map. We note that coefficient $A(x,t)$ make the variational approach more difficult, as we have to look for critical points of the functional

$$\mathcal{J}(u) = \frac{1}{2} \int_{\Omega} A(x,u) |\nabla u|^2 dx - \int_{\Omega} G(x,u) dx - \int_{\Omega} h(x)u(x) dx$$

in the Banach space $X = H_0^1(\Omega) \cap L^\infty(\Omega)$ (here, $G(x,t) = \int_0^t g(x,s) ds$). Anyway, we prove that, if $A(x, \cdot)$ and $G(x, \cdot)$ are even and satisfy “good” growth assumptions, infinitely many solutions exist in spite of the lack of symmetry. A suitable supercritical growth is allowed for the nonlinear term $G(x,t)$. We use a variant of the variational perturbation techniques introduced by Rabinowitz but by means of a weak version of the Cerami–Palais–Smale condition. (Joint works with Anna Maria Candela and Giuliana Palmieri)

A GEOMETRIC METHOD FOR INFINITE-DIMENSIONAL CHAOS: SYMBOLIC DYNAMICS FOR THE KURAMOTO-SIVASHINSKY PDE ON THE LINE

Piotr Zgliczyński Uniwersytet Jagiellonski

We discuss a method for rigorous study of dynamics of dissipative PDEs. The method is then applied to certain Poincaré map of the Kuramoto-Sivashinsky PDE on the line with odd and periodic boundary conditions and with parameter $\nu = 0.1212$. We give a computer-assisted proof of the existence of symbolic dynamics and countable infinity of periodic orbits with arbitrary large periods. This is a joint work with Daniel Wilczak

EXISTENCE RESULTS FOR THE SECOND ORDER PERIODIC BOUNDARY VALUE PROBLEM

Mirosława Zima University of Rzeszów

The talk is based on the joint paper with José Ángel Cid, Gennaro Infante and Milan Tvrdý [3]. We will discuss sufficient conditions for existence and localization of positive solutions for $x''(t) + ax'(t) = r(t)x^\alpha - s(t)x^\beta$ subject to periodic boundary conditions $x(0) = x(T)$, $x'(0) = x'(T)$. The problem is related with the valveless pumping phenomenon. Some results from [1], [2], and [4] will be also presented.

- [1] J. A. Cid, G. Propst and M. Tvrdý, On the pumping effect in a pipe/tank flow configuration with friction, *Phys. D* 273-274 (2014), 28–33.
- [2] J. A. Cid, G. Infante, M. Tvrdý and M. Zima, J. A. Cid, G. Infante, M. Tvrdý and M. Zima, *A topological approach to periodic oscillations related to the Liebau phenomenon*, *J. Math. Anal. Appl.* 423 (2015), 1546–1556.
- [3] J. A. Cid, G. Infante, M. Tvrdý and M. Zima, M. Zima, New results for the Liebau phenomenon via fixed point index, *Nonlinear Anal. RWA* 35 (2017), 457–469.
- [4] F. Wang, J. A. Cid and M. Zima, Lyapunov stability for regular equations and applications to the Liebau phenomenon, *Discrete Contin. Dyn. Systems - A* 38 (2018), 4657–4674.

13

Variational Problems and Nonlinear PDEs

Organizers:

Jarosław Mederski (Polish Academy of Sciences)

Pietro d'Avenia (Politecnico di Bari)

FRACTIONAL PERIODIC PROBLEMS WITH CRITICAL GROWTH

Vincenzo Ambrosio Ecole Polytechnique Federale de Lausanne

In this talk we present some existence results for a class of fractional periodic problems involving the fractional critical exponent. These results are obtained by combining suitable variational methods and a variant of the extension technique in the periodic setting.

FRACTIONAL SCHRÖDINGER EQUATIONS WITH SINGULAR POTENTIALS AND SIGN-CHANGING NONLINEARITIES

Bartosz Bieganowski Nicolaus Copernicus University

We look for ground state solutions to the following nonlinear (fractional) Schrödinger equation

$$(-\Delta)^{\alpha/2}u + \left(V(x) - \frac{\mu}{|x|^\alpha} \right) u = f(x, u) - \Gamma(x)|u|^{q-2}u \text{ on } \mathbb{R}^N, \quad 0 < \alpha < 2,$$

where $V = V_{per} + V_{loc} \in L^\infty(\mathbb{R}^N)$ is the sum of a periodic potential V_{per} and a localized potential V_{loc} , $\Gamma \in L^\infty(\mathbb{R}^N)$ is periodic and $\Gamma(x) \geq 0$ for a.e. $x \in \mathbb{R}^N$ and $2 < q < 2_\alpha^*$. We assume that

$$\text{ess inf } V(x) > 0$$

and f has the subcritical growth but higher than $\Gamma(x)|u|^{q-2}u$, however the nonlinearity $f(x, u) - \Gamma(x)|u|^{q-2}u$ may change sign. We investigate the existence of ground state solutions being minimizers on the Nehari manifold.

MAXIMAL AND MINIMAL KINETIC ENERGY IN THE CLASS OF VORTEX SHEETS SATISFYING PRANDTL'S SIMILITUDE LAWS

Tomasz Cieślak IMPAN

I will consider the problem of minimization as well as maximization of kinetic energy in the class of 2d vortex sheet flows satisfying the so-called Prandtl similitude laws. I will find the maximal and minimal possible values and the objects on which they are attained. I will discuss the meaning of the above exercise for understanding of self-similar vortex spirals. The talk is based on a common work with K.Oleszkiewicz, M.Preisner and M.Szumanska.

ON SPECTRAL STABILITY OF AHARONOV-BOHM OPERATORS WITH MOVING POLES

Veronica Felli Università di Milano-Bicocca

In this talk, I will present some results in collaboration with L. Abatangelo (Milano-Bicocca), L. Hillairet (Orléans), C. Léna (Lisboa), B. Noris (Amiens), and M. Nys, concerning the behavior of the eigenvalues of Aharonov-Bohm operators with one moving pole or two colliding poles. In both cases of poles moving inside the domain and approaching the boundary, the rate of the eigenvalue variation is estimated in terms of the vanishing order of some limit eigenfunction. An accurate blow-up analysis for scaled eigenfunctions will be presented too.

REGULARITY RESULTS FOR A WIDE CLASS OF PARABOLIC SYSTEMS.

Teresa Isernia Università Politecnica delle Marche

In this talk we discuss some recent regularity results of solutions to parabolic systems in divergence form where the leading term has Uhlenbeck structure of Orlicz type. Using a difference quotient method and Moser type iteration, we prove that the solution is locally bounded.

BUFURCATION FROM INFINITY OF BOUND STATES OF A NONLINEAR SCHRÖDINGER EQUATION

Wojciech Kryszewski Nicolaus Copernicus University in Toruń, Poland

We shall consider the externally driven *nonlinear Schrödinger equation* of the form

$$i\psi_t = -\Delta\psi + V(x)\psi - W'(x, \psi), \quad (13.1)$$

where $W : \mathbb{R}^N \times \mathbb{C} \rightarrow \mathbb{R}$. One usually assumes that $W(x, z)$ depends on x and $|z|$ only, i.e. $W(x, z) = H(x, |z|)$ where $H : \mathbb{R}^N \times [0, +\infty) \rightarrow \mathbb{R}$ with $H(x, s) = \int_0^s h(x, \xi) d\xi$ for $x \in \mathbb{R}^N$ and $s \geq 0$, and $h : \mathbb{R}^N \times \mathbb{R} \rightarrow \mathbb{R}$ is a Carathéodory function. Problems concerning (13.1) play important role in different physical contexts, especially in the description of macroscopic quantum systems as, for instance, plasma physics, nonlinear optics and others. Depending on a model it has different interpretations: in the classical model of linear quantum mechanics of a particle $H = 0$ and $|\psi(t, x)|^2$ is the probability of finding the particle at a given point in the space-time. In the theory of Bose-Einstein condensates, for example, one speaks of the Gross-Pitaevski equation and $W(x, z) = \frac{1}{4}|z|^4$, $z \in \mathbb{C}$; $|\psi|^2$ is interpreted as the mass density of bosons. For appropriate choice of h the equation (13.1) has *standing wave* solutions, i.e. satisfying the ansatz

$$\psi(t, x) = e^{-i\lambda t} u(x), \quad t \geq 0, \quad x \in \mathbb{R}^N, \quad (13.2)$$

with the time-independent profile $u \in H^1(\mathbb{R}^N)$ and $\lambda \in \mathbb{R}$. By the use of topological tools (such as the infinite dimensional Conley index theory) we shall show that if $\lambda_0 \in \sigma(-\Delta + V(x))$ satisfies some technical assumptions and $f(x, u) := h(x, u) \frac{u}{|u|}$ is subject to the so-called Landesman-Lazer conditions, then (13.1) has a sequence of bound states ψ_n of the form (13.2) with $\lambda = \lambda_n \rightarrow \lambda_0$, $u = u_n \in H^1$ such that $\|u_n\|_{H^1} \rightarrow \infty$ and the *energy*

$$E(\psi_n) := \frac{1}{2} \int_{\mathbb{R}^N} (|\nabla \psi_n|^2 + V(x)|\psi_n|^2) dx dt - \int_{\mathbb{R}^N} W(x, \psi) dx \rightarrow \infty$$

as $n \rightarrow \infty$.

THE LANE-EMDEN EQUATION ON A PLANAR DOMAIN

Angela Pistoia Sapienza Università di Roma

I will review some old and new results concerning existence, multiplicity and asymptotic behaviour of solutions to the classical Lane-Emden equation on a planar domain when the exponent of the non-linearity is large.

A SEMILINEAR CURL-CURL PROBLEM IN \mathbb{R}^3

Jacopo Schino Institute of Mathematics of the Polish Academy of Sciences

We look for nontrivial solutions to the semilinear problem

$$\nabla \times \nabla \times E = f(x, E) \text{ in } \mathbb{R}^3,$$

where $E : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ and $f = \nabla_E F$. We discuss the discreteness of Cerami sequences and apply a deformation argument based on the Krasnoselskii genus and the Lusternik-Schnirelman values. We prove the existence of a ground state solution as well as infinitely many geometrically distinct solutions.

The main difficulties are due to working in an unbounded domain and the infinite dimension of the kernel of the energy functional (i.e. the space of gradient vector fields).

This talk is based on a work in progress with J. Mederski and A. Szulkin.

LEAST ENERGY RADIAL SIGN-CHANGING SOLUTION FOR A SCHRÖDINGER-POISSON SYSTEM WITH ASYMPTOTICALLY CUBIC NONLINEARITY

Gaetano Siciliano Universidade de São Paulo

In this talk we consider a Schrödinger-Poisson system in the whole space under a nonlinearity which behaves cubically at infinity. In this case the nonlinearity is in a strict competition with the nonlocal term. Previous results in the literature deal with the “super cubic” case.

By using variational methods we are able to show the existence of a nodal solution which has minimal energy among all the solutions which change sign. Moreover it changes sign exactly once.

A SEMILINEAR CURL-CURL PROBLEM IN \mathbb{R}^3 , PART I

Andrzej Szulkin Department of Mathematics, Stockholm University

We look for nontrivial solutions to the semilinear problem

$$\nabla \times \nabla \times E = f(x, E) \text{ in } \mathbb{R}^3,$$

where $E : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ and $f = \nabla_E F$. We discuss the discreteness of Cerami sequences and apply a deformation argument based on the Krasnoselskii genus and the Lusternik-Schnirelman values. We prove the existence of a ground state solution as well as infinitely many geometrically distinct solutions.

The main difficulties are due to working in an unbounded domain and the infinite dimension of the kernel of the energy functional (i.e. the space of gradient vector fields).

This talk is based on a work in progress with J. Mederski and J. Schino.

MAXIMAL SOLUTION OF THE LIOUVILLE EQUATION IN DOUBLY CONNECTED DOMAIN

Giusi Vaira Università della Campania "L. Vanvitelli"

In this paper we consider the Liouville equation $\Delta u + \lambda^2 e^u = 0$ with Dirichlet boundary conditions in a two dimensional, doubly connected domain Ω . We show that there exists a simple, closed curve $\gamma \in \Omega$ such that for a sequence $\lambda_n \rightarrow 0$ and a sequence of solutions u_n it holds $\frac{u_n}{\log \frac{1}{\lambda_n}} \rightarrow H$, where H is a harmonic function in $\Omega \setminus \gamma$ and $\frac{\lambda_n^2}{\log \frac{1}{\lambda_n}} \int_{\Omega} e^{u_n} dx \rightarrow 2\pi c_{\Omega}$, where c_{Ω} is a constant depending on Ω only.

14

Nonlinear Variational Methods with Applications

Organizers:

Anna Zatorska (University of Warsaw)

Sofia Giuffrè (Mediterranea University of Reggio Calabria)

GEOMETRIC ASPECTS OF FINSLER P-CAPACITARY POTENTIALS

Chiara Bianchini Università degli Studi di Firenze

In the Euclidean case it is well known that the geometry of the p-capacitary potential of a convex set Ω influences the symmetry of Ω itself. An analogous holds true also in the Finsler setting, where the space R^N is endowed with an anisotropic norm. In particular in this talk we will characterize the shape of Ω in terms of geometric properties of its capacitary potential, as its concavity and the shape of its level sets. To this aim we will consider some non-standard overdetermined problems, where the governing operator is the Finsler p-Laplacian. This is a joint work with P. Salani and G. Ciraolo.

DUALITY AND OPTIMALITY CONDITIONS IN OPTIMAL TRANSPORT AND ROCKAFELLAR THEOREM

Luigi De Pascale Università di Firenze

The story begins with a theorem of R.T. Rockafellar (70's) saying that a set is maximal and cyclically monotone if and only if it is contained in the sub-differential of a convex function. Later on, the condition of cyclical monotonicity appeared as necessary and sufficient for the optimality of a probability measure in Kantorovich's relaxation of optimal transport problems. Duality helped to connect the two ideas, giving spectacular developments and allowing the connection with the Monge-Ampere equation. More recently, mostly because of the applications in Quantum Chemistry, some unbounded transport costs attracted the attention of the mathematical community. For these costs duality and optimality conditions require some care. I will present recent results in this direction.

FOKKER-PLANCK TYPE EQUATIONS WITH SINGULAR DRIFT TERM

Fernando Farroni Dipartimento di Matematica e Applicazioni "R. Caccioppoli" -
Università degli Studi di Napoli Federico II

We consider weak solutions to a nonlinear evolution diffusion-convection equation with an unbounded convection field. We provide existence and uniqueness results as well as the asymptotic behaviour of solutions. The initial value-boundary problem has a strong relationship with the class of Fokker-Planck type equations, naturally arising in the theory of the so-called mean field games.

ON PARAMETER IDENTIFICATION IN VARIATIONAL INEQUALITIES

Joachim Gwinner Universität der Bundeswehr München

In this talk we address the identification of various parameters, in particular ellipticity and friction parameters in unilateral contact, modelled as variational inequalities of first and second kind. First we investigate the dependence of the solution of the forward problem on these parameters and prove Lipschitz estimates. Then we formulate two optimization approaches to these parameter identification problems. We extend the output least squares (OLS) approach, provide an existence result and establish a convergence result for finite dimensional

approximation. Further we investigate the modified output least squares (MOLS) approach which is based on an energy functional. This talk is based on the papers [1 – 3].

SOME REFERENCES

[1] J. Gwinner, An optimization approach to parameter identification in variational inequalities of second kind, *Optim. Lett.* 12 1141–1154, 2018.

[2] J. Gwinner, On two-coefficient identification in elliptic variational inequalities, 21 pp, to appear in *Optimization*.

[3] J. Gwinner, B. Jadamba, A.A.Khan, M. Sama, Identification in variational and quasi-variational inequalities, *J. Convex Anal.* 25(2) 545–569, 2018.

ON THE DIRICHLET PROBLEM IN CYLINDRICAL DOMAINS FOR EVOLUTION OLEINIK–RADKEVIC PDE’S: A THYCHONOV-TYPE THEOREM

Alessia Kogoj University of Urbino

We are concerned with linear second order PDE’s of the type:

$$\mathcal{L} = \mathcal{M} - \partial_t := \sum_{i,j=1}^n \partial_{x_i} (a_{i,j} \partial_{x_j}) - \sum_{j=1}^n b_j \partial_{x_j} - \partial_t.$$

We assume \mathcal{L} with nonnegative characteristic form and satisfying the Oleinik-Radkevic rank hypoellipticity condition. These hypotheses allow to construct Perron-Wiener solutions of the Dirichlet problems for \mathcal{L} and \mathcal{M} on bounded open subsets of \mathbb{R}^{n+1} and of \mathbb{R}^n , respectively, by using Potential Theory in σ^* -Harmonic spaces.

Our Main result is the following Thychonov-type Theorem:

Let $O := \Omega \times]0, T[$ be a bounded cylindrical domain of \mathbb{R}^{n+1} , $\Omega \subset \mathbb{R}^n$, $x_0 \in \partial\Omega$ and $0 < t_0 < T$. Then $z_0 = (x_0, t_0) \in \partial O$ is \mathcal{L} -regular for O if and only if x_0 is \mathcal{M} -regular for Ω .

As an application of our Main Theorem we show some regularity criteria for the boundary point in the Dirichlet problem for degenerate Ornstein-Uhlenbeck operators, as consequences of analogous criteria for Kolmogorov-Fokker-Planck equations.

The results presented are obtained in collaboration with Ermanno Lanconelli.

DISSIPATIVE AND NON-DISSIPATIVE EVOLUTIONARY QUASI-VARIATIONAL INEQUALITIES WITH DERIVATIVE CONSTRAINTS

Carlos Rautenberg Humboldt-Universität zu Berlin - Weierstrass Institute

We consider evolutionary quasi-variational inequality (QVI) problems of dissipative and non-dissipative nature with pointwise constraints on the gradient. The talk is motivated by a variety of recent mathematical models arising in physics. A semi-discretization in time is employed for the study of the problem class and the derivation of a solution algorithm. Convergence of the discretization procedure is proven and properties of the original problem, such as existence, extra regularity and non-decrease in time, are obtained. The talk is finalized with a report on numerical tests obtained involving different nonlinearities and types of constraints.

UNIQUENESS AND REGULARITY FOR MINIMIZERS OF RELAXED FUNCTIONALS IN BV VIA COMPARISON PRINCIPLES

Giulia Treu Dipartimento di Matematica "Tullio Levi-Civita" Università di Padova

We prove the validity of Comparison Principle for BV minimizers of relaxed functionals and we use them to prove both local Lipschitz regularity of minimizers and uniqueness of the minimizers under very weak assumptions on the boundary datum. The results are contained in a joint paper with Alice Fiaschi.

15

Advances in Kinetic Theory

Organizers:

Ewelina Zatorska (University College London)

Andrea Tosin (Politecnico di Torino)

BOLTZMANN-TYPE OPTIMAL CONTROL PROBLEMS

Giacomo Albi University of Verona

We are interested in a Boltzmann-type framework to deal with the optimal control of large particle systems. We will start reviewing suboptimal approaches based on the control of binary interaction dynamics. Secondly we will tackle directly the optimal control of the Boltzmann equation, in particular showing its relation with respect to mean-field optimal control problems. Finally we will propose a stochastic hybrid algorithm, able to mitigate the numerical complexity of these problems. Different numerical examples will be presented in the context of consensus dynamics, and swarming models.

BGK KINETIC APPROXIMATION FOR GAS MIXTURES

Marzia Bisi University of Parma, Italy

Dilute gaseous systems involving particles of different species may be rigorously studied by means of suitable integro-differential Boltzmann equations, but also through simpler relaxation-time approximations of BGK-type, preserving positivity, correct collision invariants and equilibria. Various approaches may be adopted for building up the BGK operators, according to the selected constraints for the auxiliary parameters (preservation of correct exchange rates for species macroscopic fields, of global collision invariants, of Prandtl number, ...). In this talk we present a recent model for inert mixtures, well suited for various intermolecular collision laws, not only of Maxwell-molecule or hard-spheres type. Moreover, we also mention BGK descriptions taking into account even non-elastic interactions, as chemical reactions or internal energy transitions in polyatomic gases.

ON A KINETIC MODEL FOR BEHAVIOURAL ASSET PRICING

Bertram Düring University of Sussex

We propose a kinetic model for behavioural asset pricing in which a large number of interacting agents price an asset (e.g. a stock or commodity), leading to an inhomogeneous Boltzmann-type equation. The microscopic dynamics depend on the estimated value of the asset and the rationality or irrationality of the agents. The interaction rules take into account the interplay of the agents with sources of public information and herding phenomena, depending on the (ir)rationality of the agents. We derive a nonlinear nonlocal Fokker-Planck equation with incomplete diffusion in a suitable limit and prove the existence of global-in-time weak solutions for the initial-boundary-value problem. Numerical experiments for the inhomogeneous Boltzmann-type equation shows how herding may lead to strong trends with low volatility of the asset price, but eventually also to abrupt corrections.

THE BOLTZMANN EQUATION FOR A MULTI-SPECIES MIXTURE CLOSE TO GLOBAL EQUILIBRIUM

Esther S. Daus Vienna University of Technology

In this talk I will present our joint work with Marc Briant on the Cauchy theory for a multi-species dilute gaseous mixture with different masses modeled by the multi-species Boltzmann equation close to equilibrium on the torus. The physically most relevant space for such a Cauchy theory is the space of density functions that only have finite mass and energy. Thus, the ultimate aim of this work is to obtain existence, uniqueness and exponential trend to equilibrium of solutions to the multi-species Boltzmann equation in $L_v^1 L_x^\infty(m)$, where $m \sim (1 + |v|^k)$ is a polynomial weight and $k > k_0$, recovering the optimal physical threshold of finite energy $k_0 = 2$ in the particular case of a multi-species hard spheres mixture with same masses. Our strategy is to combine and adapt several very recent methods, combined with new hypocoercivity estimates, in order to develop a new constructive approach that allows to deal with polynomial weights without requiring any spatial Sobolev regularity. We emphasize that dealing with different masses induces a loss of symmetry in the Boltzmann operator which prevents the direct adaptation of standard mono-species methods (e.g. Carleman representation, Povzner inequality).

NEW RESULTS FOR POLYATOMIC RAREFIED GASES

Laurent Desvillettes Université Paris Diderot

The modeling of polyatomic rarefied gases is important because diatomic species represent most of the molecules present in the upper atmosphere.

We present in the talk a new estimate for the smoothing properties of the positive part of a typical collision kernel used for polyatomic gases, first introduced by Jean-François Bourgat, LD, Patrick Le Tallec and Benoît Perthame, and generalized later by LD, Roberto Monaco and Francesco Salvarani.

The talk is based on a work performed in collaboration between Yemin Chen, LD, and Lingbing He.

ASYMPTOTIC PRESERVING AND TIME DIMINISHING SCHEMES FOR RAREFIED GAS DYNAMIC.

Giacomo Dimarco University of Ferrara

In this talk, we present a new class of numerical schemes for solving kinetic-type equations. The method is based on a micro-macro decomposition (Euler-kinetic or Navier-Stokes-kinetic) for which the macro part is solved using a finite volume method whereas the micro part uses a Monte Carlo method. In particular, the particle method which solves the microscopic part is designed in such a way that the global scheme becomes computationally less expensive as the solution approaches the equilibrium state as opposed to standard methods for kinetic equations. At the same time, the statistical error due to the particle part of the solution decreases as the system approaches the equilibrium state. This causes the method to degenerate to the sole solution of the macroscopic hydrodynamic equations in the limit of infinite number of collisions. Thus, the method is automatically costly diminishing without imposing any artificial transition to pass from the microscopic to the macroscopic model at the contrary to domain decomposition techniques in which a transition region should be artificially imposed. In this sense, the schemes proposed realize an automatic transition from kinetic to hydrodynamic which only depends on the real physics and not on numerical artifacts.

VARIATIONAL SOLUTIONS OF THE LINEARIZED BOLTZMANN EQUATION FOR GAS FLOWS IN MICROCHANNELS

Silvia Lorenzani Politecnico di Milano

Since rarefied gas flows occur in many micro-electro-mechanical systems (MEMS), a correct prediction of these flows is important to design and develop MEMS. In spite of their apparently complex structure, the basic constituent of a real MEMS device is the microchannel, the region between two parallel plates that can reveal many specific features of the low speed internal flows in microdevices. Therefore, an important aspect of the matter is to have an approximate closed form solution for gas flow rates in microchannels in order to use it in applications. To develop an accurate formula directly from kinetic theory there is a particularly useful technique, the variational method proposed by C. Cercignani in 1969, which applies to the integrodifferential form of the Boltzmann equation and can be used for any linearized Boltzmann model. A practical advantage of the variational principle is that it allows to write down simple approximate equations to be used in practical design by suitable analytical manipulations.

In the current investigation, the variational technique is used to compute gas flow rates in microchannels in a wide range of Knudsen numbers, by considering the true linearized Boltzmann collision operator and general models of boundary conditions. The variational approach has the enormous advantage of reducing drastically the computational CPU time giving at one time the same accuracy for each value of the Knudsen number progressing from free molecular, through transitional, to continuum regions.

COUPLED SELF-ORGANIZED HYDRODYNAMICS AND STOKES MODELS FOR SUSPENSIONS OF ACTIVE PARTICLES

Sara Merino-Aceituno University of Sussex

We derive macroscopic dynamics for collective motion in a fluid. The starting point is a coupled Vicsek-Stokes system. The Vicsek model describes self-propelled agents interacting through alignment. It provides a phenomenological description of steric interactions between agents at high density. Stokes equations describe a low Reynolds number fluid.

FROM KINETIC TO CONTINUOUS MODELS - THE PERSPECTIVE OF NUMERICAL SIMULATIONS

Piotr Minakowski Otto-von-Guericke-Universität Magdeburg

The main focus of the talk is to describe the results of numerical simulation of hydrodynamical models that arise from the kinetic framework.

We will discuss the possible dynamics in different scales: meso- and macroscopic. The first one is relevant for systems with very large number of the particles, while the macroscopic (hydrodynamical) level becomes convenient when condensation of particles is too large to distinguish single evolution of any one of them. Moreover, the computational cost of the second one is lower.

In particular we will show what kind of the particle-type of behaviour can be reproduced on the macroscopic level. As example we consider continuous limits of kinetic models i.e the compressible-incompressible two phase Euler system and pressureless Navier-Stokes-type system. This fluid equations are used to model

pedestrian motion and traffic, or collective behaviour that corresponds to particular choice of alignment kernel in Cucker-Smale model.

CELL MIGRATION: VELOCITY-JUMP PROCESS OR MICROSCOPIC ALGORITHMS?

Loy Nadia Politecnico di Torino

Cell migration is the process that is at the basis of the formation and the maintenance of multicellular organisms. Cells move by run and tumble, that is a kind of dynamics in which the cell alternates runs over straight lines and re-orientations. During this erratic motion, cells may interact with each other and with the external environment in which there may be chemical signals, nutrients or fibers of the extra-cellular matrix. Such factors may bias the choice of the velocity after a re-orientation. In mathematical biology the run and tumble has been successfully described by velocity jump processes implemented in kinetic equations: in this framework, the transition probability of assuming a certain post-tumbling velocity is the key element of the model. It is widely known that kinetic models also allow to describe binary collisions between individuals of the same population and individuals of different populations through a binary interactions approach in which microscopic rules are included in a collision kernel. We want to establish a parallelism between velocity jump processes and collisional models and to see which analytical and numerical techniques may be used in the two approaches.

- [1] Nadia Loy, Andrea Tosin, *Kinetic models for cell migration: a parallelism between velocity jump processes and binary interaction algorithms*, in preparation.
- [2] A. Chauviere, N. Loy and L. Preziosi, *Kinetic models with non-local sensing for chemotaxis*, in preparation.
- [3] T. Hillen, N. Loy and L. Preziosi, *Transport equations for mesenchymal motion with direction-dependent turing rate*, in preparation.

HOMOENERGETIC SOLUTIONS FOR THE BOLTZMANN EQUATION

Alessia Nota Institute for Applied Mathematics, University of Bonn

We consider a particular class of solutions of the Boltzmann equation, known as homoenergetic solutions, which are useful to describe the dynamics of Boltzmann gases under shear, expansion or compression in nonequilibrium situations. While their well posedness theory has many similarities with the theory of homogeneous solutions of the Boltzmann equation, their long time asymptotics differs completely, due to the fact that these solutions describe far from equilibrium phenomena. Indeed, the long time asymptotics cannot always be described by Maxwellian distributions. For several collision kernels the asymptotics of homoenergetic solutions is given by particle distributions which do not satisfy the detailed balance condition. In this talk I will describe different possible long time asymptotics of homoenergetic solutions of the Boltzmann equation, as well as some open problems in this direction. (This is a joint work with R.D. James and J.J.L. Velázquez).

VARIANCE REDUCTION TECHNIQUES IN UNCERTAINTY QUANTIFICATION FOR KINETIC EQUATIONS

Lorenzo Pareschi University of Ferrara, Mathematics and Computer Science Department

Kinetic equations play a major role in modeling large systems of interacting particles. Uncertainties may be due to various reasons, like lack of knowledge on the microscopic interaction details or incomplete informations at the boundaries. Recently, these equations found novel applications in socio-economic and life sciences, where processes characterized by large groups of agents exhibit spontaneous emergence of social structures. Due to the lack of fundamental principles, since physical forces are replaced by empirical social forces which are at best known in terms of statistical information, the presence of random inputs characterizing the parameters uncertainty should be considered as an essential feature in the modeling process. These uncertainties, however, contribute to the curse of dimensionality and the development of efficient numerical methods is a challenge. In this paper we consider the construction of novel Monte Carlo collocation methods for such problems which are capable to reduce the variance of standard Monte Carlo collocation techniques.

MODELLING OF DYNAMICAL NETWORKS: FROM MICRO- TO MACRO-DESCRIPTIONS

Diane Peurichard INRIA Paris

In this talk we study the derivation of kinetic and macroscopic models (PDE) from agent-based models for complex dynamical networks of interconnected particles. The agent-based model features particles (2D spheres) having the ability to link/unlink with its close neighbors by creating springs of given equilibrium length. In the limit of large number of particles, we formally obtain a kinetic system of two equations: one for the distribution function of individual particles and one describing the pairs of linked particles. In the large scale limit and under scaling assumptions, we obtain an aggregation diffusion equation which numerically match the limiting behavior of the particle model. The linear and non-linear stability analysis of the homogeneous states of the macroscopic model enable to identify precise criteria linking the aggregative capacity of the model to few key model parameters. This model is then extended to the two-species case and enable the study of cell segregation and border sharpening in biological systems.

CONSERVATION OF ENERGY FOR SYSTEMS OF CONSERVATION LAWS. AROUND ONSAGER'S CONJECTURE.

Agnieszka Świerczewska-Gwiazda University of Warsaw

A common feature of systems of conservation laws of continuum physics is that they are endowed with natural companion laws which are in such case most often related to the second law of thermodynamics. This observation easily generalizes to any symmetrizable system of conservation laws. They are endowed with nontrivial companion conservation laws, which are immediately satisfied by classical solutions. Not surprisingly, weak solutions may fail to satisfy companion laws, which are then often relaxed from equality to inequality and overtake a role of a physical admissibility condition for weak solutions. We want to discuss what is a critical regularity of weak solutions to a general system of conservation laws to satisfy an associated companion law as

an equality. An archetypal example of such result was derived for the incompressible Euler system by Constantin et al. ([1]) in the context of the seminal Onsager's conjecture. This general result can serve as a simple criterion to numerous systems of mathematical physics to prescribe the regularity of solutions needed for an appropriate companion law to be satisfied. The talk is based on papers [1,3,4].

[1] C. Bardos, P. Gwiazda, A. Świerczewska-Gwiazda, E. S. Titi, E. Wiedemann. On the Extension of Onsager's Conjecture for General Conservation Laws , arXiv:1806.02483.

[2] P. Constantin, W. E. and E. S. Titi. Onsager's conjecture on the energy conservation for solutions of Euler's equation. *Comm. Math. Phys.*, 165(1):207–209, 1994.

[3] E. Feireisl, P. Gwiazda, A. Świerczewska-Gwiazda, E. Wiedemann. Regularity and Energy Conservation for the Compressible Euler Equations, *Arch. Ration. Mech. Anal.* 223 (2017), no. 3, 1375–1395

[4] P. Gwiazda, M. Michálek, A. Świerczewska-Gwiazda. A note on weak solutions of conservation laws and energy/entropy conservation, *Arch. Ration. Mech. Anal.*, 229 (2018) 1223-1238

ON A KINETIC ELO RATING MODEL FOR PLAYERS WITH DYNAMICAL STRENGTH

Marco Torregrossa University of Pavia

The ELO rating system evaluates the relative strength level of players or teams in several sports. It was introduced by the American physicist Arnold Elo in 1950 for chess competition and now it is adopted by several sports federations, among them FIFA. The Elo rating system assigns each player an initial rating. After each encounter, the system updates the ratings of each player depending on the outcome of the encounter and on the difference of the ratings of the players. A system is valid if the rating becomes less dependent on the initial value and converges to the player's strength. In 2014, Junca and Jabin proved that the rating converges to the intrinsic strength of the player, that is fixed in time. In this talk, I will present a kinetic model in which rating and strength are both updated after each encounter. I focus on the existence of solutions to the corresponding Fokker-Planck equation and on its behaviour in the longtime limit. This is a joint work with B. Düring and M.-T. Wolfram.

KINETIC MODELS FOR INVERSE PROBLEMS

Giuseppe Visconti RWTH Aachen University

The main goal of this talk is to present a way to link modern ensemble filter methods for solving inverse problems with kinetic theory. To this end, first we propose the mean-field limit of the Ensemble Kalman Filter (EnKF) which is a method to solve parameter estimation problems and we compute the related moment equation. Then, we write the kinetic equation corresponding to the Bayes' formula and we show that the moment equation is equivalent to the one derived from the EnKF when suitable approximations are considered. Finally, we discuss how the regularization of a solution of an inverse problem enters in the kinetic formulation.

ON BOLTZMANN MEAN FIELD GAME MODELS FOR KNOWLEDGE GROWTH

Marie-Therese Wolfram University of Warwick

In this talk we present a Boltzmann mean field game model for knowledge growth, which was proposed by the economists R. Lucas and B. Moll. In this model agents exchange knowledge in binary interactions and at the same time determine their optimal interaction frequency. The corresponding mean-field game system consists of a Boltzmann type equation describing the evolution of agents and a Hamilton-Jacobi-Bellman equation for their optimal strategy. We focus on the existence of special solutions, so called balanced growth path, which relate to exponential growth of the overall economy. The rich dynamics of this system will be illustrated with numerical simulations.

HYDRODYNAMIC MODELS OF COLLECTIVE BEHAVIOR WITH DAMPING AND NONLOCAL INTERACTIONS

Aneta Wróblewska-Kamińska Department of Mathematics, Imperial College London

Hydrodynamic systems for interacting particles where attraction is taken into account by nonlocal forces derived from a potential and repulsion is introduced by local pressure arise in swarming modelling. We focus on the case where there is a balance between nonlocal attraction and local pressure in presence of confinement in the whole space. Under suitable assumptions on the potentials and the pressure functions, we show the global existence of solutions for the compressible Navier-Stokes system with linear damping and nonlocal interaction force. Moreover, we show that global weak solutions converge for large times to the set of these stationary solutions in a suitable sense. In particular cases, we can identify the limiting density uniquely as the global minimizer of the free energy with the right mass and center of mass. This is a joint result with Jose A. Carrillo and Ewelina Zatorska.

STOCHASTIC GALERKIN METHODS FOR KINETIC EQUATIONS OF THE COLLECTIVE BEHAVIOR

Mattia Zanella Politecnico di Torino

Kinetic equations play a major role in the modelling of large systems of interacting particles/agents with a proved effectiveness in describing real world phenomena ranging from plasma physics to socioeconomic dynamics. Their formulation has often to deal with social forces deduced empirically and of which we have at most statistical information (Tosin-Zanella, CMS, to appear). Hence, to produce realistic descriptions of the underlying phenomena it is of paramount importance to consider the presence of random inputs in the form of uncertain parameters as a structural feature of the kinetic models and to develop suitable numerical methods to capture admissible states of the systems.

In this talk we concentrate on stochastic Galerkin methods for the uncertainty quantification of Vlasov-Fokker-Planck (VFP) equations with nonlocal flux. In particular, we develop methods that preserve their large-time solution (Dimarco-Pareschi-Zanella, SEMA-SIMAI, 2017) and we introduce the so-called Monte Carlo generalized polynomial chaos (MCgPC) methods (Carrillo-Pareschi-Zanella, CiCP, to appear). In contrast to a direct application of stochastic Galerkin methods, which are highly accurate but lead to the loss of positivity, the proposed schemes are capable to achieve high accuracy in the random space without losing nonnegativity of the solution.

16

Mathematical Modelling for Complex Systems: Seeking New Frontiers

Organizers:

Mirostaw Lachowicz (University of Warsaw)

Elena De Angelis (DISMA- Politecnico di Torino)

A PERSONALIZED MATHEMATICAL TOOL FOR NEURO-ONCOLOGY

Abramo Agosti Department of Mathematics, Politecnico di Milano

In this talk I will describe a diffuse-interface model based on mixture theory used to model the patient-specific evolution of glioblastoma multiforme. Using thermodynamics principles, a Cahn-Hilliard type equation with degenerate mobility is obtained, in which the spreading dynamics of the multiphase tumour is coupled through a growth term with a parabolic equation describing a nutrient species dynamics.

I will show some analytical results, and I will present a class of finite element approximations which preserve the analytical properties of the continuous solutions.

The model is fed by clinical neuroimaging data that provide the anatomical and microstructural characteristics of a patient brain. I will compare the results of numerical simulations with the clinical data from one patient collected at given times of key clinical interest, from the first diagnosis of a giant glioblastoma to its surgical removal and the subsequent radiation therapies. A good accordance with the data is observed, and the ground-breaking potential of the model for delivering accurate patient-specific predictions is highlighted.

This is a joint work with Pasquale Ciarletta, Chiara Giverso, Maurizio Grasselli.

THE GROUND STATE OF A BOSE-EINSTEIN CONDENSATION ON BRANCHED STRUCTURES

Riccardo Adami Politecnico di Torino

TBA

SOME LIMIT BEHAVIOURS OF DYNAMICAL SYSTEMS ON NETWORKS

Jacek Banasiak University of Pretoria

We consider an ensemble of dynamical systems on edges of a network connected by Kirchhoff type conditions at the nodes. The main result is that if the dynamics on the edges is fast while the redistribution outgoing flow among the edges is slow, the network dynamics on its attractor can be approximated by a system of ordinary differential equations.

CONSERVATION LAWS IN CROWD AND CONSENSUS DYNAMICS

Rinaldo M. Colombo University of Brescia

TBA

CONTROL AND ESTIMATION OF TRAFFIC FLOW USING AUTONOMOUS VEHICLES

Maria Laura Delle Monache Inria

In this talk, we will consider different mathematical frameworks for traffic flow control and estimation. In particular, we will show the evolution of traffic control and estimation from classical strategies (for example ramp-metering) to more modern approaches using autonomous vehicles. We will introduce different ways to describe mathematically the phenomenon by using systems of ODEs, scalar conservation laws and coupled PDE-ODE models, modeling traffic dynamics with a macroscopic approach and presenting different control techniques that could be useful in several real situations. We will show theoretical, numerical and experimental results.

TRAFFIC CONTROL THROUGH AUTONOMOUS VEHICLES

Mauro Garavello University of Milano Bicocca

We introduce a new mathematical model for controlling car traffic through special vehicles, i.e. autonomous and/or connected ones. Car traffic is described by a macroscopic model, while the dynamics of the special vehicles is described by a microscopic model. More precisely, the Lighthill-Whitham-Richards model

$$\partial_t \rho + \partial_x f(\rho) = 0 \quad (16.1)$$

describes the evolution of traffic in a single road. Here $\rho = \rho(t, x)$ denotes the density of traffic at time t and at position x , and $f : \mathbb{R} \rightarrow \mathbb{R}$ is the flux. Assume that a vehicle (or more vehicles), whose position at time t is described by the function $y = y(t)$, aims at controlling the behavior of traffic. The evolution of such a vehicle is described by the ODE

$$\dot{y}(t) = u(t) \quad (16.2)$$

where $u = u(t)$ is a control function, which selects the desired speed. Following the model proposed by Delle Monache and Goatin, we consider the following control system

$$\begin{cases} \partial_t \rho + \partial_x f(\rho) = 0 & t > 0, x \in \mathbb{R} \setminus \{y(t)\} \\ \dot{y}(t) = u(t) & t > 0 \\ \rho(t, y(t)) [v(\rho(t, y(t))) - \dot{y}(t)] \leq F(\dot{y}(t)) & t > 0 \\ \rho(0, x) = \rho_0(x) & x \in \mathbb{R} \\ y(0) = y_0. \end{cases} \quad (16.3)$$

Here, ρ_0 and y_0 are the initial condition, v is the average speed, while the function F in the third equation gives the reduction of the flux due to the presence of the special vehicle.

In the talk, we discuss about the concept of solution for such a system and we show that, given a control function u , a solution exists. The proof is based on the wave-front tracking technique.

This is a joint work with P. Goatin, T. Liard, and B. Piccoli.

A MEAN FIELD KINETIC THEORY MODELLING OF FLUID FLOWS WITH PHASE CHANGE

Livio Gibelli University of Warwick

In spite of the increasing number of experimental, theoretical and numerical studies, many aspects of non-equilibrium fluid flows involving phase change are still unclear with numerous unresolved issues. The talk focuses on their kinetic theory modeling based on the Enskog-Vlasov equation. This equation provides an approximate description of the microscopic behavior of the fluid composed of spherical molecules interacting by Sutherland potential, but it has the capability of handling both the liquid and vapor phase, including the interface region. As such, it is a useful bridge between the continuum approaches which fail to properly deal with the complexities of interfacial phenomena and the Molecular Dynamics (MD) simulations which require a huge computational effort. The evaporation of a single component liquid into near vacuum conditions is discussed as example of application.

SOLID - LIQUID PHASE TRANSITION IN FLOWS

Maria Gokieli ICM, University of Warsaw

We consider a flow in a domain depending upon a phase transition occurring inside it. Examples are the water - ice transition, the formation of biofilm on the boundaries of the domain or the gasification of biomass. In any case, we have a coupling between the flow and some other process and a moving interface between phases. We present an approach based on variational inequalities so as to model such a situation and present first mathematical results.

UNIQUENESS RESULTS FOR CONSERVATION LAWS WITH DISCONTINUOUS FLUXES

Graziano Guerra Università degli Studi di Milano-Bicocca

Scalar conservation laws $\partial_t u + \partial_x f(t, x, u) = 0$ with the flux f discontinuous with respect to the time-space variables (t, x) arise in many applications where the conservation laws describe physical models in rough media. For example, traffic flows with rough road conditions and polymer flooding models in porous media. An extensive body of recent literature has dealt with fluxes which are discontinuous over a finite number of curves in the (t, x) plane. We are interested in the existence and uniqueness of solutions obtained via vanishing viscosity approximations i.e. solutions to $\partial_t u + \partial_x f(t, x, u) = \varepsilon \partial_{xx} u$ when $\varepsilon \rightarrow 0$ for highly discontinuous (regulated) fluxes.

We give a definition of regulated functions in two variables which generalizes the corresponding definition for functions of one variable. Then we show the existence and uniqueness of the vanishing viscosity limit for regulated fluxes. The result is obtained using three main arguments: comparison results related to viscous Hamilton-Jacobi equations in one space variable, a known result for fluxes with a single discontinuity at the origin and a compensated compactness argument.

The results of this talk are obtained in collaboration with Alberto Bressan and Wen Shen

INFINITE POPULATIONS OF INTERACTING ENTITIES AS COMPLEX SYSTEMS: MULTISCALE MARKOV DYNAMICS

Jurij Kozicki Maria Curie-Skłodowska University, Institute of Mathematics

In many applications, one deals with systems of entities characterized by a trait x belonging to a topological space X . It is believed that a given entity with trait x interacts mostly (even entirely) with those entities whose traits belong to a neighborhood of x . Such interactions form the local structure of the system. The main aim of the theory is to explain how the local structure determines the global behavior of the whole infinite system. In particular, this applies to the Markov evolution of the system's states defined as probability measures on the corresponding configuration spaces. In view of the mentioned complexity, this evolution is considered in different scales that opens the possibility to get a deeper insight into its properties. This approach will be demonstrated in a number of models.

ANALYSIS OF DEEP-WATER WAVES

Henryk Leszczyński University of Gdańsk

The velocity potential satisfies the Laplace equation with nonlocal boundary conditions on a free surface. The Fourier transform is applied to the differential problem. The Fourier transform images of boundary functions are approximated by Picard's iterations and the method of lines. This is a joint work with Jan Jankowski (Polish Register of Shipping).

STOCHASTIC DYNAMICS IN EVOLUTIONARY GAMES INVOLVING SOCIAL DILEMMAS

Jacek Miękiś University of Warsaw

One of the fundamental problems in evolutionary biology is to understand mechanisms promoting altruistic behaviour in animal and human populations. We address it within three simple models of evolutionary game theory.

We construct various Markov processes of evolving populations of individuals who may choose to cooperate or to defect. We show that stationary states of these processes depend on details of models, in particular on methods of sampling players.

In the Stag-hunt game, we are concerned with fixing probabilities of two absorbing cooperation and defection states. We formulate two simple and intuitive criteria for evolutionary stability of pure Nash equilibria in finite populations. In particular, we show that the 1/3 law of evolutionary games follows from a more general mean-potential law [1].

In the Snowdrift game, we show that a method of matching individuals have a dramatic influence on the long-run behavior of evolving populations. In particular, we show that lattice structure is always beneficial to cooperation as compared to random-matching models of well-mixed populations [2].

In the Prisoners Dilemma game played on the random scale-free Barabási-Albert graphs, we introduce a cost of maintaining a link between interacting players. We show that when the cost increases, the population of players undergoes a sharp transition from an ordered state, where almost all players cooperate, to a state in which both cooperators and defectors coexist. At the critical cost, the population oscillates in time between these two states [3].

References

1. P. Nałęcz-Jawecki and J. Miękiś, Mean-potential law in evolutionary games, *Phys. Rev. Lett.* 120: 028101 (2018).
2. C. Hauert and J. Miękiś, Interaction and competition in evolutionary games, submitted (2018).
3. J. Miękiś, J. Łącki, and B. Sułkowski, Phase transitions in the Prisoners Dilemma game on scale-free networks, preprint (2017).

SHAPLEY VALUE REDISTRIBUTION OF SOCIAL WEALTH FOSTERS COOPERATION IN SOCIAL DILEMMAS

Tadeusz Płatkowski University of Warsaw

We consider multiplayer social dilemma games played in a large population. The members of the population interact in randomly formed coalitions. Each coalition generates a social wealth (value), which is distributed among the coalition members according to their Shapley values. Evolution of the whole population is governed by the replicator equation. We demonstrate that application of the Shapley value fosters the time asymptotic cooperation in populations for various types of multiplayer social dilemmas.

ON THE LONG TIME BEHAVIOR FOR THE FORMATION OF SWARMS

Martin Parisot Inria Paris, project-team ANGE

This work deals with the modeling of formation and destruction of swarms using a nonlinear Boltzmann-like equation. The model contains parameters characterizing the attractiveness or repulsiveness of individuals, leading to gregarious and solitary behaviors. A mathematical analysis of the long time existence is realized in the space homogeneous case and we identify relevant hydrodynamic limits on a formal way for both gregarious and solitary behaviors. On the numerical point of view, several strategies of the collision operator (agent interactions) are proposed that have to be adapted in the case of global existence or not. The need for an anti-diffusive scheme for the space transport is also highlighted thanks to numerical illustrations.

MATHEMATICAL MODELS FOR PEDESTRIAN DYNAMICS

Marie-Therese Wolfram University of Warwick

In this talk we discuss different modeling approaches for pedestrian dynamics, starting on the microscopic level before continuing with meso- and macroscopic approaches. In particular we focus on the formal derivation of PDE models from cellular automata and show how simple interaction rules can initiate segregation. For example, side stepping may lead to lane formation in bidirectional flows. We analyse the corresponding PDE system, and illustrate the dynamics with computational experiments.

17

Advances in Nonlinear Elliptic and Parabolic PDEs: from local to nonlocal problems

Organizers:

Miłosz Krupski (Uniwersytet Wrocławski)

Bruno Volzone (Università degli studi di Napoli "Parthenope")

Matteo Bonforte (Universidad Autónoma de Madrid)

Piotr Biler (University of Wrocław)

FRACTIONAL DIFFUSION LIMIT OF A LINEAR KINETIC TRANSPORT EQUATION IN A BOUNDED DOMAIN

Pedro Aceves-Sanchez Imperial College

In recent years, the study of evolution equations featuring a fractional Laplacian has received many attention due the fact that they have been successfully applied into the modelling of a wide variety of phenomena, ranging from biology, physics to finance. The stochastic process behind fractional operators is linked, in the whole space, to an α -stable processes as opposed to the Laplacian operator which is linked to a Brownian stochastic process.

In addition, evolution equations involving fractional Laplacians offer new interesting and very challenging mathematical problems. There are several equivalent definitions of the fractional Laplacian in the whole domain, however, in a bounded domain there are several options depending on the stochastic process considered.

In this talk we shall present results on the rigorous passage from a velocity jumping stochastic process in a bounded domain to a macroscopic evolution equation featuring a fractional Laplace operator. More precisely, we shall consider the long-time/small mean-free path asymptotic behaviour of the solutions of a re-scaled linear kinetic transport equation in a smooth bounded domain.

MAXIMUM PRINCIPLES FOR SOME PARABOLIC EQUATIONS

Mohammad Almahameed Irbid National University

We introduce maximum principles for some uniformly parabolic operators with some applications.

PERIODIC SOLUTIONS FOR THE ONE-DIMENSIONAL FRACTIONAL LAPLACIAN

Begoña Barrios Barrera Universidad de La Laguna

Along this talk we establish the existence of periodic solutions of the nonlocal problem $(-\Delta)^s u = f(u)$ in \mathbb{R} , where $(-\Delta)^s$ stands for the s -Laplacian, $s \in (0, 1)$. We introduce a suitable framework which allows linking the searching of such solutions into the existence of the ones of a semilinear problem in a suitable Hilbert space. Then with the usual tools of nonlinear analysis, we get the existence theorems which are lately enlightened with the analysis of some examples like the Benjamin-Ono equation.

BEHAVIOR OF NONLOCAL MINIMAL SURFACES FOR SMALL VALUES OF THE FRACTIONAL PARAMETER

Claudia Bucur University of Melbourne

Nonlocal minimal surfaces are defined as boundary of sets that minimize the fractional perimeter in a bounded open set $\Omega \subset \mathbb{R}^n$ among all sets with fixed data outside of Ω . It is known that if the fixed exterior data is a half-space, the only s -minimal set is the half-space. On the other hand, if one removes (even from far away) some small set from the half-space, for s small enough the s -minimal set completely sticks to the boundary, that is, the s -minimal set is empty inside Ω . Starting from this example, in this talk we present the behavior of s -minimal surfaces when the fractional parameter is small. We classify the behavior of s -minimal surfaces with respect to the fixed exterior data. So, for s small and depending on the data at “infinity”, the s -minimal set can be either empty in Ω , fill all Ω , or possibly develop a wildly oscillating boundary.

FRACTIONAL PATLAK-KELLER-SEGEL SYSTEM

Jan Burczak University of Oxford/Polish Academy of Sciences

Consider the following fractional generalization of the classical Patlak-Keller-Segel equation

$$u_t + (-\Delta)^{\frac{\alpha}{2}} u = -\chi \operatorname{div}(u \nabla (-\Delta)^{-1}(u - \langle u \rangle)) + f(u),$$

motivated by “Levy flight foraging hypothesis”, widely present in biology.

I will discuss a series of regularity results, obtained in collaboration with Rafael Granero-Belinchón, including
 (i) in case of “critical” dissipation: A disproof of a finite-time blowup conjecture for $f(s) \equiv 0$, $\alpha = 1$ on a circle
 (ii) in case of a weaker than “critical” dissipation, but with a damping: classical solvability and boundedness of solutions for $f(s) \equiv rs(1 - s)$ on d -dimensional torus

ANCIENT SOLUTIONS OF SUPERLINEAR HEAT EQUATIONS ON RIEMANNIAN MANIFOLDS

Daniele Castorina John Cabot University (Rome, Italy)

We study the qualitative properties of ancient solutions of superlinear heat equations in a Riemannian manifold, with particular attention to positivity and triviality in space. This is joint work with Carlo Mantegazza (Napoli Federico II)

THEORETICAL AND NUMERICAL ASPECTS FOR NONLOCAL (AND LOCAL) EQUATIONS OF POROUS MEDIUM TYPE

Félix del Teso Basque Center for Applied Mathematics

We consider the following problem of porous medium type:

$$\begin{cases} \partial_t u(x,t) - (L^\sigma + \mathcal{L}^\mu)[\varphi(u)](x,t) = f(x,t), & (x,t) \in \mathbb{R}^N \times (0, \infty), \\ u(x,t) = u_0(x), & x \in \mathbb{R}^N, \end{cases} \quad (17.1)$$

where $\varphi : \mathbb{R}^N \rightarrow \mathbb{R}$ is continuous and nondecreasing, and

$$\begin{aligned} L^\sigma[v](x) &= \operatorname{Tr}(\sigma \sigma^T D^2 v(x)), & \sigma &\in \mathbb{R}^{N \times p}, \quad p \in \mathbb{N}, \\ \mathcal{L}^\mu[v](x) &= \text{P.V.} \int_{|z|>0} (v(x+z) - v(x)) d\mu(z), \\ \mu &\equiv \text{Symmetric Radon measure s.t } \int \min\{|z|^2, 1\} d\mu(z) < +\infty, \text{ (nonlocal diffusion)} \end{aligned} \quad (17.2)$$

We will present a general overview of some of the results obtained in collaboration with J. Endal and E.R. Jakobsen:

- Uniqueness of distributional solutions.
- Continuous dependence on $L^\sigma + \mathcal{L}^\mu$, φ and u_0 .
- Unified theory of monotone numerical schemes of finite difference type. Here we use the fact that operators in the class of discretizations of $L^\sigma + \mathcal{L}^\mu$. This fact allows us to use a pure PDE approach.
- We also propose a branch of discretizations and schemes and analyze their accuracy.
- As a consequence of numerics, we obtain existence of distributional solutions together with interesting properties like L^1 - $C([0, T], L^1_{\text{loc}}(\mathbb{R}^N))$ regularity, energy estimates, ...

(17.3)

ON HIGHER DIMENSIONAL SINGULARITIES FOR THE FRACTIONAL YAMABE PROBLEM.

Azahara DelaTorre Freiburg University

We consider the problem of constructing solutions to the fractional Yamabe problem that are singular at a given smooth sub-manifold, for which we establish the classical gluing method of Mazzeo and Pacard for the scalar curvature in the fractional setting. This proof is based on the analysis of the model linearized operator, which amounts to the study of a fractional order ODE, and thus our main contribution here is the development of new methods coming from conformal geometry and scattering theory for the study of non-local ODEs. Note, however, that no traditional phase-plane analysis is available here. Instead, first, we provide a rigorous construction of radial fast-decaying solutions by a blow-up argument and a bifurcation method. Second, we use conformal geometry to rewrite this non-local ODE, giving a hint of what a non-local phase-plane analysis should be. Third, for the linear theory, we use complex analysis and some non-Euclidean harmonic analysis to examine a fractional Schrödinger equation with a Hardy type critical potential. We construct its Green's function, deduce Fredholm properties, and analyze its asymptotics at the singular points in the spirit of Frobenius method. Surprisingly enough, a fractional linear ODE may still have a two-dimensional kernel as in the second order case.

This is a work done in collaboration with Weiwei Ao, Hardy Chan, Marco Fontelos, Manuel del Pino, María del Mar González and Juncheng Wei.

LONG-TIME ASYMPTOTICS FOR NONLOCAL POROUS MEDIUM EQUATION WITH ABSORPTION OR CONVECTION

Filomena Feo University of Naples "Parthenope"

A large variety of models for conserved quantities in continuum mechanics or physics are described by the continuity equation $u_\tau + \nabla \cdot (u\mathbf{v}) = 0$, where the density distribution $u(y, \tau)$ evolves in time τ following a velocity field $\mathbf{v}(y, \tau)$. According to Darcy's law, the velocity \mathbf{v} is usually derived from a potential p in the form $\mathbf{v} = -\mathcal{D}\nabla p$ for some tensor \mathcal{D} . In porous media, the power-law relation $p = u^m$ is commonly proposed. Although local constitutive relations like $p = u^m$ were successful in numerous practical models, there are situations where the

potential (or pressure) p depends non-locally on the density distribution u . The simplest prototypical example is $p = (-\Delta)^{-s}u$, expressed as the Riesz potential of u . The resulting evolution equation then becomes

$$u_\tau - \nabla \cdot (u \nabla (-\Delta)^{-s}u) = 0,$$

and basic questions like existence, uniqueness and regularity of solutions have been studied thoroughly in papers by L. A. Caffarelli and J.L. Vázquez. While in general it is difficult to obtain quantitative properties of solutions to non-local nonlinear equations, this equation possesses special features that enable one to study the long term behaviours in terms of its self-similar solution. Using similarity variables motivated from the scaling relations, the transformed equation has an entropy function so that the convergence towards the self-similar profile in one dimension can be established by the well-known entropy method.

In this talk, we consider two variants of this equation with an absorption term or a convection term.

This talk is based on a joint work with Y. Huang and B. Volzone.

NONLINEAR DIFFUSION ON NEGATIVELY CURVED MANIFOLDS

Gabriele Grillo Politecnico di Milano

We report on some recent results on the asymptotic behaviour of solutions to the porous medium equation on negatively curved manifolds. The results depend crucially on the growth assumptions on curvature at infinity. In fact if curvature is superquadratic a surprising connection with an associated elliptic problem, which is shown to have a solution in that case, arise.

THE LIOUVILLE THEOREM AND LINEAR OPERATORS SATISFYING THE MAX PRINCIPLE: CLASSIFICATION IN THE CONSTANT COEFFICIENT CASE.

Espen R. Jakobsen Norwegian University of Science and Technology NTNU

I will present a classification of linear operators \mathcal{L} satisfying the Liouville theorem: Bounded solutions of $\mathcal{L}u = 0$ are constant. Our results give necessary and sufficient conditions for all the generators of Levy processes, or in other words, the constant coefficient linear operators satisfying the max principle. Some examples of such generators are the Laplace, fractional Laplace, and discrete finite differences operators. The main novelty is the inclusion of the nonlocal part of such operators. Our proofs are short and natural and differs from most proofs e.g. for the fractional Laplacian.

This is joint work with Nathael Alibaud (Besancon, France), Felix del Teso, and Jørgen Endal (both NTNU, Norway)

ON A NONLINEAR NONLOCAL DIFFUSION EQUATION

Grzegorz Karch Uniwersytet Wrocławski

In the talk, I will describe an abstract framework for non-local non-linear diffusion, by which we mean a phenomenon with properties strongly associated to diffusive processes such as the conservation of mass, the maximum principle, and the comparison principle. This framework encompasses some of the known examples of equations like the fractional porous medium equation or the equation with the fractional p -Laplacian, but it also opens up the space for new examples to be constructed and studied.

RENORMALIZED SOLUTIONS OF NONLOCAL SEMILINEAR EQUATIONS WITH MEASURE DATA

Tomasz Klimsiak Nicolaus Copernicus University

We generalize the notion of renormalized solution to semilinear elliptic and parabolic equations involving operator associated with general (possibly nonlocal) regular Dirichlet form and smooth measure on the right-hand side. We show that under mild integrability assumption on the data a quasi-continuous (quasi-càdlàg) function u is a renormalized solution to an elliptic (or parabolic) equation iff u is its probabilistic solution, i.e. u can be represented by a suitable nonlinear Feynman-Kac formula. Next we present the existence results for a broad class of local and nonlocal semilinear equations.

FROM STOCHASTIC COMPLETENESS TO NONLINEAR DIFFUSIONS AND BACK

Matteo Muratori Politecnico di Milano

We prove that the conservation of mass for the heat semigroup on a complete Riemannian manifold M (namely stochastic completeness), hence a linear property, is equivalent to uniqueness for nonlinear evolution equations of fast diffusion type on M , in the class of nonnegative bounded solutions. This connection was well known only in the linear framework, that is for the heat equation itself. More precisely, we consider equations of the form $u_t = \Delta\phi(u)$, where ϕ is an arbitrary nonnegative, concave, increasing function, regular outside the origin and satisfying $\phi(0) = 0$. We stress that either property is also shown to be equivalent to nonexistence of nontrivial, nonnegative bounded solutions to the semilinear elliptic equation $\Delta W = \phi^{-1}(W)$, with the same ϕ as above. As a consequence, we provide sharp criteria for uniqueness or nonuniqueness of nonnegative bounded solutions to fast diffusion-type equations and existence or nonexistence of nontrivial, nonnegative bounded solutions to the associated semilinear elliptic equations on M , which seem to be completely new in the literature. In particular, our results show that there is a wide class of manifolds where uniqueness of bounded solutions to the fast diffusion equation $u_t = \Delta u^m$, with $m \in (0, 1)$, fails. This is in clear contrast with the Euclidean setting, in which uniqueness does hold for merely L^1_{loc} solutions due to the seminal paper by Herrero and Pierre.

This is a joint work with G. Grillo and K. Ishige.

STABILITY OF THE LOG-SOBOLEV INEQUALITY FOR THE GAUSSIAN MEASURE

Maria Rosaria Posteraro Università di napoli federico II

We study the deficit in the logarithmic Sobolev Inequality and in the Talagrand transport-entropy Inequality for the Gaussian measure, in any dimension. We obtain a sharp lower bound using a distance introduced by Bucur and Fragalà. Thereafter, we investigate the stability issue with tools from Fourier analysis.

HEAT EQUATIONS FOR ANISOTROPIC NONLOCAL OPERATORS WITH SINGULAR FORCING

Fernando Quiros Universidad Autonoma de Madrid

We prove existence, uniqueness and regularity for bounded weak solutions of nonlocal heat equations associated to stable diffusion operators. The main features are that the right-hand side has very few regularity and that the spectral measure can be singular in some directions.

Joint work with Arturo de Pablo and Ana Rodriguez.

FUNCTIONAL FRAMEWORK AND TOPOLOGICAL DEFECTS IN NEMATIC SHELLS

Antonio Segatti Dipartimento di Matematica, Università di Pavia, Italy

In this talk I will report on a series of joint results obtained in collaboration with Giacomo Canevari (Bilbao) and Marco Veneroni (Pavia) on nematic liquid crystals smeared on curved substrates (nematic shells). This structures offer an interesting playground where modelling, Analysis of PDEs, Calculus of Variations, Topology and Geometry meet. In this talk I will first discuss how the topology of the shell influences the choice of a proper functional setting of the problem. Secondly, I will discuss how the defects emerge and I will present their energetics.

HARNACK AT LARGE FOR DEGENERATE/SINGULAR OPERATORS

Vincenzo Vespri University of Florence

In his celebrated paper “A Harnack Inequality for Parabolic Differential Equations, Comm. Pure Appl. Math., 17, (1964), 101 - 134” Moser, in Theorem 1, extended the Harnack inequality to linear parabolic operators with elliptic and measurable coefficients. Moser focussed his attention on pointwise estimates of the solutions. More specifically, in Theorem 2 and estimate (1.7), he proved that there exists a positive constant c such that, for any x and y in \mathbb{R}^N , for any $0 < s < t < T$ and for any nonnegative solution u of

$$\partial_t u = \sum_{i,j=1}^N D_i(a_{ij}(x,t)D_j u)$$

in $\mathbb{R}^N \times (0, \infty)$, we have

$$u(x, t) \geq u(y, s) \left(\frac{s}{t}\right)^c e^{-c\left(1 + \frac{|x-y|^2}{t-s}\right)}.$$

Let us remark that these estimates, even if not optimal especially in time variable, give the idea of the strong connection between Harnack estimates and the well known exponential behavior of the fundamental solution.

Moser proved this estimate by using a technique called Harnack chain which consists in iterating the Harnack estimates. However, it is known that this technique produces non optimal estimates. By using Nash techniques, many authors proved sharp estimates from above and from below for linear operators in different settings, cf. Li and Yau, Auscher and Coulhon, Grigor'yan and Telcs and Saloff-Coste. In "A new proof of Moser's parabolic Harnack inequality via the old ideas of Nash Arch. Rat. Mech. Anal. 96 (1986), 327–338", Fabes and Stroock proved that the Gaussian estimate are equivalent to a parabolic Harnack inequality.

In his book, "Aspects of Sobolev-type inequalities (1992)", Saloff-Coste gave a proof of Moser's estimate with the right coefficients.

In this talk we investigate the connection between the fundamental solution estimates and the parabolic Harnack inequality also in the case of quasilinear degenerate/singular parabolic operators.

THE ABILITY TO MOVE GIVES BENEFITS IN A TWO SPECIES COMPETITION MODEL WITH NUTRIENT TAXIS.

Dariusz Wrzosek University of Warsaw

In a joint work with Piotr Krzyżanowski (Warsaw) and Michael Winkler (Paderborn) a model describing the competition of two species for a common nutrient is studied. It is assumed that one of the competitors is motionless while the other has the ability to move upwards gradients of the nutrient density. It is proved that under suitable assumptions on the initial data, in the long time perspective the ability to move turns out to be a crucial feature providing competitive advantage irrespectively of a possible difference between the species with regard to their rates of nutrient uptake and proliferation.

18

Nonlinear Partial Differential Equations and Related Function Spaces

Organizers:

Iwona Skrzypczak (IMPAN)

Angela Alberico (National Research Council)

ON THE LARGE TIME BEHAVIOUR AND BLOW-UP OF SOLUTIONS FOR SOME SYSTEMS OF FRACTIONAL EVOLUTION EQUATIONS

Mohamed Berbiche University of Mohamed Khider Biskra Algeria

We consider the Cauchy problem for a coupled semi-linear system of fractional wave equations with polynomial nonlinearity in multi-dimensional space R^N . We see under some conditions on the exponents, the order of time fractional derivative and on the dimension, there is a critical value in which small data global existence, and a finite time blow-up of solution are separated. The asymptotic behavior of solution is also discussed.

NONLINEAR AND NONLOCAL DEGENERATE DIFFUSIONS ON BOUNDED DOMAINS

Matteo Bonforte Universidad Autonoma de Madrid, Spain

We investigate quantitative properties of nonnegative solutions $u(t, x) \geq 0$ to the nonlinear fractional diffusion equation, $\partial_t u + \mathcal{L}F(u) = 0$ posed in a bounded domain, $x \in \Omega \subset R^N$, with appropriate homogeneous Dirichlet boundary conditions. As \mathcal{L} we can use a quite general class of linear operators that includes the three most common versions of the fractional Laplacian $(-\Delta)^s$ with $s \in (0, 1)$, in a bounded domain with zero Dirichlet boundary conditions; many other examples are included. The nonlinearity F is assumed to be increasing and is allowed to be degenerate, the prototype being $F(u) = |u|^{m-1}u$, with $m > 1$.

We will shortly present some recent results about existence, uniqueness and a priori estimates for a quite large class of very weak solutions, that we call weak dual solutions.

We will devote special attention to the regularity theory: decay and positivity, boundary behavior, Harnack inequalities, interior and boundary regularity, and asymptotic behavior. All this is done in a quantitative way, based on sharp a priori estimates. Although our focus is on the fractional models, our techniques cover also the local case $s = 1$ and provide new results even in this setting.

A surprising instance of this problem is the possible presence of nonmatching powers for the boundary behavior: for instance, when $\mathcal{L} = (-\Delta)^s$ is a spectral power of the Dirichlet Laplacian inside a smooth domain, we can prove that, whenever $2s \geq 1 - 1/m$, solutions behave as $dist^{1/m}$ near the boundary; on the other hand, when $2s < 1 - 1/m$, different solutions may exhibit different boundary behaviors even for large times. This unexpected phenomenon is a completely new feature of the nonlocal nonlinear structure of this model, and it is not present in the semilinear elliptic case, for which we will shortly present the most recent results. The above results are contained on a series of recent papers in collaboration with A. Figalli, Y. Sire, X. Ros-Oton and J. L. Vazquez.

AN INVERSE SPECTRAL PROBLEM

Barbara Brandolini Università degli Studi di Napoli Federico II

Let Ω be a convex, possibly unbounded, domain in \mathbb{R}^2 and denote by $\mu_1(\Omega)$ the first nontrivial Neumann eigenvalue of the Hermite operator in Ω . It is known that $\mu_1(\Omega) \geq 1$. The inequality is sharp since equality sign holds if Ω is any strip. Moreover it can be read as an optimal Poincaré-Wirtinger inequality for functions belonging to the weighted Sobolev space $H^1(\Omega, \gamma)$, where γ is the 2-dimensional Gaussian measure. We study the equality case and we show that $\mu_1(\Omega) = 1$ if and only if Ω is any strip.

FULLY ANISOTROPIC ELLIPTIC EQUATIONS WITH MERELY INTEGRABLE, OR MEASURE DATA

Andrea Cianchi Università di Firenze

Dirichlet problems for nonlinear elliptic equations whose growth is governed by a general convex function of the gradient are discussed. This convex function does not necessarily depend on the gradient just through its modulus, nor have a power type growth. Moreover, it is not required to satisfy the so called Δ_2 or ∇_2 conditions in the theory of (anisotropic) Orlicz spaces, the natural function space framework for this kind of problems. A minimal integrability assumption on the right-hand side of the equation ensuring existence and uniqueness of a weak solution is exhibited. The existence and optimal regularity of a suitably defined solution via approximations are then established when the right-hand side is just a measure. Uniqueness of this kind of solutions is also proved if the right-hand side is an integrable function. This is a joint work with Angela Alberico, Iwona Chlebicka (Skrzypczak) and Anna Zatorska-Goldstein.

SHARP INTEGRABILITY RESULTS FOR SOLUTIONS TO EQUATIONS OF N-LAPLACIAN TYPE

Luigi D'Onofrio Università degli Studi di Napoli "Parthenope"

I will present a joint work with Paolo Baroni, Roberta Schiattarella and Gabriella Zecca. We prove optimal gradient integrability results of Calderon-Zygmund type for equations modeled on the n -Laplacian, in the scales of Orlicz and rearrangement invariant spaces. We allow for measurable coefficients and we both inquire the case of energy solutions and of very weak solutions.

ENERGY CONSERVATION FOR THE COMPRESSIBLE EULER EQUATIONS WITH VACUUM

Tomasz Dębiec University of Warsaw

Differential equations modelling the evolution of a physical system usually come with one or several conserved quantities, like the energy. However, when the solution is sufficiently irregular, the additional conservation law may not be satisfied, and the question arises precisely what regularity is needed to guarantee conservation.

We will consider the compressible isentropic Euler equations on $\mathbb{T}^d \times [0, T]$. The pressure, p , is a given function of the density, and in many practical applications is of the form $p(\rho) = \kappa \rho^\gamma$ with $\gamma \in [1, 2)$. It has been shown (by Feireisl, Gwiazda, Świerczewska-Gwiazda and Wiedemann) that if a weak solution belongs to an appropriate Besov space, then a local energy equation is satisfied, provided one has $p \in C^2$. This means that in the physically relevant case of the polytropic pressure law one has to exclude vacuum states.

We present an extension of the said result. We will discuss under what additional assumptions can one relax the C^2 assumption on the pressure. Finally, we extend our result to the case when the equations are posed on $\Omega \times [0, T]$ where Ω is a (smooth) bounded domain.

This talk is based on a joint work with I. Akramov, J. Skipper and E. Wiedemann (LU Hannover).

REGULARITY RESULTS FOR MINIMIZERS OF ASYMPTOTICALLY CONVEX INTEGRALS WITH DISCONTINUOUS COEFFICIENTS

Flavia Giannetti Università degli Studi di Napoli "Federico II"

I will talk about some regularity results for the minimizers of integral functionals whose integrands are uniformly convex and have a radial structure with respect to the gradient variable only at infinity. Concerning the dependence on the variable x , we assume a Sobolev type regularity.

PARABOLIC EQUATIONS IN TIME AND SPACE DEPENDENT ANISOTROPIC MUSIELAK-ORLICZ SPACES

Piotr Gwiazda Institute of Mathematics, Polish Academy of Sciences

The talk will concentrate on general nonlinear parabolic equation on a Lipschitz bounded domain in R^n ,

$$\begin{cases} \partial_t u - \operatorname{div} A(t, x, \nabla u) = f(t, x) & \text{in } (0, T) \times \Omega, \\ u(t, x) = 0 & \text{on } (0, T) \times \partial\Omega, \\ u(0, x) = u_0(x) & \text{in } \Omega, \end{cases}$$

with $f \in L^\infty((0, T) \times \Omega)$ and $u_0 \in L^\infty(\Omega)$. The growth of the monotone vector field A is controlled by a generalized fully anisotropic N -function $M : [0, T) \times \Omega \times R^n \rightarrow [0, \infty)$ inhomogeneous in time and space, and under no growth restrictions on the last variable. It results in the need of the integration by parts formula which has to be formulated in an advanced way. Existence and uniqueness of solutions are proven when the Musielak-Orlicz space is reflexive OR in absence of Lavrentiev's phenomenon. To ensure approximation properties of the space we impose natural assumption that the asymptotic behaviour of the modular function is sufficiently balanced. Its instances are log-Hölder continuity of variable exponent or optimal closeness condition for powers in double phase spaces.

The noticeable challenge of these considerations is changing along time of a non-reflexive and inhomogeneous fully anisotropic space. The presented results come from the following papers:

- [1] Youssef Ahmida, Piotr Gwiazda, Iwona Skrzypczak, Ahmed Youssfi. Gossez's approximation theorems in the Musielak-Orlicz-Sobolev spaces, arXiv:1711.06145, to appear in J. Funct. Anal.
 - [2] Piotr Gwiazda, Iwona Skrzypczak, Anna Zatorska-Goldstein. Well-posedness of parabolic equations in the non-reflexive and anisotropic Musielak-Orlicz spaces in the class of renormalized solutions arXiv:1707.06097, to appear in J. Differential Equations
 - [3] Piotr Gwiazda, Iwona Skrzypczak, Anna Zatorska-Goldstein. Parabolic equation in time and space dependent anisotropic Musielak-Orlicz spaces in absence of Lavrentiev's phenomenon, arXiv:1806.06711
 - [4] Piotr Gwiazda, Iwona Skrzypczak, Anna Zatorska-Goldstein. Renormalized solutions to parabolic equations in time and space dependent anisotropic Musielak-Orlicz spaces in absence of Lavrentiev's phenomenon, arXiv:1807.06464
-

RAYLEIGH-BÉNARD PROBLEM FOR MICROPOLAR FLUIDS - THE CASE OF LARGE AND INFINITE PRANDTL NUMBER

Piotr Kalita Jagiellonian University

We study the Rayleigh-Bénard convection for micropolar and incompressible fluids. We present two results: we demonstrate that if the Prandtl number is large, then, although the weak solution can be possibly nonunique, after some time, given uniformly with respect to L^2 bounded sets of initial data, the solutions become uniformly H^1 bounded. This fact, together with weak-strong uniqueness, implies the existence of the global attractor, invariant and compact in H^1 on which the dynamics is single valued. The second result we discuss concerns the infinite Prandtl number case. Here, we prove that the upper bound on the Nusselt number is given by the cube root of the Rayleigh number multiplied by the constant which is a monotone decreasing function of the micropolar damping and microrotation viscosity. Thus we demonstrate the stabilizing influence of micropolar effects for the infinite Prandtl number case. This is joint work with Grzegorz Łukaszewicz, Jose A. Langa, and Matteo Caggio.

OPTIMAL DOMAIN SPACES IN ORLICZ-SOBOLEV EMBEDDINGS

Vít Musil Charles University

We deal with Orlicz-Sobolev embeddings in open subsets of \mathbb{R}^n . A necessary and sufficient condition is established for the existence of an optimal, i.e. largest possible, Orlicz-Sobolev space continuously embedded into a given Orlicz space. Moreover, the optimal Orlicz-Sobolev space is exhibited whenever it exists. Parallel questions are addressed for Orlicz-Sobolev embeddings into Orlicz spaces with respect to a Frostman measure, and, in particular, for trace embeddings on the boundary.

BOUNDEDNESS OF CLASSICAL OPERATORS ON REARRANGEMENT-INVARIANT SPACES

Luboš Pick Charles University, Prague

We study the action of classical operators of interest in harmonic analysis such as the Hardy-Littlewood maximal operator (including the fractional version), Hilbert or Stieltjes transform, Laplace transform or Riesz potential on rearrangement-invariant function spaces. We focus on the sharpness of the spaces. We present characterizations of the optimal domain or range partner spaces. We use duality techniques and reduction principles to Hardy and supremum operators.

FAST DIFFUSION EQUATIONS WITH CAFFARELLI-KOHN-NIRENBERG WEIGHTS: HARNACK INEQUALITIES AND HOLDER CONTINUITY

Nikita Simonov Universidad Autonoma de Madrid

We study a priori estimates for a class of non-negative local weak solution to the weighted fast diffusion equation $u_t = |x|^\gamma \nabla \cdot (|x|^{-\beta} \nabla u^m)$, with $0 < m < 1$ posed on cylinders of $(0, T) \times \mathbb{R}^N$. The weights $|x|^\gamma$ and $|x|^{-\beta}$, with $\gamma < N$ and $\gamma - 2 < \beta \leq \gamma(N - 2)/N$ can be both degenerate and singular and need not to belong to the Mouckenhaupt class \mathcal{A}_2 , a typical assumption for these kind of problems. This range of parameters is optimal for the validity of a class of Caffarelli-Kohn-Nirenberg inequalities, which play the role of the standard Sobolev inequalities in this more complicated weighted setting.

The weights that we consider are not translation invariant and this causes a number of extra difficulties: for instance, the scaling properties of the equation change when considering the problem around the origin or far from it. We will present quantitative upper and lower estimates for local weak solutions, focussing our attention where a change of geometry appears. Such estimates fairly combine into forms of Harnack inequalities of forward, backward and elliptic type. As a consequence, we obtain Holder continuity of the solutions.

This is a joint work with M. Bonforte (UAM, Spain).

Quantitative a Priori Estimates for Fast Diffusion Equations with Caffarelli-Kohn-Nirenberg weights. Harnack inequalities and Holder continuity <http://arxiv.org/abs/1804.03537>

NON-NEWTONIAN FLUIDS AND ABSTRACT PROBLEMS: APPLICATIONS OF ORLICZ SPACES IN THE THEORY OF NONLINEAR PDE

Aneta Wróblewska-Kamińska Department of Mathematics, Imperial College
London

We are interested in the existence of solutions to strongly nonlinear partial differential equations. We concentrate mainly on problems which come from dynamics of non-Newtonian fluids of a nonstandard rheology, more general than of power-law type, and also on some abstract theory of elliptic and parabolic equations. In considered problems the nonlinear highest order term (stress tensor) is monotone and its behaviour – coercivity/growth condition – is given with help of some general convex function. In our research we would like to cover both cases: sub- and super-linear growth of nonlinearity (shear thickening and shear thinning fluids) as well its anisotropic and non-homogenous behaviour. Such a formulation requires a general framework for the function space setting, therefore we work with non-reflexive and non-separable anisotropic Orlicz and Musielak-Orlicz spaces. Within the presentation we would like to emphasise problems we have met during our studies, their reasons and methods which allow us to achieve existence results.

19

Nonlinear Diffusion Problems

Organizers:

Jan Goncerzewicz (Wrocław University of Science and Technology)

Alberto Tesei (Università Sapienza di Roma)

THE QUASILINEAR PARABOLIC EQUATION OF KIRCHHOFF TYPE.

Łukasz Dawidowski University of Silesia in Katowice

Consider the Dirichlet problem for quasilinear generalized degenerate Kirchhoff equation

$$u_t - (1 + \|\nabla u\|_{L^2(\Omega)}^2)\Delta u + g(u, x) = 0 \quad (19.1)$$

with initial condition

$$u(0, x) = u_0(x), x \in \Omega,$$

and boundary condition of the Dirichlet type

$$u|_{\partial\Omega} = 0,$$

We will assume that $u_0 \in H^2(\Omega)$ and $\Omega \subseteq \mathbb{R}^N$ is a domain of the class C^2 .

The existence of solution of problem (19.1) under some assumptions will be studied using the Leray – Schauder principle.

THE POROUS MEDIA EQUATION IN AN INFINITE CYLINDER, BETWEEN TWO INFINITE PARALLEL PLATES, AND THE LIKE

Brian Gilding Kuwait University

This is joint work with Jan Goncerzewicz. The subject is the initial-boundary value problem for the porous media equation, with initial data that have bounded support, and homogeneous Dirichlet boundary data. The first part of the talk is concerned with the problem in a spatial domain that corresponds to an infinite or semi-infinite strip in two space dimensions, and to an infinite or semi-infinite cylinder in three space dimensions. Building upon results of J. L. Vázquez, the large-time behaviour of a solution and the interface marking the extent of the support of the solution is characterized in rather fine detail. As much as the approach resembles that used to prove the large-time behaviour of solutions of the porous media equation in a standard domain, a number of specific technical difficulties have to be overcome. These are related to the existence and uniqueness theory being less well-developed, and to lacunas in the analysis of the self-similar behaviour that emerges. The key to the characterization is the establishment of a suitable invariance principle for solutions of the problem. The second part of the talk focusses on the same topics for a spatial domain that, in three space dimensions, corresponds to the void between two infinite parallel plates, or such a void with the exclusion of a bounded portion. The appropriate invariance principle is proven. Coupling this with intuitive arguments, the large-time behaviour of a solution and its interface can be ascertained. A rigorous proof of this behaviour is lacking.

BLOW-UP AND GLOBAL EXISTENCE FOR THE POROUS MEDIUM EQUATION WITH REACTION ON A CLASS OF CARTAN-HADAMARD MANIFOLDS

Fabio Punzo Politecnico di Milano

The talk is concerned with the porous medium equation with power-type reaction terms up on negatively curved Riemannian manifolds, and solutions corresponding to bounded, nonnegative and compactly supported data. If $p > m$, small data give rise to global-in-time solutions while solutions associated to large data blow up in infinite

time. If $p < m$, large data blow up at worst in infinite time, and under the stronger restriction $p \in (1, (1+m)/2]$ all data give rise to solutions existing globally in time, whereas solutions corresponding to large data blow up in infinite time. The results are in several aspects significantly different from the Euclidean ones, as has to be expected since negative curvature is known to give rise to faster diffusion properties of the porous medium equation.

EXISTENCE OF MEASURE-VALUED SOLUTIONS TO A CLASS OF PARABOLIC EQUATIONS WITH DEGENERATE COERCIVITY AND SINGULAR INITIAL DATA

Flavia Smarrazzo Campus Bio-Medico di Roma

Initial-boundary value problems for nonlinear parabolic equations having Radon measures as initial data have been widely investigated, looking for solutions which for positive times take values in some function space. On the other hand, if the diffusivity degenerates too fast at infinity, it is well known that function-valued solutions may not exist and it looks very natural to consider solutions which, for positive times, describe an orbit in the space of the finite Radon measures. We study existence of measure-valued solutions to the homogeneous Dirichlet initial-boundary value problem for a class of parabolic equations without strong coercivity. The notion of solution is natural, since it is obtained by a suitable approximation procedure which also relies on regularization of the initial measure. Moreover, qualitative properties of the constructed solutions concerning the evolution of their singular part with respect to the Lebesgue measure will be discussed, as well as conditions (depending on the initial data and on the strength of degeneracy of the diffusivity at infinity) under which the constructed solutions are in fact function-valued or not.

Joint works with Marco Papi, Maria Michaela Porzio and Alberto Tesi.

20

Geometric Properties of Solutions of PDEs and Nonlocal Equations

Organizers:

Tadeusz Kulczycki (Wrocław University of Science and Technology)

Paolo Salani (DiMaI – Università di Firenze)

CONVEXITY OF SOLUTIONS TO ANISOTROPIC BERNOULLI TYPE PROBLEMS

Chiara Bianchini Università degli Studi di Firenze

The classical Bernoulli free-boundary problem is an overdetermined problem in a ring shape domain $\Omega \setminus \bar{K}$, where the set K is given and Ω has to be determined in such a way that the capacity potential of the ring has constant gradient on $\partial\Omega$. It is well known that if K is convex then the solution Ω is unique and in particular it is convex.

The Bernoulli problem offers a model for a 2-plate condenser which maintains a constant potential difference and where the intensity of the electrostatic field has to be constant on the exterior plate. In this talk we investigate the analogous situation when an anisotropic dielectric is considered between the two plates. The corresponding Bernoulli problem contains, as the governing operator, the so called Finsler Laplacian which is related to the anisotropy of the dielectric.

EIGENVALUES AND INEQUALITIES

Giuseppina di Blasio Università degli Studi della Campania "L. Vanvitelli"

We discuss about optimal lower and upper bounds for functionals involving the first Dirichlet eigenvalue $\lambda_F(p, \Omega)$ of the anisotropic p -Laplacian, $p \in (1, +\infty)$. Our aim is to enhance how, by means of the \mathcal{P} -function method, it is possible to get several sharp estimates for $\lambda_F(p, \Omega)$ in terms of several geometric quantities associated to the domain. The \mathcal{P} -function method is based on a maximum principle for a suitable function involving the eigenfunction and its gradient. This is a joint work with F. Della Pietra and N. Gavitone.

INEQUALITIES FOR ONE-DIMENSIONAL CAUCHY-TYPE MEASURES

Tomasz Byczkowski Wrocław University of Science and Technology

Gaussian measures occupy central place in various areas of Mathematics and Physics. We have some important and well-known inequalities for these measures: Prekopa-Lindler, Borell, Ehrhard and Landau-Shepp. The aim of our research was to find appropriate analogues of these inequalities for rotationally invariant, standard Cauchy measures. The first step consisted in examining of the one-dimensional case. Even here the situation is different than in Gaussian case, as half-lines are no longer minimal sets (in the sense of perimeter). It turns out that there are three types of minimal sets, depending on the measures. Further on, we considered one-dimensional sections of our n -dimensional Cauchy measures (we call them "Cauchy-type measures") and applied the Steiner-Ehrhard symmetrization procedure, which is the first step in the direction of n -dimensional setting.

GEOMETRICAL PROPERTIES FOR SOLUTIONS OF SUBELLIPTIC NONLINEAR PDES

Federica Dragoni Cardiff University

Geometrical properties for solutions to elliptic and parabolic PDEs is a classic mathematical problem. In particular we will focus on starshapedness, a geometrical notion deeply connected to convexity but weaker than that. For elliptic PDEs it is known that the capacity potential defined on a starshaped ring has starshaped level sets. We generalise these results in the setting of subelliptic PDEs, in particular to PDEs associated to Carnot groups. Carnot groups are non-commutative nilpotent Lie groups which are not isomorphic to the Euclidean space at any scale. Going into these more degenerate geometries some unexpected phenomena appear: e.g. there are different possible notions of starshapedness and they may not be all equivalent. We use a notion associated to the natural scaling in Carnot groups (dilations) and show that this geometrical property is inherited by the level sets of a large class of nonlinear PDEs. Joint work with Nicola Garofalo and Paolo Salani.

EIGENVALUES OF THE FRACTIONAL LAPLACE OPERATOR IN THE UNIT BALL

Bartłomiej Dyda Wrocław University of Science and Technology

We will present explicit formulae for the fractional Laplace operator acting on a certain class of functions. As an application of this result, we will produce a complete system of eigenfunctions of the operator $(1 - |x|^2)_+^{\alpha/2} (-\Delta)^{\alpha/2}$ with the Dirichlet boundary conditions outside of the unit ball. Finally, we will show how to use the latter result to estimate the eigenvalues of the fractional Laplace operator in the unit ball. Joint work with Alexey Kuznetsov and Mateusz Kwaśnicki.

NON-LOCAL SUBLINEAR PROBLEMS: EXISTENCE, COMPARISON, AND RADIAL SYMMETRY

Antonio Greco University of Cagliari

In a recent paper [Greco, Servadei (2016)] a non-autonomous overdetermined problem associated to the fractional torsion equation $(-\Delta)^s u = 1$ has been considered. To be more specific, let Ω be a bounded domain in \mathbb{R}^N containing the origin, and let ν be the inner normal to $\partial\Omega$. The authors prove that if there exists a solution of

$$\begin{cases} (-\Delta)^s u = 1 & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \\ (\partial_\nu)^s u(z) = q(|z|) & \text{for } z \in \partial\Omega, \end{cases}$$

where $q(r)$ is a given positive function that increases fast enough (not less than r^s), then the domain Ω is a ball centered at the origin. Counterexamples show that the last assumption cannot be removed. The result is related to previous papers [Dalibard, Gérard-Varet (2013)] and [Fall, Jarohs (2015)] where the case $q = \text{const.}$ is considered. In a work in press in collaboration with V. Mascia, the semilinear equation $(-\Delta)^s u = f(x, u)$ is investigated under the assumption that the ratio $f(x, t)/t$ is strictly decreasing in $t \in (0, +\infty)$ for almost every $x \in \Omega$. A convenient assumption on the function q is established, so that the statement given above continues to hold. The proof relies on a comparison principle for semilinear equations and on a boundary-point lemma for linear equations that may have an independent interest. In particular, the comparison principle in [Leonori, Peral, Primo, Soria (2015)] is extended to the case of two domains $\Omega_1 \subset \Omega_2$.

WEIGHTED HÖLDER REGULARITY FOR THE FRACTIONAL p -LAPLACIAN

Antonio Iannizzotto University of Cagliari

We investigate some properties of the weak solutions to the Dirichlet problem driven by the fractional p -Laplacian operator:

$$(-\Delta)_p^s u = f \text{ in } \Omega, \quad u = 0 \text{ in } \Omega^c,$$

where Ω is a bounded domain with $C^{1,1}$ boundary, $f \in L^\infty(\Omega)$, $p > 1$ and $s \in (0, 1)$ are real numbers, and the leading operator is defined for sufficiently smooth $u : \mathbf{R}^N \rightarrow \mathbf{R}$, by

$$(-\Delta)_p^s u(x) = \lim_{\varepsilon \rightarrow 0^+} \int_{B_\varepsilon(x)} \frac{|u(x) - u(y)|^{p-2} (u(x) - u(y))}{|x - y|^{N+ps}} dy.$$

Mainly focusing on regularity, we prove that $u/\text{dist}^s(\Omega^c)$ is Hölder continuous up to the boundary, with an exponent $\alpha \in (0, s]$ and a uniform estimate of the $C^\alpha(\overline{\Omega})$ -norm in terms of $\|f\|_\infty^{1/(p-1)}$. Such weighted Hölder regularity is a substitute of the $C^{1,\alpha}$ regularity proved by Lieberman for the p -Laplacian, since the quotient $u/\text{dist}^s(\Omega^c)$ is involved in strong maximum principles and Hopf type lemmas for the fractional p -Laplacian. In such sense, we believe that our result contributes not only to the regularity theory for nonlocal nonlinear operators, but also to the geometric theory and variational methods for such operators.

Work in collaboration with S. Mosconi (University of Catania) and MŠquassina (Catholic University, Brescia).

CRITICAL SCHRÖDINGER PERTURBATIONS OF FRACTIONAL LAPLACIAN

Tomasz Jakubowski Wrocław University of Science and Technology

We consider the semigroup generated by the fractional Laplacian with the Hardy potential $\kappa|x|^{-\alpha}$. We give sharp estimates of the density of this semigroup for $\kappa \leq \kappa_0$, where κ_0 is the Hardy constant. We also discuss the blow-up phenomena for $\kappa > \kappa_0$.

MAXIMUM PRINCIPLES IN NONLOCAL PROBLEMS AND APPLICATIONS

Sven Jarohs Goethe Universität Frankfurt

Maximum principles are one of the key ingredients in proving geometric properties of solutions. In this talk I discuss maximum principles for nonlocal equations and explain the differences to the local situation. Moreover, I present some results on geometric properties of solutions to nonlocal equations gained by such maximum principles.

DIRICHLET'S PROBLEM FOR CRITICAL HAMILTON-JACOBI FRACTIONAL EQUATION

Agnieszka Kałamańska University of Warsaw

Using an extended approach of Dan Henry, we study solvability of the Dirichlet problem on a bounded smooth domain for the Hamilton-Jacobi equation with critical nonlinearity posed in Sobolev spaces:

$$\begin{cases} u_t + (-\Delta)^{1/2}u + H(u, \nabla u) = 0, t > 0, x \in \Omega, \\ u(t, x) = 0, t > 0, x \in \partial\Omega, \\ u(0, x) = u_0, x \in \Omega. \end{cases}$$

We will also discuss the additional regularity and uniqueness of the limiting weak solution. The talk will be based on joint work with Tomasz Dłotko.

STEFAN COHN-VOSSEN AND SOME CONJECTURES HE STATED

Bernd Kawohl University of Cologne

Many mathematicians know the book of Hilbert and Cohn-Vossen on “Geometry and the Imagination” (Polish translation: Geometria pogladova, Italian translation: Geometria intuitiva). Not so many know that Cohn-Vossen (as well as Richard Courant and Heinz Hopf) grew up in Breslau. His life took a tragic turn when he was dismissed from teaching at Cologne University in 1933. He emigrated to the Soviet Union and died 1936 at the age of 34. I shall present some conjectures from an unpublished manuscript of Cohn-Vossen and report on (partial) solutions and related results on Ulam floating, sets of constant width and an isoperimetric partitioning of plane convex sets. My lecture will be dedicated to his memory. The URL www.mi.uni-koeln.de/cohn-vossen (<http://www.mi.uni-koeln.de/cohn-vossen>) leads to more information on him.

SERRIN'S TYPE OVERDETERMINED PROBLEMS IN CONVEX CONES

Alberto Roncoroni University of Pavia

We consider a Serrin's type overdetermined problem in convex cones for a class of possibly degenerate operators in the Euclidean space. When the operator is the Laplacian, F. Pacella and G. Tralli recently proved a rigidity result of Serrin's type by showing that the existence of a solution implies that the domain is a spherical sector. We show that this result holds for more general and possibly degenerate operators (such as the mean curvature operator and the p-Laplacian). Moreover, we consider a suitable generalization of the Laplace operator to space forms, i.e. the hyperbolic space and the (hemi)-sphere, and we prove a rigidity result. This is a joint work with G. Ciraolo.

SHARP SPECTRAL INEQUALITIES

Cristina Trombetti Università di Napoli (Italy)

We present some recent sharp estimates for Laplacian eigenvalues with various boundary conditions.

INTERIOR AND BOUNDARY REGULARITY OF NONLOCAL MINIMAL SURFACES

Enrico Valdinoci Università di Milano

We recall the notion of nonlocal minimal surfaces and we discuss their qualitative and quantitative interior and boundary behavior. In particular, we present some optimal examples in which the surfaces stick at the boundary. This phenomenon is purely nonlocal, since classical minimal surfaces do not stick at the boundary of convex domains.

21

New Perspectives in Singular Hamiltonian Systems

Organizers:

Joanna Janczewska (Gdańsk University of Technology)

Alessandro Portaluri (University of Turin)

Marek Izydorek (Gdańsk University of Technology)

ON THE EXISTENCE OF HOMOCLINIC TYPE SOLUTIONS OF INHOMOGENOUS LAGRANGIAN SYSTEMS.

Jakub Ciesielski Politechnika Gdańska

We study the existence of homoclinic type solutions for second order Lagrangian systems of the type $\ddot{q}(t) - q(t) + a(t)\nabla G(q) = f(t)$, where $t \in \mathbb{R}$, $q \in \mathbb{R}^n$, $a: \mathbb{R} \rightarrow \mathbb{R}$ is a continuous positive bounded function, $G: \mathbb{R}^n \rightarrow \mathbb{R}$ is a C^1 -smooth potential satisfying the Ambrosetti-Rabinowitz superquadratic growth condition and $f: \mathbb{R} \rightarrow \mathbb{R}^n$ is a continuous bounded square integrable forcing term. A homoclinic type solution is obtained as limit of $2k$ -periodic solutions of an approximative sequence of second order differential equations.

HOMOCLINIC ORBITS FOR AN ALMOST PERIODICALLY NEWTONIAN SYSTEM IN \mathbb{R}^3

Robert Krawczyk Gdańsk University of Technology

We will be concerned with the existence of homoclinic solutions for a Newtonian system $\ddot{q}(t) + a(t)W(q(t)) = 0$, where $t \in \mathbb{R}$, $q \in \mathbb{R}^3$. It is assumed that there is a line $l \in \mathbb{R}^3 \setminus \{0\}$ such that potential $W \in C^2(\mathbb{R}^3 \setminus l, \mathbb{R})$ has a global maximum at the origin and the line l consists of singular points. Moreover, W satisfies the “strong-force” condition in a neighbourhood of l and $a: \mathbb{R} \rightarrow \mathbb{R}$ is a continuous almost periodic function. The existence of at least two solutions will be discussed.

APPLICATIONS OF DIFFERENTIAL GALOIS METHODS FOR STUDY INTEGRABILITY OF HAMILTONIAN SYSTEM

Andrzej J. Maciejewski University of Zielona Gora

Joint work with Janusz Gill Institute of Astronomy, University of Zielona Góra, Poland.

Last two decades show amazing power of differential Galois theory in investigations of integrability of differential equation. In the context of Hamiltonian system this approach is called the Morales-Ramis theory.

In my talk I will present an overview of the most important results obtained till know. Among others I will show a simple proof of non-integrability of the three body problem which I have found with my collaborators.

I will show examples of general theorems which give necessary conditions for partial and super-integrability of specific classes of Hamiltonian systems.

THE EXISTENCE OF MAXIMAL BOUNDED INVARIANT SETS

Maciej Starostka Ruhr Universität Bochum

We discuss sufficient conditions for the existence of isolated invariant sets, i.e. sets for which the Conley index or Morse theoretical methods can be applied. As an application we give a negative answer to the question asked by A. Parusinski. Namely, we show that there exist two gradient proper maps on \mathbb{R}^n , which are homotopic in the category of proper maps, but not homotopic in the category of proper gradient maps.

ON THE FREDHOLM LAGRANGIAN GRASSMANNIAN, SPECTRAL FLOW AND ODES IN HILBERT SPACES

Nils Waterstraat University of Kent

We consider homoclinic solutions for Hamiltonian systems in symplectic Hilbert spaces and generalise spectral flow formulas that were proved by Pejsachowicz and the author in finite dimensions some years ago. Roughly speaking, our main theorem relates the spectra of infinite dimensional Hamiltonian systems under homoclinic boundary conditions to intersections of their stable and unstable spaces. We make use of striking results by Abbondandolo and Majer to study Fredholm properties of infinite dimensional Hamiltonian systems.

22

Ergodic Theory and Topological Dynamics

Organizers:

Tomasz Downarowicz (Wrocław University of Science and Technology)

Anna Giordano Bruno (Università di Udine)

Stafano Marmi (Scuola Normale Superiore)

Krzysztof Frączek (Nicolaus Copernicus University)

ON DYNAMICS OF QUADRATIC STOCHASTIC OPERATORS

Wojciech Bartoszek Politechnika Gdańska

We discuss asymptotic behaviour of iterates of quadratic stochastic operators (nonlinear Markov processes) that are mean monotonic. They are defined on the set of probability measures concentrated on weakly compact order intervals of a fixed Banach lattice. We are focused on conditions guaranteeing regularities of iterates and find limit distributions.

ON MINIMAL HOMEOMORPHISMS AND NON-INVERTIBLE MAPS PRESERVING FOLIATIONS

Andrzej Biś University of Łódź

We apply the decomposition theory of a manifold, developed by Daverman, to study of a decomposition space of a foliated manifold. We consider properties of a minimal homeomorphism preserving foliation on a compact manifold. As a consequence of our main result we obtain a generalization of the result of Kolyada, Snoha and Trofimchuk concerning a minimal skew-product homeomorphism of the 2-torus. The talk is based on a joint paper with Wojciech Kozłowski.

A GARDEN OF EDEN THEOREM FOR ALGEBRAIC DYNAMICAL SYSTEMS

Tullio Ceccherini-Silberstein Università del Sannio

The Garden of Eden theorem (originally proved by E.F.Moore and J.Myhill for the free abelian groups \mathbb{Z}^d) states that given a finite alphabet set A and an amenable group G , SURJECTIVITY of a cellular automaton (continuous G -equivariant map) $\tau: A^G \rightarrow A^G$ is equivalent to a weak form of injectivity, called PREINJECTIVITY (TCS-A.Machi-F.Scarabotti). Following M.Gromov's suggestion, this result was considered in other settings beyond symbolic dynamics in TCS-M.Coornaert (for Anosov diffeomorphisms of tori) and TCS-M.Coornaert-H.Li (for algebraic dynamical systems in the sense of K.Schmidt: this is based on Pontryagin duality for (locally) compact groups).

NORMED SEMIGROUP ENTROPY OF AMENABLE SEMIGROUP ACTIONS

Dikran Dikranjan Udine University

A *norm* on a commutative semigroup $(S, +)$ is a map $v: S \rightarrow \mathbf{R}_{\geq 0}$ such that $\{v(x), v(y)\} \leq v(x+y) \leq v(x) + v(y)$ for all $x, y \in S$; a *normed semigroup* is a commutative semigroup provided with a norm (see [3]).

If G is a cancellative right-amenable semigroup with a left action $G \curvearrowright^{\alpha} S$ on a normed semigroup (S, v) by semigroup morphisms one can define the *semigroup entropy* $h_{\mathfrak{S}}(\alpha)$ as follows. For $x \in S$ and for $F \in [G]^{<\omega}$, let $T_F(\alpha, x) = \sum_{g \in F} gx$. The function

$$f_x: [G]^{<\omega} \rightarrow \mathbb{R}, \quad F \mapsto v(T_F(\alpha, x))$$

is subadditive, right-subinvariant and uniformly bounded on singletons, in the sense of the celebrated Ornstein-Weiss Lemma for amenable group actions. A counterpart for cancellative left-amenable semigroups of this result was recently obtained in [1]. It ensures that the limit $H_{\mathfrak{S}}(\alpha, x) = \lim_{i \in I} \frac{f_i(F_i)}{|F_i|}$ exists for every right-Følner net $(F_i)_{i \in I}$ of G . The *semigroup entropy* of α is

$$h_{\mathfrak{S}}(\alpha) = \sup\{H_{\mathfrak{S}}(\alpha, x) : x \in M\}.$$

The aim of the talk is to discuss the properties of the entropy $h_{\mathfrak{S}}$ and the possibility to obtain various known entropies of amenable semigroup actions (e.g., measure entropy, topological entropy, or algebraic entropy, the last one introduced and studied in [2]) by means of appropriate functors $F : \mathfrak{X} \rightarrow \mathfrak{S}$ from the relevant category \mathfrak{X} (compact spaces, probability measure spaces or abelian groups) to \mathfrak{S} . Needless to say, many other entropies can be sheltered under this umbrella (the case of \mathbb{N} -actions is studied in full detail in [3]).

- [1] T. Ceccherini-Silberstein, M. Coornaert, F. Krieger, An analogue of Fekete's lemma for subadditive functions on cancellative amenable semigroups, *J. Anal. Math.* 124 (2014) 59–81.
- [2] D. Dikranjan, A. Fornasiero, A. Giordano Bruno, Algebraic entropy for amenable semigroup actions, preprint.
- [3] D. Dikranjan, A. Giordano Bruno, Entropy on normed semigroups, submitted.

THE COMPARISON PROPERTY OF AMENABLE GROUPS

Tomasz Downarowicz Wrocław University of Science and Technology, Faculty of Pure and Applied Mathematics

Let a countable amenable group G act on a zero-dimensional compact metric space X . For two clopen subsets A and B of X we say that A is *subequivalent* to B (we write $A \preceq B$), if there exists a finite partition $A = \bigcup_{i=1}^k A_i$ of A into clopen sets and there are elements g_1, g_2, \dots, g_k in G such that $g_1(A_1), g_2(A_2), \dots, g_k(A_k)$ are disjoint subsets of B . We say that the action *admits comparison* if for any clopen sets A, B , the condition, that for every G -invariant probability measure μ on X we have the sharp inequality $\mu(A) < \mu(B)$, implies $A \preceq B$. Comparison has many desired consequences for the action, such as the existence of tilings with arbitrarily good Følner properties, which are factors of the action. We also study a purely group-theoretic notion of comparison: if every action of G on any zero-dimensional compact metric space admits comparison then we say that G has the *comparison property*. Classical groups \mathbb{Z} and \mathbb{Z}^d enjoy the comparison property, but in the general case the problem remains open. Our main result states that groups whose every finitely generated subgroup has subexponential growth have the comparison property. This is joint work with Guohua Zhang.

A SIMPLE APPLICATION OF A THEOREM OF RATNER

Livio Flaminio Université de Lille

One of the first rigidity theorems of Marina Ratner characterises the measurable isomorphisms of time-changes of horocycle flows.

A. Kanigowski asked when, for positive distinct integers p and q and a time change V_t of the classical horocycle flow, there exists a non trivial joining of the sped up flows V_{pt} and V_{qt} . Ratner's theorem (M. Ratner. Rigid reparametrizations and cohomology for horocycle flows. *Invent. Math.*, 88(2):341–374, 1987) together with some considerations on the cohomological equation derived from our joint work with G. Forni (L. Flaminio

and G. Forni. Invariant distributions and time averages for horocycle flows. *Duke Math. J.*, 119(3):465–526, 2003) implies that this is the case only if time change is cohomologous to a constant plus an harmonic one-form. This is a joint work with G. Forni.

DOUBLY STOCHASTIC OPERATORS WITH ZERO ENTROPY

Bartosz Frej Wrocław University of Science and Technology

Doubly stochastic operators naturally generalize measure preserving maps. This statement is justified by the fact that the Koopman operator of a measure preserving map is doubly stochastic. Moreover, every doubly stochastic operator (on a standard space) comes from dynamics induced on the underlying space by a transition probability. After introducing the notion of operator entropy, I will try to carry to the world of such operators three famous theorems: the Rokhlin's theorem on genericity of zero entropy, the Kushnirenko's theorem on equivalence of discrete spectrum and nullity, and the Halmos-von Neumann's theorem on representation of maps with discrete spectrum as group rotations.

ALGEBRAIC ENTROPY FOR AMENABLE SEMIGROUP ACTIONS

Anna Giordano Bruno Università di Udine

Recently, T. Ceccherini-Silberstein, M. Coornaert and F. Krieger extended Ornstein-Weiss Lemma to cancellative left-amenable semigroups; this result allows them to generalize the topological entropy h_{top} of R.L. Adler, A.G. Konheim and M.H. McAndrew to left actions of cancellative left-amenable semigroups on compact topological spaces.

Analogously, we generalize the notion of algebraic entropy h_{alg} of J. Peters and M.D. Weiss to every right action $S \curvearrowright^\alpha A$ of a cancellative left-amenable semigroup S on a discrete abelian group A . We prove several properties of this invariant, in particular the so-called Addition Theorem in case A is torsion, that is, if B is an α -invariant subgroup of A , then $h_{alg}(\alpha) = h_{alg}(\alpha_B) + h_{alg}(\alpha_{A/B})$, where $S \curvearrowright^{\alpha_B} B$ and $S \curvearrowright^{\alpha_{A/B}} A/B$ are the actions induced by α on B and A/B respectively. Moreover, we extend the so-called Bridge Theorem of M.D. Weiss, that connects the algebraic entropy to the topological entropy: if A is torsion, then $h_{alg}(\alpha) = h_{top}(\widehat{\alpha})$, where $S \curvearrowright^{\widehat{\alpha}} \widehat{A}$ is the action induced by α on the compact Pontryagin dual group \widehat{A} of A .

SARNAK'S CONJECTURE IMPLIES CHOWLA'S CONJECTURE ALONG A SUBSEQUENCE

Dominik Kwietniak Jagiellonian University in Kraków

We prove that the Moebius disjointness of zero entropy dynamical systems implies the existence of an increasing sequence of integers along which the Chowla conjecture on autocorrelations of the Moebius function holds.

The proof uses Tao's result on the equivalence of logarithmic versions of conjectures of Sarnak and Chowla, together with some new results on the set of limit points of the sequence of harmonic empirical measures

$$\mathcal{E}^{log}(x, N) = \frac{1}{1 + 1/2 + \dots + 1/N} \sum_{n=1}^N \frac{1}{n} \delta_{T^{n-1}(x)},$$

where $\delta_{T^n(x)}$ stands for the Dirac measure concentrated at the point $T^n(x)$, X is a compact metric space, $x \in X$, and $T: X \rightarrow X$ is a continuous map.

The talk is based on a joint work with A. Gomiłko and M. Lemańczyk.

RECURRENCE AND NON-ERGODICITY IN GENERALIZED WIND-TREE MODELS

Krzysztof Frączyk Nicolaus Copernicus University

We consider generalized wind-tree models and \mathbb{Z}^d -covers over compact translation surfaces. We deal with the recurrence and the ergodicity of the translation flow in a generic direction. The talk is based on joint work with Corinna Ulcigrai and Pascal Hubert

ZERO ENTROPY SYSTEMS AND MULTIPLICATIVE FUNCTIONS

Mariusz Lemańczyk Nicolaus Copernicus University

I will describe two strategies whose aim is to prove Sarnaks conjecture on Moebius disjointness

AN INTRODUCTION TO COARSE ENTROPY AND ITS RELATION WITH THE ALGEBRAIC ENTROPY

Nicolò Zava University of Udine

Coarse geometry is the study of large-scale properties of spaces. The objects that are considered in this theory are called coarse spaces, which are generalizations of metric spaces introduced by Roe, and the morphisms between them are bornologous maps. In particular, every group has an inner coarse structure, called group coarse structure. If the group is finitely generated, its group coarse structure coincides with the one induced by the word metric associated to any finite symmetric generating set. Moreover, every homomorphism between groups is a bornologous map if the groups are endowed with their group coarse structures. The aim of this talk is to introduce the notion of coarse entropy, which measures the chaos that a bornologous self-map creates in a locally finite coarse space. After briefly discussing the needed background, we will provide some basic properties and examples. Furthermore, for every surjective endomorphism of a group, we show that its coarse entropy (if we endow the group with the group coarse structure) coincides with its algebraic entropy.

23

Hutchinson-Barnsley Theory of Fractals

Organizers:

Grażyna Horbaczewska (University of Łódź)

Emma D'Aniello (Università degli Studi della Campania "Luigi Vanvitelli")

Magdalena Nowak (Jan Kochanowski University in Kielce)

Filip Strobil (Łódź University of Technology)

ON SOME METRIC AND TOPOLOGICAL DIMENSION FUNCTIONS OF PEANO CONTINUA

Taras Banach Jan Kochanowski University in Kielce

On some metric and topological dimension functions of Peano continua; abstract: We shall discuss properties of dimension functions S-Dim and Hö-Dim of metric Peano continua and their topological versions S-dim and Hö-dim, analyze the relation between these dimension functions and classical fractal dimensions (such as the box-counting dimension). Also we discuss applications of the dimension functions S-dim and Hö-dim to the (Kameyama) problem of recognizing Peano continua that are homeomorphic to deterministic fractals.

ON TWO ONE PARAMETER FAMILIES OF CANTORVALS

Artur Bartoszewicz Łódź University of Technology

We consider two families of Cantorvals. The first one consists of Cantorvals connected with ternary Cantor sets with different selfsimilarity ratios. The second family consists of generalized Guthrie-Nymann-Jones Cantorvals. We show that all considered Cantorvals are attractors of Affine Iterated Function Systems but not all are the sets of subsumes of series.

VALUATION THEORY, GENERALIZED IFS ATTRACTORS AND FRACTALS

Franz-Viktor Kuhlmann University of Szczecin

Using valuation rings and valued fields as examples, we discuss in which ways the notions of “topological IFS attractor” and “fractal space” can be generalized to cover more general settings. This question arose in joint work with Katarzyna Kuhlmann on spaces of real places (this notion will be explained in my talk). Already in very basic examples of such spaces we observed rich self-similarities and wondered whether it can be justified to call the space “fractal”, and if so, which insights on its structure could then be inferred.

We use the definition of “fractal” that is based on iterated function systems, both in metric and in topological spaces. Then discrete valuation rings with finite residue fields are fractal, both metric and topological. But when it comes to more complicated valuation theoretical objects, one has to introduce more general definitions to cover them. To this end, we introduce two generalizations of the notion “IFS attractor”, discuss some of their basic properties and some examples.

This is joint work with Jan Dobrowolski.

FAST BASINS OF ATTRACTORS

Krzysztof Leśniak Nicolaus Copernicus University in Toruń, Faculty of Mathematics and Computer Science

Fast basin of an attractor of the IFS is a new kind of fractal. It is a flat projection of a branched fractal manifold. We will discuss some properties of fast basins.

TOPOLOGICAL PROPERTIES OF FRACTALS OF GENERALIZED ITERATED FUNCTION SYSTEMS OF INFINITE ORDER

Łukasz Maślanka Łódź University of Technology

Miculescu and Mihail in 2008 introduced a concept of a generalized iterated function system (GIFS in short), a particular extension of classical IFS. Instead of families of selfmaps of a metric space X , they considered families of mappings defined on finite Cartesian product X^m . They obtained a counterpart of a Banach fixed point theorem with convergence to the fixed point of some natural sequence of “iterates”. It turned out that a great part of the classical Hutchinson-Barnsley theory has natural counterpart in the GIFSs’ case.

Recently, Secelean extended these considerations to mappings defined on the space $\ell_\infty(X)$ of all bounded sequences of elements of X and obtained version of the Hutchinson-Barnsley theorem for appropriate families of such functions (which we call GIFSs $_\infty$).

I will present other, more restrictive approach to mappings defined on $\ell_\infty(X)$ for which solutions of many problems are more natural. In particular, I am going to discuss the connection between attractor of GIFS $_\infty$ and its code space, investigate the link between attractors of GIFSs’ and attractors of GIFSs $_\infty$ and present that typical (in the Baire category sense) nonempty compact set of the Euclidean space is the attractor of some GIFS $_\infty$ (which is interesting since it is known that typical compact set is not attractor of any IFS).

EMBEDDING IFS-ATTRACTORS IN THE EUCLIDEAN SPACE

Magdalena Nowak Jan Kochanowski University in Kielce

We study a problem which compact sets can be embed in the Euclidean space as an IFS-attractor. We also consider a topological version of such fractals: (X, \mathcal{F}) where X is a compact Hausdorff topological space and \mathcal{F} is a finite topologically contracting system of continuous self-maps such that $X = \bigcup_{f \in \mathcal{F}} f(X)$. For such fractals we consider a Kameyama (pseudo)metric $p_\lambda^{\mathcal{F}}$ generated by family \mathcal{F} and depend on positive constant λ . All maps from \mathcal{F} become Lipschitz functions with Lipschitz constant $< \lambda$. Using $p_\lambda^{\mathcal{F}}$ we obtain some results connected to embedding fractals in the Euclidean space.

We will also show that each finite-dimensional compact metrizable space containing an open uncountable zero-dimensional subset Z is homeomorphic to an IFS-attractor in a Euclidean space \mathbb{R}^d . This is a generalization of result proved by Duvall and Husch in 1992, where Z is a copy of the Cantor set.

ZERO-DIMENSIONAL COMPACT METRIZABLE SPACES AS ATTRACTORS OF (GENERALIZED) ITERATED FUNCTION SYSTEMS (JOINT WORK WITH Ł. MAŚLANKA)

Filip Strobin Institute of Mathematics, Łódź University of Technology

In the last years, the problem of considering zero-dimensional compact metrizable spaces as attractors of iteration function systems has been undertaken by several authors.

In the first part of my talk I will recall results obtained recently in this field. In particular, I will quote that such a space X is a fractal generated by some IFS if and only if X is uncountable or X is countable but the Cantor-Bendixon height of X is successor ordinal.

Then I will move to generalized IFSs (GIFSs in short), introduced by Mihail and Miculescu in 2008. The idea is that, instead of families of selfmaps of a metric space X , GIFSs consist of maps defined on a finite

Cartesian product X^m with values in X (in such a case we say that a GIFS is of order m ; clearly, GIFSs of order 1 are classical IFSs). It turned out that a great part of the classical Hutchinson–Barnsley theory has natural counterpart in this GIFSs’ framework. On the other hand, there are known only few examples of fractal sets which are generated by GIFSs, but which are not IFSs’ fractals. I will show that each zero-dimensional compact metrizable space X (in particular, countable with limit Cantor-Bendixon height) is homeomorphic to the attractor of some GIFS of order 2 on the real line. Moreover, I will justify that for any $m \geq 2$, such a space X can be embedded into the real line \mathbb{R} as attractor of a GIFS of order m and (in the same time) nonattractor of any GIFS of order $m - 1$, as well as it can be embedded as nonattractor of any GIFS. Finally, I will show that a relatively simple modifications of countable zero dimensional space of height ω deliver spaces whose each connected component is “big”, and which are GIFS’s fractals not homeomorphic to IFS’s fractals.

Organizers:

Błażej Wróbel (University of Wrocław)

Andrea Carbonaro (University of Genova)

Jacek Dziubański (University of Wrocław)

Fulvio Ricci (Scuola Normale Superiore, Pisa)

SOBOLEV AND BESOV SPACES ON LIE GROUPS

Tommaso Bruno Politecnico di Torino

Let G be a noncompact connected Lie group and \mathbf{X} be a family of left-invariant vector fields on G satisfying Hörmander's condition. Denote with ρ a right Haar measure on G , with χ a positive character of G and consider the measure μ_χ whose density is χ with respect to ρ . In this talk, I will introduce inhomogeneous Sobolev spaces $L_\alpha^p(\mu_\chi)$ ($1 < p < \infty$, $\alpha \geq 0$) and Besov spaces $B_\alpha^{p,q}(\mu_\chi)$ ($1 \leq p, q \leq \infty$, $\alpha \geq 0$) adapted to μ_χ and \mathbf{X} and illustrate some embedding theorems and algebra properties of these spaces.

The talk is based on joint work with M. Peloso, A. Tabacco and M. Vallarino.

SPECTRAL MULTIPLIERS ON STRATIFIED GROUPS

Mattia Calzi Scuola Normale Superiore

In 1984, A. Hulanicki showed that the spectral calculus associated with a positive Rockland operator on a graded group maps Schwartz multipliers into Schwartz convolution kernels. In 2010, A. Martini generalized the preceding result to more general families of differential operators including the commutative finite families of homogeneous left-invariant differential operators one of which is Rockland.

Focusing on the setting of stratified Lie groups, I will consider:

- (S) a converse of the aforementioned result, that is: given a Schwartz kernel, does it correspond to a Schwartz multiplier?
 - (RL) an analogue of the Riemann–Lebesgue lemma: given an integrable kernel, does it correspond to a continuous multiplier?
-

ON EXPONENTIAL DECAY OF BERGMAN KERNELS ON COMPLETE HERMITIAN MANIFOLDS

Gian Maria Dall'Ara University of Vienna

Joint work with Franz Berger and Duong Ngoc Son of the University of Vienna. Given a smooth positive measure on a complete Hermitian manifold with Ricci curvature bounded from below, we prove a pointwise Agmon-type bound for the corresponding Bergman kernel, under rather general conditions involving the coercivity of an associated complex Laplacian on $(0,1)$ -forms. Our results extend several well-known bounds in the literature to the case in which the manifold is neither assumed to be Kähler nor of “bounded geometry”. The key ingredients of our proof are a localization formula for the complex Laplacian (of the kind used in the theory of Schrödinger operators) and a mean value inequality for subsolutions of the heat equation on Riemannian manifolds due to Li, Schoen, and Tam.

MAXIMAL DIRECTIONAL OPERATORS ALONG ALGEBRAIC VARIETIES

Francesco Di Plinio University of Virginia

This is joint work with Ioannis Parissis (U Basque Country). We establish the sharp growth order of the L^2 norm of the maximal directional averaging operator along finite subsets of an algebraic subvariety of R^n of arbitrary dimension k . Our result is fully sharp in the case of the two-dimensional sphere. This extends results of Cordoba, Katz ($k = 1$) and of Demeter (for the particular case of the 2-sphere) and implies new L^2 estimates for Keakeya-Nykodym maximal functions with tubes pointing along polynomial directions. Our proof technique is novel and relies in particular on a new scheme of polynomial partitioning on algebraic varieties in the vein of Guth, Guth-Katz et. al.

THE p -ELLIPTICITY CONDITION AND APPLICATIONS

Oliver Dragičević University of Ljubljana

For $p > 1$ we introduce the concept of p -ellipticity of bounded complex matrix functions on open subsets in euclidean spaces. It extends the classical ellipticity condition, which corresponds to the case of $p = 2$. We show that this notion is closely related to several phenomena in analysis and PDE, in particular semigroup contractivity, holomorphic functional calculus, maximal regularity and reverse Hölder inequalities for weak solutions to elliptic equations. The latter example is due to Dindoš and Pipher, while our contractivity results extend earlier work by Cialdea and Maz'ya. The p -ellipticity condition is a result of a study of the so-called bilinear embedding theorems for divergence-form operators with complex coefficients.

The talk is based on joint works with Andrea Carbonaro.

LP-MULTIPLIERS ON NILPOTENT LIE GROUPS SENSITIVE TO THE GROUP STRUCTURE

Paweł Głowacki Institute of Mathematics, University of Wrocław

We propose new sufficient conditions for L^p -multipliers on homogeneous nilpotent groups. The multipliers generalise the flag multipliers of Nagel-Ricci-Stein-Wainger, but the approach and the techniques applied are entirely different. Our multipliers are better adapted to the specific commutation rules on the Lie algebra of the given group. The proofs are based on a new symbolic calculus fashioned after Hormander. We also take advantage of Cotlar-Stein lemma and Littlewood-Paley theory in the spirit of Duoandikoetxea-Rubio de Francia.

HARMONIC ANALYSIS IN THE RATIONAL DUNKL SETTING

Agnieszka Hejna University of Wrocław

Dunkl theory is a generalization of Fourier analysis and special function theory related to root systems and reflections groups. The Dunkl operators T_j , which were introduced by C. F. Dunkl in 1989, are deformations of directional derivatives by difference operators related to the reflection group. The Dunkl Laplacian is defined by $\Delta = \sum_{j=1}^N T_j^2$. We shall discuss the objects of harmonic analysis related to the Dunkl Laplacian, i.e. Dunkl kernel, Dunkl transform, Dunkl translations, Dunkl heat and Poisson semigroups, harmonic and conjugate harmonic

functions, Hardy spaces, and Dunkl multipliers. In the one-dimensional case and in the product case the heat kernel can be expressed explicitly in terms of classical special functions. In the general case considered in this talk, no such precise information is available.

This talk is based on the joint articles with J-Ph. Anker and J. Dziubański.

AN ENDPOINT ESTIMATE FOR MAXIMAL SINGULAR INTEGRAL WITH ROUGH KERNEL

Petr Honzík Charles University Prague

It is an open question if the maximal singular integral with rough kernel is of the weak type 1-1. We give an endpoint estimate showing that the operator maps the space $L \log \log^{2+\varepsilon} L$ to $L^{1,\infty}$.

RECENT PROGRESS ON THE WEIGHTED L^p -CONJECTURE

Mateusz Krukowski Technical University of Łódź

Let $p > 1$ and let $f \star g$ denote the convolution of two complex-valued functions defined on a locally compact Hausdorff group G . The L^p -conjecture states that if

$$L^p(G) \star L^p(G) \subset L^p(G),$$

then the group G must be compact. The history of this claim dates back to the papers of Urbanik and Zelazko. However, it wasn't until 1963 when the conjecture was formulated by Rajagopalan in his PhD thesis. Thus began a long walk towards the complete solution. In the next 25 years, the problem has been studied in a variety of locally compact Hausdorff groups: discrete, nilpotent, solvable etc. At last, the conjecture was resolved affirmatively in 1990 by Sadashiro Saeki.

Year 2010 witnessed the revival of the conjecture due to 3 mathematicians: Abtahi, Nasr-Isfahani and Rejali. They considered the Banach space $L_\omega^p(G)$ of measurable functions $f : G \rightarrow \mathbb{C}$ such that

$$\int_G (f \cdot \omega)^p d\mu < \infty,$$

where $\omega : G \rightarrow \mathbb{R}_+$ is a measurable and submultiplicative function. Again, the question is: if

$$L_\omega^p(G) \star L_\omega^p(G) \subset L_\omega^p(G),$$

does it follow that the group G must be compact? We provide a positive answer if the weight is of reasonable growth or when the group G is nilpotent.

THE HAUSDORFF-YOUNG INEQUALITY ON LIE GROUPS

Alessio Martini University of Birmingham

While the best constants in the Hausdorff-Young inequality on \mathbf{R}^n have long been known, the corresponding problem on general locally compact groups is still open. In joint work with Michael Cowling, Detlef Müller and Javier Parcet (arXiv:1807.04670), we establish a sharp local central version of the inequality for compact Lie groups, and extend known results for the Heisenberg group. In addition, we prove a universal lower bound to the best constants for general Lie groups.

MARCINKIEWICZ TYPE MULTIPLIERS ON REDUCIBLE NONCOMPACT SYMMETRIC SPACES

Stefano Meda Università di Milano-Bicocca

Symmetric spaces of the noncompact are important examples of homogeneous manifolds with nonpositive curvature. They may be realized as $X := G/K$, where G is a connected noncompact semisimple Lie group and K a maximal compact subgroup thereof. The analysis of the G -invariant operators on $L^p(G/K)$ has been the object of several investigations in the last forty years. In this talk we focus on *reduced* symmetric spaces of the form $X_1 \times X_2$, where $X_1 = G_1/K_1$ and $X_2 = G_2/K_2$ are symmetric spaces of the noncompact type and real rank one.

It is a special case of a well known fact due to Clerc and Stein that if T is a $(G_1 \times G_2)$ -invariant operator bounded on $L^p((G_1 \times G_2)/(K_1 \times K_2))$, then the corresponding spherical multiplier m is a bounded holomorphic function on an appropriate tube \mathbf{T}_p in \mathbf{C}^2 . Our main result is that if $p \in (1, 2)$ and m satisfies Marcinkiewicz type conditions of order N on \mathbf{T}_p , with N big enough, then the corresponding invariant operator T is bounded on $L^q(G/K)$ for all q in $[p, p']$. To the best of our knowledge, this is the first occurrence of Marcinkiewicz-type conditions in this setting.

This is joint work with Błażej Wróbel.

GENERALIZED SPHERICAL MEAN RADON TRANSFORM ACTING ON RADIAL FUNCTIONS

Adam Nowak Institute of Mathematics, Polish Academy of Sciences

We consider a generalization of the classical spherical means and investigate in detail its action on radially symmetric functions. The results we get have applications related to the wave equation and more general PDEs.

This is a joint work with Óscar Ciaurri (Universidad de La Rioja) and Luz Roncal (Basque Center for Applied Mathematics).

THE INVERSION PROBLEM FOR MEASURE AND FOURIER-STIELTJES ALGEBRAS

Przemysław Ohrysko Chalmers University of Technology and Gothenburg University

In my talk I will present recent progress on the quantitative version of the inversion problem for measure and Fourier-Stieltjes algebras. This area was investigated by N. Nikolski in the paper “In search of the invisible spectrum” and the results from my speech supplements the original work and provide solutions to some open problems. In brief, let G be a locally compact Abelian group and let $M(G)$ denote the Banach algebra of all complex Borel measures. Suppose that $\mu \in M(G)$ satisfies $\|\mu\| \leq 1$ and $|\widehat{\mu}| \geq \delta > 0$ where $\widehat{\mu}$ is the Fourier-Stieltjes transform of μ and δ is a fix constant. The classical formulation of the Wiener-Pitt phenomenon shows that the aforementioned assumptions does not imply the invertibility of μ . However, by the results of N. Nikolski, if $\delta > \frac{1}{\sqrt{2}}$ then μ is invertible and $\|\mu^{-1}\| \leq \frac{1}{2\delta^2-1}$. I will show that for any $\delta > \frac{1}{2}$ the invertibility of μ is also assured and in quite general case $\|\mu^{-1}\| \leq \frac{1}{2\delta-1}$. Nevertheless, the problem in full generality remains open. I will also discuss analogous results for Fourier-Stieltjes algebras.

FRACTIONAL PALEY-WIENER SPACES IN ONE AND SEVERAL VARIABLES

Marco M Peloso Università degli Studi di Milano

We introduce and study spaces of entire functions in several complex variables whose restriction to a strongly pseudoconvex manifold satisfy some integrability conditions. For these spaces we prove Paley–Wiener type theorems and some other structural properties. These spaces are a natural generalisation of the classical Paley–Wiener space in variable but also define a new class of spaces in the complex plane. We study their relation with solutions to the Schrodinger equation, determine some of their structural properties and study the question of characterising sampling, interpolating and complete interpolating sequences. This is report on joint work with A. Monguzzi and M. Salvatori.

RIESZ TRANSFORMS ON A CLASS OF NON-DOUBLING MANIFOLDS

Adam Sikora Macquarie University

We consider a class of manifolds \mathcal{M} obtained by taking the connected sum of a finite number of N -dimensional Riemannian manifolds of the form $(\mathbb{R}^{n_i}, \delta) \times (\mathcal{M}_i, g)$, where \mathcal{M}_i is a compact manifold, with the product metric. The case of greatest interest is when the Euclidean dimensions n_i are not all equal. This means that the ends have different “asymptotic dimension”, and implies that the Riemannian manifold \mathcal{M} is not a doubling space. We completely describe the range of exponents p for which the Riesz transform on \mathcal{M} is a bounded operator on $L^p(\mathcal{M})$. Namely, under the assumption that each n_i is at least 3, we show that Riesz transform is of weak type $(1, 1)$, is continuous on L^p for all $p \in (1, \min_i n_i)$, and is unbounded on L^p otherwise.

**DIMENSION-FREE ESTIMATES FOR MAXIMAL FUNCTIONS OVER CONVEX BODIES;
FROM THE CONTINUOUS TO A DISCRETE SETTING.****Błażej Wróbel** University of Wrocław

The topic of dimension-free L^p estimates for Hardy-Littlewood maximal functions over convex bodies in \mathbb{R}^d had been mainly developed in the 80' and 90' in the work of Stein, Bourgain, Carbery, and Müller. The interest in the topic has been recently renewed due to recent progress by Aldaz 2011 and Bourgain 2014. However, up to now, nothing has been done in the discrete context, i.e. when \mathbb{R}^d is replaced by \mathbb{Z}^d .

In this talk we present first dimension-free results for discrete Hardy-Littlewood maximal functions. We prove a comparison principle that allows us to deduce partial results in the discrete case from the continuous one. Then we provide an example showing that the phenomenon of dimension-free estimates in the discrete context is not as robust as in the continuous case. Finally, we focus on the discrete cube, where we were able to obtain some positive results.

The talk is based on joint work with J. Bourgain, M. Mirek, and E.M. Stein.

25

Nonlinear Operators, Approximation Algorithms for Operator Equations and Related Problems

Organizers:

Agnieszka Chlebowicz (Rzeszów University of Technology)

Giuseppe Marino (Università della Calabria)

Tomasz Zajęc (Rzeszów University of Technology)

ON THE INTEGRAL EQUATIONS OF ERDÉLYI-KOBER TYPE

Agnieszka Chlebowicz Rzeszów University of Technology

We discuss the solvability of a quadratic integral equation of Erdélyi-Kober type of the following form:

$$x(t) = h(t) + \frac{g(t, x(t))}{\Gamma(\alpha)} \int_0^t \frac{ms^{m-1}s^p f(t, s, x(s))}{(t^m - s^m)^{1-\alpha}} ds \quad (1)$$

in the class of functions defined and continuous on the real half-axis with the growth bounded by a suitable power function. In our study we apply the technique of measures of noncompactness and the Darbo fixed point theorem. Moreover, we discuss a special case of the equation (1) with $g(t, x(t)) = 1$, in which we use simpler tools to prove the existence of the solution.

SOLUTIONS OF VOLTERRA-STIELTJES INTEGRAL EQUATIONS IN THE CLASS OF FUNCTIONS CONVERGING AT INFINITY

Agnieszka Dubiel Rzeszów University of Technology

The aim of the talk is to present results concerning the solvability of a nonlinear Volterra-Stieltjes integral equation in the class of real functions defined, bounded and continuous on the real half-axis \mathbb{R}^+ and having finite limits at infinity.

The considered class of integral equations contains a few types of nonlinear integral equations, such as the Volterra-Hammerstein integral equation, the Volterra-Chandrasekhar integral equation, the Volterra-Wiener-Hopf integral equation and others.

We will utilize the concepts related to the theory of functions of bounded variation, the classical Schauder fixed point principle and a suitable criterion for relative compactness in the Banach space of real functions defined, bounded and continuous on \mathbb{R}^+ .

ON SOME RELATIONSHIP BETWEEN MEASURES OF NONCOMPACTNESS AND REFLEXIVITY OF THE BANACH SPACE

Szymon Dudek Rzeszów University of Technology

We consider the function defined by formula

$$\mu_{si}(A) := \sup\{\|x\| : x \in A\} - \inf\{\|y\| : y \in \overline{\text{co}}(A)\}, \quad (25.1)$$

where A is a bounded subset of some Banach space. We recall some considerations given by D. Ariza-Ruiz and J. Garcia-Falset and concerning properties of Banach space which guarantee that the function (25.1) is the measure of noncompactness. Moreover, we give an answer to the open question asked by these authors. This answer states some connection between Kadec-Klee property, strictly convexity, reflexivity of the Banach space and properties of the function (25.1).

Moreover, the above mentioned authors noticed that every reflexive space has μ -fixed point property for every measure of noncompactness μ and asked the following open question:

If a Banach space E has μ -fixed point property for every measure of noncompactness μ , is E reflexive?

We give the answer using the classical James theorem.

- [1] S. DUDEK, L. OLSZOWY, *On generalization of Darbo-Sadovskii's type fixed point theorems for iterated mappings in Fréchet spaces*, J. Fixed Point Theory Appl. (in press).

A MEASURE OF NONCOMPACTNESS IN THE SPACE OF FUNCTIONS WITH INCREMENTS TEMPERED BY THE MODULUS OF CONTINUITY AND ITS APPLICATION TO THE THEORY OF INTEGRAL EQUATIONS

Rafał Nalepa Rzeszów University of Technology

In our lecture, we present a measure of noncompactness in the space of functions with increments tempered by the modulus of continuity. Then, using a technique associated with a measure of noncompactness and classic fixed point theory of Darbo type we prove an existence theorem concerning some integral equation. In our considerations, we will study two kinds of nonlinear quadratic integral equations. One of them will be considered in the space of function satisfying the Hölder condition on the closed interval and the second one in the space of function satisfying the Hölder condition on the half-axis.

THE SUPERPOSITION OPERATOR IN THE SPACE OF FUNCTIONS CONTINUOUS AND CONVERGING AT INFINITY ON THE REAL HALF-AXIS

Beata Rzepka Rzeszów University of Technology

We will consider the so-called superposition operator in the space $CC(\mathbb{R}_+)$ of real functions defined, continuous on the real half-axis \mathbb{R}_+ and converging to finite limits at infinity. We will assume that the function $f = f(t, x)$ generating the mentioned superposition operator is uniformly continuous with respect to the variable x , uniformly for $t \in \mathbb{R}_+$. Moreover, we require that the function $t \rightarrow f(t, x)$ satisfies the Cauchy condition at infinity uniformly with respect to the variable x . Under the above indicated assumptions a few properties of the superposition operator in question are derived. Examples illustrating our considerations will be also included.

REPRESENTATION THEOREMS FOR LOCAL OPERATORS

Małgorzata Wróbel Institute of Mathematics, Częstochowa University of Technology

Let X be a topological space and Y, Z be arbitrary nonempty sets. Denote by $\mathcal{G} = \mathcal{G}(X, Y)$ and $\mathcal{H} = \mathcal{H}(X, Z)$ two classes of functions $\varphi : X \rightarrow Y$ and $\phi : X \rightarrow Z$, respectively. A mapping $K : \mathcal{G} \rightarrow \mathcal{H}$ is said to be a *topologically locally defined operator* (briefly: a *locally defined operator* or a $(\mathcal{G}, \mathcal{H})$ -local operator), if for every open set $U \subset X$ and for all functions $\varphi, \psi \in \mathcal{G}$ the following implication holds true:

$$\varphi|_U = \psi|_U \Rightarrow K(\varphi)|_U = K(\psi)|_U.$$

The mappings of this type are sometimes called the *operators with memory*. Every Nemytskij (sometimes also called composition or superposition) operator $H : \mathcal{G} \rightarrow \mathcal{H}$ generated by the function of two variables h in the following way:

$$H(\varphi)(x) = h(x, \varphi(x)), \quad \varphi \in \mathcal{G} \quad (x \in X),$$

is locally defined. It turns out that in the case when the topological space X , the sets Y, Z , and the classes of functions $\mathcal{G} = \mathcal{G}(X, Y)$ and $\mathcal{H} = \mathcal{H}(X, Y)$ are more special ones, this definition of Nemytskij operator is not sufficient to describe the possible forms of the locally defined operators. In the talk we give the representation theorems for locally defined operators mapping some function spaces into itself.

WELL-POSED MINIMIZATION PROBLEMS IN THE SENSE OF FURI AND VIGNOLI

Tomasz Zajac Rzeszów University of Technology

The idea of the well-posedness of minimization problems comes back to A. N. Tikhonov (1966) and was subsequently developed by E. S. Levitin and B. T. Polyak (1966) and next by M. Furi and A. Vignoli (1970). The main aim of this presentation is to give some examples of well-posed minimization problems in the sense of Furi and Vignoli.

26

Noncommutative Harmonic Analysis, Noncommutative Probability and Quantum Groups

Organizers:

Marek Bożejko (Polish Academy of Science)

Franco Fagnola (Politecnico di Milano)

Janusz Wysoczański (Uniwersytet Wrocławski)

ON VACUUM DISTRIBUTION FOR SUMS OF POSITION OPERATORS IN MONOTONE AND WEAKLY MONOTONE FOCK SPACES

Vitonofrio Crismale Dipartimento di Matematica, Università degli studi di Bari Aldo Moro

We provide a recurrence formula to compute atoms and weights for the (discrete) vacuum distribution of sums of creation and annihilation operators $s_i := a_i + a_i^\dagger$ in monotone Fock space. The result is obtained in a direct way, without using monotone convolution, and exploiting some properties of palindromic polynomials. Moreover, we show the law above is a basic measure on the spectrum of the unital C^* -algebra generated by $\sum_{i=1}^n s_i$. This allows us to achieve the norm for any finite sum of gaussian operators as the right endpoint of the support of its vacuum distribution.

In the weakly monotone case, we establish the s_i are monotone independent, and moreover any of them has the distribution given by the Wigner semi-circle law. We explicitly compute the law for the sum of two position operators, i.e the monotone convolution of the Wigner law by himself, as an absolutely continuous measure w.r.t. the Lebesgue one. Moreover, we state that for $m \geq 3$ the vacuum law is indeed absolutely continuous, symmetric and compactly supported on symmetric intervals.

The talk is based on joint works with Y. G. Lu, M. E. Griseta and J. Wysoczanski.

FOURIER ANALYSIS FOR TYPE III REPRESENTATIONS OF THE NONCOMMUTATIVE TORUS

Francesco Fidaleo Dept. Mathematics, Univ. Tor Vergata, Roma

For the noncommutative 2-torus, we define and study Fourier transforms arising from representations of states with central supports in the bidual, exhibiting a possibly nontrivial modular structure (i.e. type III representations).

We then prove the associated noncommutative analogous of Riemann-Lebesgue Lemma and Hausdorff-Young Theorem. In addition, the L_p -convergence result of the Cesaro means (i.e. the Fejer theorem), and the Abel means reproducing the Poisson kernel are also established, providing inversion formulae for the Fourier transforms in L_p -spaces, p in $[1,2]$.

Finally, in $L^2(M)$ we show how such Fourier transforms “diagonalise” appropriately some particular cases of modular Dirac operators, the latter being part of a one parameter family of modular spectral triples naturally associated to the previously mentioned non type II_1 representations.

LIMIT DISTRIBUTIONS OF RANDOM MATRICES. HILBERT SPACE APPROACH WITH APPLICATIONS.

Romuald Lenczewski Wydział Matematyki, Politechnika Wrocławska

We present a Hilbert space approach to the limit joint $*$ -distributions of complex independent random matrices, using direct sums and direct integrals of Hilbert spaces. The limit operators are expressed in terms of canonical creation and annihilation operators living in the discrete or continuous fibers of the considered Hilbert space of Fock type. We give two applications of this approach: (1) a bijective proof of the formula for $*$ -moments of the triangular operator of Dykema and Haagerup, using the enumeration formula of Chauve, Dulucq and Rechnitzer for alternating ordered rooted trees, (2) computation of limit moments of a general class of matrices of Wishart type in terms of generalized multivariate Fuss-Narayana polynomials.

POSITIVE DEFINITE FUNCTIONS ON COXETER GROUPS

Wojciech Młotkowski Uniwersytet Wrocławski

For an element w of a Coxeter group (W, S) , with a reduced representation $w = s_{i_1} s_{i_2} \dots s_{i_n}$, we define its length $|w| := n$, colour $S(w) := \{s_{i_1}, s_{i_2}, \dots, s_{i_n}\}$ and colour-length $\|w\| := \#S(w)$. We are going to study positive definite functions on W which are constant on elements of the same length, colour and colour length respectively. The talk is based on a joint work with Światosław Gal and Marek Bożejko.

THE ROLE OF THE ATOMIC DECOHERENCE-FREE SUBALGEBRA IN THE STUDY OF QUANTUM MARKOV SEMIGROUPS

Emanuela Sasso Genova University

In this talk we want to explain the relationships between the atomicity of the decoherence-free subalgebra, environmental decoherence, ergodic decomposition of the trace class operators, and the structure of fixed points. In particular, we show that, for a Quantum Markov Semigroup (QMS) with a faithful normal invariant state, the atomicity of the decoherence-free subalgebra and environmental decoherence are equivalent. Moreover, we characterize the set of reversible states and explicitly describe the relationship between the decoherence-free subalgebra and the fixed point subalgebra for QMSs with the above equivalent properties. Loosely speaking one can say that, for QMSs with a faithful invariant state, the same conclusions can be drawn replacing finite dimensionality of the system Hilbert space by atomicity of the decoherence-free subalgebra.

FREE CANTELLI PROBLEM AND SUBORDINATION FOR FREE CONVOLUTIONS

Kamil Szpojankowski Warsaw University of Technology

Cauchy transform of a probability measure μ (or a random variable X with distribution μ) is defined by

$$G_X(z) = \int_{\mathbb{R}} \frac{1}{z-x} d\mu.$$

Consider two freely independent random variables X and Y and its sum $X + Y$. Subordination of free additive convolution is a property of the Cauchy transform of the sum which says that

$$G_{X+Y}(z) = G_X(\omega(z)),$$

where ω is an analytic function such that $1/\omega(z)$ is again a Cauchy transform of some probability measure. Subordination of free convolution was discovered by Voiculescu and then generalized by Biane. It is known that similar subordination phenomenon holds for multiplicative free convolution. However nothing was known for more complicated non-commutative polynomials in two free random variables. I will present a new example of subordination in free probability which we discovered in context of studying free version of Cantelli problem. Free Cantelli problem is a conjecture which says that for free random variables X, Y both having Wigner semicircle law random variable

$$X + f(X)Yf^*(X)$$

has semicircle distribution if and only if f is constant. I will present an effective tool for calculating the distribution of $X + f(X)Yf^*(X)$.

Talk based on joint work with Franz Lehner (TU Graz, Austria).

LEVY PROCESSES AND COMPACT QUANTUM GROUPS

Anna Wysoczańska-Kula Uniwersytet Wrocławski

The notion of Levy process can be transported from the classical probability to noncommutative setting; in particular, it can be defined on compact quantum groups. Such processes are still in correspondence with convolution semigroup of states and, consequently, with their generators. However, it turns out that the analogue of Levy-Khinchine formula does not need to hold. Also, Levy processes on compact quantum groups link noncommutative probability and noncommutative geometry: symmetric processes give rise to Dirichlet forms and spectral triples.

27

Topology and Logic in Algebra and Functional Analysis

Organizers:

Jerzy Kąkol (A. Mickiewicz University)

Libor Vesely (Università degli Studi di Milano)

Clemente Zanco (Università degli Studi di Milano)

Grzegorz Plebanek (Instytut Matematyczny UWr)

INTERSECTION PROPERTIES OF THE UNIT BALL IN BANACH SPACES

Carlo Alberto De Bernardi Università Cattolica del Sacro Cuore

Let X be a real Banach space with the closed unit ball B_X and the dual X^* . We say that X has the intersection property (I) [generalized intersection property (GI), respectively] if, for each countable family [for each family, respectively] $\{B_i\}_{i \in A}$ of equivalent closed unit balls such that $B_X = \bigcap_{i \in A} B_i$, one has $B_{X^{**}} = \bigcap_{i \in A} B_i^{\circ\circ}$, where $B_i^{\circ\circ}$ is the bipolar set of B_i , that is, the bidual unit ball corresponding to B_i . During the talk, we present some recent results, obtained in collaboration with L. Vesely, concerning the relations between the properties (I) and (GI), and geometric and differentiability properties of Banach spaces.

ON BANACH SPACES OF CONTINUOUS FUNCTIONS ON FINITE PRODUCTS OF SEPARABLE COMPACT LINES

Artur Michalak Adam Mickiewicz University, Poznań, Poland

We study properties of Banach spaces $C(L, X)$ of all continuous functions from a finite product of compact lines L into a Banach space X . We show that if $L_1, \dots, L_k, K_1, \dots, K_n$ are nonmetrizable separable compact lines and X, Y are separable Banach spaces, then (1) the space $C(L_1 \times \dots \times L_k, X)$ is not isomorphic to any subspace of $C(K_1 \times \dots \times K_n, Y)$ whenever $k > n$,

(2) does not exist any continuous linear surjection from $C(L_1 \times \dots \times L_k, X)$ onto $C(K_1 \times \dots \times K_n, Y)$ whenever $n > k$. As a straightforward consequences of these results we obtain that if L is a nonmetrizable separable compact line and $k \neq n$, then spaces $C(L^n)$ and $C(L^k)$ are not isomorphic.

DELFS-KNEBUSCH GENERALIZED TOPOLOGY

Artur Piękosz Cracow University of Technology

Parts of the talk are based on joint works with Eliza Wajch, Cenap Özel and Mohammed Al Shumrani.

The generalized topology of Delfs and Knebusch was introduced in 1985. It allows to glue infinitely many semialgebraic sets and to build a semialgebraic version of homotopy theory. Luckily, the same methods give an o-minimal homotopy theory for o-minimal expansions of (real closed) fields. But the gluing method works also for any structure with a topology on the underlying set.

We take a closer look at this generalized topology. It induces the usual topology and many bornologies, for example: the bornology of small sets. It is natural to see a generalized topological space as a bornological universe with additional structure. We consider the problem of quasi-metrization in this context. We also consider the problem of strict compactification of generalized topological spaces.

The category of generalized topological spaces and strictly continuous mappings has many interesting full subcategories. Some of them (the small spaces, the locally small spaces) have nice simple descriptions or (the partially topological spaces) lead to new branches of topological algebra. New examples of topological constructs are obtained.

We also record the use of the Axiom of Choice or its weak forms.

INSTITUTE OF MATHEMATICS, CRACOW UNIVERSITY OF TECHNOLOGY, WARSZAWSKA 24, 31-155
KRAKÓW, POLAND

E-mail address: apiekosz@pk.edu.pl

TWISTED SUMS WITH c_0

Grzegorz Plebanek Instytut Matematyczny UW

We discuss recent results on the problem if for every nonmetrizable compact space K , the Banach space $C(K)$ admits a nontrivial twisted sum with c_0 .

SYSTEMS OF COORDINATES IN WLD BANACH SPACES

Tommaso Russo Università degli Studi di Milano

One fundamental tool for the study of normed space is the investigation of systems of coordinates that a given space can be furnished with. This is, of course, true already in finite dimensions, but it is in the infinite-dimensional setting that several non-equivalent notions of a system of coordinates are available. It is therefore an important issue to understand which are the optimal systems of coordinates a given class of Banach spaces can admit.

According to a celebrated result by Kunen, there exists (under the assumption of the Continuum Hypothesis) a non-separable Banach space with virtually no (uncountable) system of coordinates, in that it admits no uncountable biorthogonal system. When considering systems of coordinates in non-separable Banach spaces it is therefore necessary to focus on some subclasses of the class of all non-separable Banach spaces, in order to achieve positive results.

In this talk, based on a joint work with Petr Hájek and Tomasz Kania, we will focus most of our attention on the class of WLD Banach spaces and we shall discuss the existence of uncountable Auerbach systems.

ON COMPACT TREES WITH THE COARSE WEDGE TOPOLOGY

Jacopo Somaglia Università degli studi di Milano & Charles University

Working in the setting of non-separable Banach spaces we study the relations between trees endowed with the coarse wedge topology, and the classes of Valdivia compacta and Plichko spaces. In particular, we describe Valdivia compact trees in terms of inner structures. Special care is given to the spaces of continuous functions on trees. We prove they are Plichko spaces whenever the height of the tree is less than $\omega_1 \cdot \omega_0$. This result is achieved using the fact that all compact trees have the property (M) .

28

Operator Theory, Operator Algebras and Applications

Organizers:

Michał Wojtylak (Jagiellonian University)

Camillo Trapani (Università degli studi di Palermo)

Piotr Niemiec (Uniwersytet Jagielloński)

INEQUALITIES OF VON NEUMANN TYPE AND ULTRA CONTRACTIVITY OF ORNSTEIN-UHLENBECK SEMIGROUPS

Marek Bożejko Polish Academy of Science

We will consider the following subjects:

1. Von Neumann inequality for arbitrary contractions on a Hilbert space.
2. Generalized Gaussian fields and functor of second quantization related to Yang-Baxter operator T .
3. The Ornstein-Uhlenbeck semigroup related to Yang-Baxter operator T is ultracontractive if $\|T\| < 1$.
4. Applications to q -gaussian processes and new von Neumann algebras factors.

ENTROPY AND RELATIVE ENTROPY IN VON NEUMANN ALGEBRAS

Andrzej Łuczak Faculty of Mathematics and Computer Science

In 1960 I. Segal introduced the notion of entropy for normal states on a semifinite von Neumann algebra, being a natural generalisation of von Neumann entropy on the algebra of all bounded linear operators on a Hilbert space. In the talk, some properties of Segal entropy are presented such as subinvariance with respect to a normal positive linear unital map and (semi-)continuity. The problem of strong subadditivity of Segal entropy analogous to the famous problem of the same kind for von Neumann entropy will also be addressed. The most general notion of relative entropy due to Araki will be compared with the notion of mutual information between states due to Umegaki.

UNBOUNDED DERIVATIONS AND *-AUTOMORPHISMS GROUPS OF BANACH QUASI *-ALGEBRAS

Maria Stella Adamo Università degli Studi di Catania

Derivations have been studied since last century because of the many links and applications on different fields of Mathematics and Physics. One of them is the connection with strongly-continuous one-parameter groups for which derivations constitute infinitesimal generators.

The aim of the talk is to discuss about analogous results for derivations on Banach quasi *-algebras properly defined, where a Banach quasi *-algebra is a mathematical structure that may be seen as completion of a normed *-algebra with a coarser norm satisfying some conditions. This kind of structure allows us to obtain well known results in a more general framework.

This is a joint work with C. Trapani.

APPROXIMATION IN VARIATION FOR SAMPLING-TYPE OPERATORS

Laura Angeloni University of Perugia

The generalized sampling series and the sampling Kantorovich series represent important families of discrete operators (see, e.g., [2,3,4]), also thanks to their interesting applications to some problems of Signal and Image Processing: for such families of operators we will present approximation results in the setting of the space of functions with bounded variation. The starting point is a relation between the derivative of the generalized sampling series based on a family of kernel functions of averaged type and the sampling Kantorovich series acting on the derivative of the function: by means of such result we are able to prove a characterization of the space of the absolutely continuous functions in terms of convergence in variation. As a consequence, we also obtain convergence in variation for the sampling Kantorovich operators in case of averaged kernels and, by means of a different technique, for classical band-limited kernels. The problem of the order of approximation for the generalized sampling series has been also faced.

References

- [1] L. Angeloni, D. Costarelli, G. Vinti, *A characterization of the convergence in variation for the generalized sampling series*, in print in: *Annales Academiae Scientiarum Fennicae Mathematica*, (2018).
- [2] L. Angeloni, D. Costarelli, G. Vinti, *A characterization of the absolute continuity in terms of convergence in variation for the sampling Kantorovich operators*, preprint, submitted (2018).
- [3] P.L. Butzer, A. Fisher, R.L. Stens, *Generalized sampling approximation of multivariate signals: theory and applications*, *Note Mat.*, **1** (10), 173–191 (1990).
- [4] C. Bardaro, P.L. Butzer, R.L. Stens, G. Vinti, *Kantorovich-Type Generalized Sampling Series in the Setting of Orlicz Spaces*, *Sampl. Theory Signal Image Process.*, **6**, 29–52 (2007).
- [5] C. Bardaro, P.L. Butzer, R.L. Stens, G. Vinti, *Prediction by Samples From the Past With Error Estimates Covering Discontinuous Signals*, *IEEE Trans. Inform. Theory*, **56** (1), 614–633 (2010).

THREE TERM RECURRENCE RELATIONS FOR ORTHOGONAL POLYNOMIALS IN SEVERAL VARIABLES - AN OPERATOR THEORY APPROACH

Dariusz Cichoń Jagiellonian University

The theory developed in [1] is a far reaching attempt to generalize the Farvard theorem which characterizes sequences of polynomials which can be orthogonalized by a measure by means of the three term recurrence relations, which in the one variable case takes the form

$$x p_k = a_k p_{k+1} + b_k p_k + a_{k-1} p_{k-1}, \quad k \geq 0 \quad (a_k, b_k \text{ are real numbers}).$$

In the multi-variable case the polynomials have to be replaced by columns of polynomials of the same degree, and the coefficients take the matrix form. Moreover, if we care for generality, the equality should be replaced by equality modulo an ideal, which allows us to consider families of orthogonal polynomials which are far from being basis of the space of all polynomials. (This becomes crucial in the multi-variable case.) As expected,

the question of existence of orthogonalizing measure becomes much more subtle in the case of two or more variables, and some solutions this problem involve operator theory. Apart from the general theory we intend to provide some new examples as particular cases.

[1] D. Cichoń, J. Stochel, F.H. Szafraniec, Three term recurrence relation modulo ideal and orthogonality of polynomials of several variables, *Journal of Approximation Theory* **134** (2005), 11-64.

SESQUILINEAR FORMS ASSOCIATED TO SEQUENCES ON HILBERT SPACES

Rosario Corso Università degli Studi di Palermo

We will discuss the possibility of defining sesquilinear forms starting from one or two sequences of elements of a Hilbert space. One can associate operators to these forms and in particular look for conditions to apply representation theorems of sesquilinear forms, such as Kato's theorems.

The associated operators correspond to classic frame operators or weakly-defined multipliers in the bounded context. In general some properties of them, such as the invertibility and the resolvent set, are related to properties of the sesquilinear forms.

We will focus on special sequences, for instance lower semi-frames, weighted Riesz basis and images of an orthonormal basis through operators.

MINIMAL INDEX AND DIMENSION FOR 2-C*-CATEGORIES WITH FINITE-DIMENSIONAL CENTERS

Luca Giorgetti Università di Roma Tor Vergata, Dipartimento di Matematica

The notion of index, in the sense we deal with in this talk, goes back to the work of Jones on subfactors of type II_1 . In the absence of a trace, one can still define the index of a conditional expectation associated to a subfactor and look for expectations that minimize the index. This value is called the minimal index of the subfactor. We deal with inclusions of von Neumann algebras (not necessarily factorial, nor finite) with finite-dimensional centers and we introduce a more fundamental invariant, the matrix dimension, whose squared l^2 -norm equals the minimal index. This recovers a result of Jones in the special case of inclusions of finite-dimensional C^* -algebras, where the matrix dimension coincides with the Bratteli inclusion matrix. We discuss the properties of this matrix dimension (additivity, multiplicativity) and show how the theory of minimal index can be formulated in the more general and purely algebraic context of 2- C^* -categories. Joint work with Roberto Longo (Roma Tor Vergata), preprint arxiv:1805.09234.

ON COMPOSITIONS AND LINEAR COMBINATIONS OF PROJECTIONS AND CONTRACTIONS IN VON NEUMANN ALGEBRAS.

Adam Paszkiewicz Uniwersytet Łódzki

For a Hilbert space H , we shall discuss trajectories $(P_n \cdots P_1 x; n \in \mathbb{N})$, $x \in H$, for sequences $P_1, P_2, \dots \in \{Q_1, \dots, Q_k\}$, for some fixed contractions Q_1, \dots, Q_k in H (or in some von Neumann algebra \mathcal{M} acting in H). We point out a number of basic technics connected, in particular, with

dilations theory in $B(H)$ and tracial states in finite von Neumann algebras.

We shall also discuss some relations between structure of the algebra \mathcal{M} and the existence of decompositions of type $A = a_1P_1 + \dots + a_kP_k$ for $P_1, \dots, P_k \in \text{Proj.}\mathcal{M}$, for any a given $A \in \mathcal{M}_h$ and possible small k .

WEAK QUASI HOPF ALGEBRAS AND CONFORMAL FIELD THEORY

Claudia Pinzari Sapienza, Università di Roma, Dipartimento di Matematica

Hopf algebras have been introduced in the 50s with different motivations, algebraic topology and duality theory for locally compact groups. There has been a major breakthrough in the mid 80s with the discovery of quantum groups in the work of Drinfeld, Jimbo, and Woronowicz and of their deep connections with several other areas including topology, geometry, functional analysis and quantum physics. My talk will focus on the connections between quantum groups and conformal field theory. In particular, I shall describe a suitable generalization of the notion of quantum group introduced in the 90s by Mack and Schomerus and motivated by algebraic quantum field theory and I shall attempt to explain the utility of the quantum group viewpoint in the two major known approaches to CFT, conformal nets and vertex operator algebras.

The talk is based on a joint work with Sebastiano Carpi and Sergio Ciamprone.

ASYMMETRIC TRUNCATED TOEPLITZ OPERATORS AND CONJUGATIONS

Marek Ptak University of Agriculture, Kraków

Let H^2 be the Hardy space on the unit disc, identified as usual with a subspace of L^2 on the unit circle. With any nonconstant inner function θ we associate the model space K_θ^2 , defined by $K_\theta^2 = H^2 \ominus \theta H^2$. In this space we can define the conjugation (antilinear, isometric, involution) $C_\theta: K_\theta^2 \rightarrow K_\theta^2$ by $C_\theta f(z) = \theta \overline{zf(z)}$.

Let us consider two nonconstant inner functions α and θ such that α divides θ . For a certain function $\varphi \in L^2$ we can define an asymmetric truncated Toeplitz operator $A_\varphi: K_\theta^2 \rightarrow K_\alpha^2$ by $A_\varphi f = P_\alpha(\varphi f)$, where $P_\alpha: L^2 \rightarrow K_\alpha^2$ is the orthogonal projection. Characterizations of bounded asymmetric truncated Toeplitz operators with L^2 symbols are given in terms of rank two operators. The relations between this characterizations and the symbol of the operator will be presented.

The results concerning asymmetric truncated Toeplitz operators with L^2 symbols between two different model spaces given by inner functions such that one divides the other will be presented. A description of the class of symbols for which the corresponding asymmetric truncated Toeplitz operator is equal to the zero operator is also given. The relation between an asymmetric truncated Toeplitz operators and natural conjugations C_θ, C_α will be investigated. The relations are different than in symmetric case $\theta = \alpha$.

Joint work with C. Câmara, K. Kliś–Garlicka.

APPROXIMATION BY SAMPLING TYPE OPERATORS AND APPLICATIONS

Gianluca Vinti University of Perugia

Some approximation results by means of sampling type operators will be presented and the results obtained will be applied to digital image processing. Finally, some applications to concrete problems in the medical and engineering fields, will be considered.

HILBERT SPACE GEOMETRY PROBLEMS OCCURING IN THE TOMITA-TAKESAKI THEORY

Laszlo Zsido Università di Roma Tor Vergata

Each normal weight on a von Neumann algebra is (by a result of U. Haagerup) the pointwise least upper bound of the majorized bounded linear functionals. This is a basic ingredient in the treatment of the fundamental facts of the Tomita-Takesaki Theory, but is not enough to reduce the case of general faithful, semi-finite, normal weights to the case of (everywhere defined) faithful normal linear functionals.

In the talk we propose a “spatial” approximation of an arbitrary faithful, semi-finite, normal weight φ on a von Neumann algebra M with bounded normal functionals. Essentially we approximate φ with its (bounded) restrictions φ_e to the reduced von Neumann algebras eMe , where $e \in M$ are projections with $\varphi(e) < +\infty$. Difficulties arise because in general we don't have $\varphi(eae) \leq \varphi(a)$ for every $a \in M^+$, and because the family of all projections of finite weight is not upward directed. We are approximating appropriately the identity operator on the Hilbert space H_φ of the GNS representation of φ with the orthogonal projections onto the Hilbert spaces of the GNS representations of the functionals φ_e (considered subspaces of H_φ) and succeed to reduce the fundamentals of the Tomita-Takesaki Theory for general faithful, semi-finite, normal weights to the case of bounded functionals.

29

Operator Semigroups: New Challenges and Applications

Organizers:

Yuri Tomilov (IM PAN)

Diego Pallara (University of Salento)

INSTABILITIES IN A COMBUSTION MODEL WITH TWO FREE INTERFACES

Davide Addona Università di Milano Bicocca

We study in a strip of R^2 a combustion model of flame propagation with stepwise temperature kinetics and zero-order reaction, characterized by two free interfaces, respectively the ignition and the trailing fronts. The latter interface presents an additional difficulty because the non-degeneracy condition is not met. We turn the system to a fully nonlinear problem which is thoroughly investigated. When the width ℓ of the strip is sufficiently large, we prove the existence of a critical value Le_c of the Lewis number Le , such that the one-dimensional, planar, solution is unstable for $0 < Le < Le_c$. Some numerical simulations confirm the analysis.

FRAGMENTATION AND FRAGMENTATION-COAGULATION PROCESSES WITH GROWTH AND DECAY WELL-POSEDNESS AND LONG TERM ASYMPTOTICS

Jacek Banasiak University of Pretoria

Motivated by problems in mathematical biology, we consider classical fragmentation-coagulation model augmented by growth and decay terms that correspond to vital processes occurring inside the clusters. Using recent results on analyticity and compactness of the fragmentation operator, we prove the AEG property for the fragmentation-growth-decay model and the existence of classical solutions for the full equation.

IRREGULAR CONVERGENCE OF MILD SOLUTIONS OF SEMILINEAR EQUATIONS

Adam Bobrowski Politechnika Lubelska

I will discuss a theorem saying that even irregular convergence of semigroups of operators implies similar convergence of mild solutions of the related semi-linear equations with Lipschitz continuous nonlinearity. This result will then be applied to three models originating from mathematical biology: shadow systems, diffusions on thin layers, and fast diffusions in regions separated by semi-permeable membranes. This is a joint work with Markus Kunze and Tomasz Lipniacki.

MAXIMAL REGULARITY FOR DIVERGENCE-FORM OPERATORS WITH NEUMANN BOUNDARY CONDITIONS IN ROUGH DOMAINS

Andrea Bruno Carbonaro University of Genova

Let $\Omega \subseteq \mathbb{R}^n$ be open and A be a complex uniformly accretive matrix function on Ω . Consider the divergence-form operator $L^A = -\operatorname{div}(A\nabla)$ with Neumann boundary conditions in Ω . We show that the associated parabolic problem $u'(t) + L^A u(t) = f(t)$, $u(0) = 0$ has maximal regularity in $L^p(\Omega)$, for all $p \in (1, +\infty)$ such that A satisfies an algebraic condition called p -ellipticity. The given range of exponents is optimal for this class of operators.

The talk is based on a work in progress with Oliver Dragičević

OPTIMAL DECAY RATE FOR THE INDIRECT STABILIZATION OF SYSTEMS OF HYPERBOLIC EQUATIONS

Roberto Guglielmi Gran Sasso Science Institute

The talk deals with the indirect stabilization of systems of hyperbolic-type equations, such as wave and plate equations with different boundary conditions, weakly coupled inside the domain or at the boundary. By energy method, we show that a single feedback allows to stabilize the full system at a polynomial rate. Furthermore, relying on an abstract result on the stability of semigroups, we exploit refined resolvent estimates in order to improve the decay rate of the energy of certain hyperbolic systems.

MAXIMAL REGULARITY, ANALYTIC SEMIGROUPS, AND GENERAL WENTZELL BOUNDARY CONDITIONS WITH A DIFFUSION TERM ON THE BOUNDARY

Davide Guidetti Università di Bologna

We consider linear parabolic problems of second order with dynamic boundary conditions on the boundary containing also a diffusion term. We prove results of maximal regularity in L^p settings, and generation of analytic semigroups. We illustrate the connection with corresponding problems with Wentzell boundary conditions.

The discussed results are joint work with G. Ruiz Goldstein, J. A. Goldstein and S. Romanelli.

THE INITIAL VALUES SPACES FOR ABSTRACT CAUCHY PROBLEMS

Sebastian Król Nicolaus Copernicus University, Toruń, Poland

Motivated by recent applications of weighted maximal regularity we study the initial values spaces corresponding to the first-order Cauchy problems. We show equivalence of the trace method and the K-method in this general setting, identify real interpolation spaces between a Banach space and the domain of a sectorial operator, and reprove an extension of Dore's theorem on the boundedness of holomorphic functional calculus. This is joint work with Ralph Chill

WEIGHTED CALDERON-ZYGMUND AND RELLICH INEQUALITIES IN L^p

Giorgio Metafune University of Salento, Lecce

We prove optimal Rellich inequalities with weights in L^p , both in the whole space and in bounded domains containing the origin. Weighted Calderon-Zygmund inequalities are deduced.

THE SPHERICAL HEAT SEMIGROUP AND GENUINELY SHARP ESTIMATES OF ITS KERNEL

Adam Nowak Institute of Mathematics, Polish Academy of Sciences

We prove sharp two-sided global estimates for the heat kernel associated with a Euclidean sphere of arbitrary dimension. Curiously enough, this seems to be a new result even for the usual sphere of dimension 2.

This is a joint work with Peter Sjögren (University of Gothenburg and Chalmers) and Tomasz Z. Szarek (Institute of Mathematics of the Polish Academy of Sciences).

GRADIENT ESTIMATES FOR SDEs WITHOUT MONOTONICITY TYPE CONDITIONS

Enrico Priola University of Turin (Italy)

We prove gradient estimates for transition Markov semigroups (P_t) associated to SDEs driven by multiplicative Brownian noise having possibly unbounded C^1 -coefficients, without requiring any monotonicity type condition. In particular, first derivatives of coefficients can grow polynomially and even exponentially. We establish pointwise estimates with weights for $D_x P_t \varphi$ of the form

$$\sqrt{t} |D_x P_t \varphi(x)| \leq c(1 + |x|^k) \|\varphi\|_\infty,$$

$t \in (0, 1]$, $\varphi \in C_b(R^d)$, $x \in R^d$. To prove the result we use two main tools. First, we consider a Feynman–Kac semigroup with potential V related to the growth of the coefficients and of their derivatives for which we can use a Bismut–Elworthy–Li type formula. Second, we introduce a new regular approximation for the coefficients of the SDE. At the end of the paper we provide an example of SDE with additive noise and drift b having sublinear growth together with its derivative such that uniform estimates for $D_x P_t \varphi$ without weights do not hold. This is a joint work with G. Da Prato (Pisa).

STOCHASTIC OPERATORS AND SEMIGROUPS: ASYMPTOTIC DECOMPOSITION AND APPLICATIONS

Ryszard Rudnicki Instytut Matematyczny PAN

Stochastic operators and semigroups have been intensively studied because they play a special role in applications [1]. They are used to investigate the long-time behaviour of the distributions of Markov processes like diffusion processes and piecewise deterministic processes. We present new results concerning the long-time behaviour of

substochastic semigroups [2,3]. Then we present some corollaries useful in studying of piecewise deterministic Markov processes [4].

[1] R. R. Stochastic operators and semigroups and their applications in physics and biology, in: Lecture Notes in Mathematics 2126, 2015. [2] K. Pichór, R. R., Asymptotic decomposition of substochastic operators and semigroups, J. Math. Anal. Appl. 2016. [3] K. Pichór, R. R., Asymptotic decomposition of substochastic semigroups and applications, Stochastics and Dynamics, 2018. [4] R.R., M.Tyran-Kamińska, Piecewise Deterministic Processes in Biological Models, SpringerBriefs in Applied Sciences and Technology, 2017.

ASYMPTOTIC BEHAVIOUR OF SOLUTION TO A CLASS OF ANISOTROPIC OPERATORS

Vincenzo Vespri University of Florence

In this talk we consider anisotropic operators of the type:

$$u_t = \sum_{k=1}^j \sum_{s=m_1}^{m_k} \operatorname{Div}(|D_{k_s} u|^{p_k-2} D_{k_s} u)$$

with $p_k > 2$ for any k . Thanks to suitable $L^1 - L^\infty$ (these estimates are fundamental for proving the ipercontractivity of the associated semigroup), we are able to study the asymptotic behaviour of the solutions. Moreover we can show that these estimates are sharp.

RIESZ TRANSFORMS AND SYMMETRIC CONTRACTION SEMIGROUPS

Błażej Wróbel University of Wrocław

In 1983 E. M. Stein proved dimension free L^p bounds for classical Riesz transforms on \mathbb{R}^d . Since then many authors studied the phenomenon of dimension free estimates for Riesz transforms defined in various contexts. In this talk we present a fairly general scheme for deducing the dimension free L^p boundedness of d -dimensional Riesz transforms from the L^p boundedness of one-dimensional Riesz transforms. The crucial tool we use is an H^∞ joint functional calculus for commuting generators of symmetric contraction semigroups.

30

Set Theory and Topology

Organizers:

Robert Rałowski (Wrocław University of Science and Technology)

Matteo Viale (University of Turin)

Paweł Krupski (Politechnika Wrocławska)

IDEAL CONVERGENT SUBSERIES AND REARRANGEMENTS OF SERIES IN BANACH SPACES

Marek Balcerzak Łódź University of Technology

Let I be a 1-shift-invariant ideal on \mathbb{N} with the Baire property. Assume that a series $\sum x_n$ with terms in a real Banach space X is not unconditionally convergent. We show that the sets of I -convergent subseries and of I -convergent rearrangements of a given series are meager in the respective Polish spaces. A stronger result, dealing with I -bounded partial sums of a series, is obtained if X is finite-dimensional. We apply the main theorem to series of functions with the Baire property, from a Polish space to a separable Banach space over \mathbb{R} , under the assumption that the ideal I is analytic or coanalytic.

Also, we discuss two methods of coding subseries of $\sum x_n$: by the choice of a sequence of increasing indices, or by the choice of parameters $t(n) \in \{0, 1\}$ multiplied by x_n . In the second approach, we study the Haar measure on $\{0, 1\}^{\mathbb{N}}$ of the set of I -convergent subseries of a divergent series in a Banach space.

- [1] M. Balcerzak, M. Popławski, A. Wachowicz, *The Baire category of ideal convergent subseries and rearrangements*, *Topology Appl.* **231** (2017), 219–230.
 - [2] M. Balcerzak, M. Popławski, A. Wachowicz, *Ideal convergent subseries in Banach spaces*, *Quaest. Math.*, in press.
-

THE CONTINUITY OF INJECTIVE DARBOUX FUNCTIONS BETWEEN MANIFOLDS

Taras Banakh Jan Kochanowski University in Kielce

A function $f : X \rightarrow Y$ between topological spaces is Darboux if the image of each connected subset of X is a connected subset of Y . It is an open problem (due to Willie Wang) if every Darboux permutation of \mathbb{R}^n is a homeomorphism. We give a partial answer to this problem proving that a Darboux injective function $f : X \rightarrow Y$ between connected metrizable spaces is continuous if one of the following conditions is satisfied: (1) Y is a 1-manifold and X is compact; (2) Y is a 2-manifold and X is a closed n -manifold of dimension $n \geq 2$; (3) Y is a 3-manifold and X is a simply-connected closed n -manifold of dimension $n \geq 3$.

A LELEK-LIKE DISCONNECTED COMPACT METRIC SPACE

Gianluca Basso Université de Lausanne, Università di Torino

Given a class of compact metric spaces and continuous surjections, the theory of projective Fraïssé limits reduces the problem of finding a universal and approximately homogeneous space for said class to a combinatorial problem: checking whether amalgamation holds for a class of finite structures and epimorphisms. We reverse such approach by proving that an interesting class of finite structures is Fraïssé and characterizing the compact metric space to which it gives rise. Such space, while disconnected, shares some similarities with the Lelek fan. We then attempt to find the optimal class of spaces and maps for which it is universal and approximately homogeneous. This is joint ongoing work with R. Camerlo.

SECRET CONNECTIONS BETWEEN BANACH SPACES AND ANALYTIC P-IDEALS

Piotr Borodulin-Nadzieja University of Wrocław

Many classical Banach spaces can be constructed by a certain procedure from families of finite sets. In a similar way, from families of finite sets, one can construct classical analytic P-ideal. Using this simple remark we may define (potentially) new examples of Banach spaces, motivated by analytic P-ideals and vice versa. We will discuss those new examples and connections between Banach spaces and analytic P-ideals defined by the same families.

THE OPEN GRAPH DICHOTOMY AND THE SECOND LEVEL OF THE BOREL HIERARCHY

Raphaël Carroy Kurt Gö del Research Center, University of Vienna

I will explain how variants of the open graph dichotomy can be used to obtain various descriptive-set-theoretical dichotomies at the second level of the Borel hierarchy. This shows how to generalise these dichotomies from analytic metric spaces to separable metric spaces by working under the axiom of determinacy.

If time allows it, I will also discuss some connections between cardinal invariants and the chromatic number of the graphs at stake.

SPLITTING CHAINS

David Chodounský Institute of Mathematics of the Czech Academy of Sciences

Splitting chains are linearly ordered splitting families in $P(\omega)/fin$. We will discuss the existence of these objects.

GENERALIZED DESCRIPTIVE SET THEORY UNDER I_0

Vincenzo Dimonte Università degli Studi di Udine

Generalized Descriptive Set Theory is the study of “simple” subsets of spaces of the form 2^κ , where κ is an uncountable cardinal. The classic way to approach this is to consider κ to be regular, but we are going to introduce a new line of research where κ is a cardinal of cofinality ω . In this setting, all the independence vagaries of the classical case disappear, and we can prove results analogous to the 2^ω case, like the Suslin and Silver dichotomies. We argue that this is because the tree-structure of subsets of 2^κ and 2^ω is very much alike, and, as a consequence, we show that postulating I_0 will yield determinacy-like results on the space 2^κ (for example proving the perfect set property for all projective sets). Finally, we notice that many results can be transferred to a generality of non-separable topological spaces. Joint work with Luca Motto Ros and Xianghui Shi.

WEAK VERSIONS OF COMPACTNESS AND THEIR PRODUCTIVITY

Mirna Dzamonja University of East Anglia

Tychonoff topology of the product of topological spaces was introduced with the purpose of making the product of compact spaces compact. We are interested in compactness-like properties that are preserved by more generous products, such as the κ -box product for an uncountable κ . This is closely related to large cardinal notions. For example, what is the large cardinal strength of the notion of square compactness studied by Hajnal and Juhasz in the 1970s? It is known to be between the weak and the strong compactness. We introduce some topological and measure-theoretic ideas to study the question.

The work presented is joint research with David Buhagiar from the University of Malta.

ON FEEBLY COMPACT (SEMI)TOPOLOGICAL SEMIGROUPS

Oleg Gutik National University of Lviv

We shall discuss some new results on feebly compact semitopological and topological semigroups whose obtained by author and his colleagues.

ON SETS WHICH CAN BE MOVED AWAY FROM SETS OF A CERTAIN FAMILY

Grażyna Horbaczewska University of Łódź

TBA

PERMUTATION GROUPS

Aleksandra Kwiatkowska University of Wrocław and University of Münster

We discuss a number of results on infinite permutation groups, that is, closed subgroups of the symmetric group on a countable set, or equivalently, automorphism groups of countable structures. We will focus on ample generics, where a group G has ample generics if for every n , the diagonal conjugacy action of G on G^n has a comeager orbit, on similarity classes, and on topological generators of permutation groups. For example, we show that for a permutation group G , under mild assumptions, for every n and an n -tuple \vec{f} in G , the countable group generated by \vec{f} is discrete, or precompact, or the conjugacy class of \vec{f} is meager. Finally, we will present results on conjugacy classes and on similarity classes of automorphism groups of structures equipped with a linear order, such as the ordered random graph, the ordered rational Urysohn metric space, or the ordered random poset, and many other extremely amenable permutation groups. This is joint work with Maciej Malicki.

LARGE CARDINALS IN THE STABLE CORE

Sandra Müller Kurt Gödel Research Center, University Vienna

The Stable Core \mathbb{S} , introduced by Sy Friedman in 2012, is a proper class model of the form $(L[S], S)$ for a simply definable predicate S . He showed that V is generic over the Stable Core (for \mathbb{S} -definable dense classes) and that the Stable Core can be properly contained in HOD. These remarkable results motivate the study of the Stable Core itself. In the light of other canonical inner models the questions whether the Stable Core satisfies GCH or whether large cardinals in V imply their existence in the Stable Core naturally arise. We answer these questions and show that GCH can fail at all regular cardinals in the Stable Core. Moreover, we show that measurable cardinals in general need not be downward absolute to the Stable Core, but in the special case where $V = L[\mu]$ is the canonical inner model for one measurable cardinal, the Stable Core is in fact equal to $L[\mu]$.

This is joint work with Sy Friedman and Victoria Gitman.

ON A CARDINAL INVARIANT RELATED TO THE HAAR MEASURE PROBLEM

Gianluca Paolini Einstein Institute of Mathematics of the Hebrew University of Jerusalem

In a recent work of Tsaban et al., given a metrizable profinite group G , a cardinal invariant of the continuum $\text{fm}(G)$ was introduced, and a positive solution to the Haar Measure Problem for G was given under the assumption that $\text{non}(\mathcal{N}) \leq \text{fm}(G)$. We prove here that it is consistent with ZFC that there is a metrizable profinite group G_* such that $\text{non}(\mathcal{N}) > \text{fm}(G_*)$, thus demonstrating that the strategy of Tsaban et al. does not suffice for a general solution to the Haar Measure Problem.

PRODUCTS OF LUZIN-TYPE SETS WITH COMBINATORIAL PROPERTIES

Piotr Szewczak Cardinal Stefan Wyszyński University in Warsaw

A topological space is Menger if for any sequence $\mathcal{U}_1, \mathcal{U}_2, \dots$ of open covers of the space, there are finite sets $\mathcal{F}_1 \subseteq \mathcal{U}_1, \mathcal{F}_2 \subseteq \mathcal{U}_2, \dots$ such that the union $\bigcup_{n \in \mathbb{N}} \mathcal{F}_n$ covers the space. If we can request that the sets $\mathcal{F}_1, \mathcal{F}_2, \dots$ are singletons, then the space is Rothberger. A subset of the real line, of cardinality at least $\text{cov}(\mathcal{M})$, is $\text{cov}(\mathcal{M})$ -Luzin if its intersection with any meager subset of the real line has cardinality strictly smaller than $\text{cov}(\mathcal{M})$. Each $\text{cov}(\mathcal{M})$ -Luzin set is Rothberger, and thus Menger. In 2003, assuming $\text{cov}(\mathcal{M}) = \mathfrak{c}$, Bartoszyński, Shelah, and Tsaban constructed two $\text{cov}(\mathcal{M})$ -Luzin sets whose all finite powers are Rothberger but its product space is not Menger. We show that such sets exist, assuming $\text{cov}(\mathcal{M}) = \text{cof}(\mathcal{M})$ and $\text{cov}(\mathcal{M})$ is regular. Our proof, in contrast to the topological construction of Bartoszyński, Shelah, and Tsaban, is purely combinatorial. We apply this result to local properties of function spaces with pointwise convergence topology. This is a joint work with Grzegorz Wiśniewski.

31

Computational Aspects of Applied Topology

Organizers:

Pawel Dlotko (Swansea University)

Massimo Ferri (Univ. Bologna)

Maurizio Garrione (Politecnico di Milano)

DRAWING CONTRACTIBLE 2-COMPLEXES IN R^4

Bruno Benedetti University of Miami

From a topological viewpoint, trees are the graphs that are contractible. It is well known that all trees are planar graphs. We will discuss whether the latter fact extends to higher dimensions: For example, can we draw any contractible complex into a Euclidean space of twice its dimension? The answer is a very strong “NO!” in dimension two, and a weaker “no” in all other dimensions. The reason comes ultimately algebra: Some presentations of the trivial group are hard to simplify.

(Joint work with K. Adiprasito)

TOPOLOGICAL ASPECTS OF COLORED TENSOR MODELS IN QUANTUM GRAVITY

Paola Cristofori University of Modena and Reggio Emilia

$(d + 1)$ -colored graphs, that is $(d + 1)$ -regular graphs endowed with a proper edge-coloration, are the objects of a long-studied representation theory (*GEM theory*) for PL d -manifolds.

Quite recently, in Mathematical Physics, a link between colored graphs and random tensors has been established through *Colored Tensor Models* (CTM), that generalize to higher dimension the 2-dimensional Quantum Gravity model of random matrices: the Feynman graphs of a d -dimensional CTM are precisely $(d + 1)$ -colored graphs.

In this context a particular significance is assumed by the *G-degree*, which is the sum of the genera of the closed surfaces where a colored graph admits a particular type of embedding. This gives rise to the definition of an invariant, again called *G-degree*, for PL manifolds, or, more generally, pseudomanifolds.

In this talk we discuss some properties of G-degree for singular manifolds and we establish classification results for all 3-pseudomanifolds and in dimension 4 for the case of manifolds.

The results presented in the talk are obtained in joint works with M.R. Casali, S. Dartois and L. Grasselli.

CLASSIFICATION OF FILTERED CHAIN COMPLEXES

Barbara Giunti Università di Pavia

Persistent homology has proven to be a useful tool to extract information from data sets. Homology, however, is a drastic simplification and in certain situations might remove too much information. This prompts us to study filtered chain complexes. We prove a structure theorem for filtered chain complexes and list all possible indecomposables. We call these indecomposables interval spheres and classify them into three types. Two types correspond respectively to finite and infinite interval modules, while the third type is unseen by homology. The structure theorem states that any filtered chain complex can be written as the unique sum of interval spheres, up to isomorphism and permutation. The proof is based on a hierarchy of full subcategories of the category of filtered chain complexes. Such hierarchy suggests an algorithm for decomposing filtered chain complexes. The implementation is ongoing, and the first results also retrieve the usual persistent barcodes.

A DISCRETE MORSE-BASED APPROACH FOR MULTIDIMENSIONAL PERSISTENT HOMOLOGY

Federico Iuricich Clemson University

Due to the constantly increasing complexity of available data, we are in need of tools for analyzing multiple filtrations (i.e., cell complexes with a vector-valued function defined on their vertices). Multidimensional Persistent Homology (MPH), a generalization of persistent homology, has been defined for this purpose. However, computing MPH is very challenging even with data of modest size and scalable algorithms are needed for extracting this information efficiently. In this talk, we will present a new approach to Multidimensional Persistent Homology computation, inspired by Discrete Morse theory, based on reducing the input complex through the definition of a discrete gradient field.

PERSISTENT HOMOLOGY OF MORSE DECOMPOSITION IN COMBINATORIAL DYNAMICS.

Michał Lipiński Jagiellonian University

We investigate combinatorial dynamical systems on simplicial complexes considered as finite topological spaces. Such systems arise in a natural way from sampling dynamics and may be used to reconstruct some features of the dynamics directly from the sample. We study the homological persistence of Morse decompositions of such systems, an important descriptor of the dynamics, as a tool for validating the reconstruction. Our approach may be viewed as a step toward applying the classical persistence theory to data collected from a dynamical system. This is a joint work with T. K. Dey, M. Juda, T. Kapela, J. Kubica and M. Mrozek.

ROUNDNESS OF CELLS IN CELLULAR MICROSTRUCTURES

Frank Lutz TU Berlin

Cellular microstructures, such as foams or polycrystalline materials, are composed of cells of varying size and shape. Typically, the cells have the combinatorial types of 3-dimensional simple polytopes, and together they tile 3-dimensional space.

Now, some of the occurring cell types in the microstructures are substantially more frequent than others - and we will see that the frequent types are “combinatorially round”. This property of cell types then gives us a starting point for a topological analysis of cellular microstructures.

TOPOLOGY AND CURVATURE OF BRAIN FUNCTIONAL NETWORKS

Giovanni Petri ISI Foundation

Topology, one of the oldest branches of mathematics, provides an expressive and affordable language progressively pervading many areas of biology, computer science and physics. Topological data analysis (TDA) tools have emerged as able to provide insights into high-dimensional, noisy and non-linear datasets coming from very different subjects. Here I will illustrate what novel insights TDA techniques are yielding, with particular

attention to the study of the functional and structural connectomes. Comparing topological observables with recent geometric observables (Ricci-Foreman curvature on simplicial complexes) I will show how topological observables capture and distinguish variations in the mesoscopic functional organization between i) drug-induced altered brain states, ii) perceptual states and the corresponding mental images, and iii) across a large age span highlighting the presence of dynamically coordinated compensation mechanisms, which cannot be reduced to local geometrical changes.

CONLEY INDEX APPROACH TO SAMPLED DYNAMICS

Mateusz Przybylski Jagiellonian University

In the talk we describe the construction of a multivalued upper semicontinuous map from experimental data. Such a map may induce a map in cohomology. Although multivalued map need not admit any continuous selector, we may use recently developed Conley theory for such maps.

We focus on the structure of an isolated invariant set S with respect to a multivalued map. Since isolating neighborhoods for multivalued maps do not necessarily admit index pairs, we work with weak index pairs. We present accurate results to detect orbits passing through the disjoint components of S in a given fashion. Conditions that guarantee their existence are expressed in terms of the index map I_P associated with a weak index pair P for S . Applying the Lefschetz-type fixed point theorem we provide sufficient conditions for the periodicity of such orbits. Moreover, we discuss the semiconjugacy with a shift dynamics of finite type.

This is a joint work with B. Batko, K. Mischaikow and M. Mrozek.

FIXED POINT ARGUMENTS FOR DYNAMICAL SYSTEMS, PERIODIC ORBITS AND BIFURCATION

Elena Queirolo VU Amsterdam

In this talk we present applications of fixed point methods to numerical validation in dynamical systems. We are in particular interested in continuation of periodic orbits of ODEs depending on parameters. After reformulating the problem in a space of quickly decaying Fourier coefficients, we turn the search of a periodic orbit into the search of the fixed point of an operator T on this space. We use the computer to rigorously prove the contractivity of our operator near a (given) numerical solution, thus enabling an existence proof for a fixed point of T , corresponding to the periodic orbit we are looking for. The step from the validation of a single periodic orbit and a branch segment is handled by extending the operator T to include a pseudo-arc length parameter, and applying a uniform contraction principle. This setup can be adapted to the rigorous validation of various types of bifurcations, including Hopf bifurcations.

ZIGZAG HOMOTOPY THEORY AND ITS RELATION TO TOPOLOGICAL DATA ANALYSIS

Jan Spaliński Warsaw University of Technology

The talk will describe how zigzag modules of Carlsson and de Silva [1] can be studied from the perspective of classical homotopy theory. This is established by proving that there is a Quillen model category structure [3] on certain related categories. The model structure is established using the methods described in the paper [2]. Consequences for topological data analysis will be discussed.

[1] G. Carlsson and V. de Silva, Zigzag Persistence, *Found. Comput. Math.* (2010), pp. 367–405.

[2] W. G. Dwyer and J. Spalinski, Homotopy theories and model categories, *Handbook of Algebraic Topology*, Elsevier (1995)

[3] D. G. Quillen, *Homotopical Algebra*, SLNM 43, Springer, (1967).

SEMANTIC RELATIONS FROM A TOPOLOGICAL VIEWPOINT

Mauricio Toro Champalimaud Foundation

Nowadays artificial neural networks (ANN) are recognised as a powerful tool for data analysis and exploration. Oftentimes, these algorithms generate high-dimensional representations of the analysed data in order to classify, regress, or organise them, by minimising an appropriate error function. Topological persistence proved to be a precious tool in discovering the intrinsic organisation of data embedded in high-dimensional spaces.

Here, we will consider the special case of word embeddings, i.e. vector spaces whose geometry is generated via an ANN by considering the particular organisation of words in large corpora of documents. First, we will show how persistent homology and the bottleneck distance can be used to measure context-dependent semantic shift of a given set of words. Second, a topological-based, multi-objective distance will be introduced in order to query a given word embedding by considering a collection of filtrating functions.

EMBEDDINGS OF SIMPLICIAL COMPLEXES IN EUCLIDEAN SPACES FROM THE ALGORITHMIC VIEWPOINT.

Stephan Zhechev IST Austria

A very classical problem, probably going back to ancient Greece, is telling whether a given graph G is planar, i.e. whether there exists an embedding of G in the plane. In the early 1930's Kuratowski gave a criterion for a graph to be planar, and in the early 1980's Hopcroft and Tarjan devised a linear-time algorithm for deciding graph planarity. A natural generalisation of graph planarity is asking whether a given k -dimensional simplicial complex K embeds in d -dimensional Euclidean space. In this talk I will give an overview of recent algorithmic results in the so called meta-stable range, which is when $2d \geq 3(k+1)$ this range, a classical theorem by Haefliger and Weber gives a necessary and sufficient condition for the existence of an embedding. In more recent work Cadek, Matousek and others provided an algorithm deciding embeddability. Moreover, if d is fixed, the algorithm runs in polynomial time on the number of simplices in K . A natural question at this point is whether an embedding could be algorithmically constructed, when it exists. While devising an algorithm for constructing explicit embeddings in the meta-stable range is still a far reaching goal, several steps towards it have been made, and will be briefly discussed.

32

Geometric Aspects of Applied Topology

Organizers:

José Carrasquel (Adam Mickiewicz University of Poznań)

Claudia Landi (Università di Modena e Reggio Emilia)

Wacław Marzantowicz (UAM, Poznań)

Ulderico Fugacci (TU Graz)

HYPERPLANES ARRANGEMENTS AND GRAPHS IN SOCIAL CHOICE AND MANAGEMENT

Gennaro Amendola eCampus University

We will describe a model introduced by Marengo and Settepanella (by means of hyperplane-arrangements) to mathematise a problem of Social Choice where agents are faced with a set of alternatives that are bundles of interdependent elements. We will describe how graph theory can be tied to Marengo and Settepanella's model, and how these mathematical tools can be applied also to study allocation of decisions on policies that are bundles of interdependent elements. We will also describe some (numerical) results achieved by means of these theories. (Joint work - in progress - with Marengo and Settepanella.)

PERSISTENT HUBS IN WEIGHTED GRAPHS

Mattia Bergomi Champalimaud Foundation

Graphs and networks occupy an essential role in both data representation and analyses (e.g., social networks and artificial neural networks respectively). We try to answer one of the most fundamental questions arising when studying these types of data: Is it possible to rank by importance every node of a given graph, with respect to a user-defined property? In other words, we aim at defining an agile tool for the exploration of weighted graphs, without losing the fundamental observer-based flavour of topological persistence.

We answer this question with a generalisation of the standard persistent homology approach to the study of weighted graphs from a combinatorial viewpoint, overcoming the the classical framework's necessity of building an "auxiliary simplicial complex". We show how it is possible to define filtrating functions capable of both identify and rank hubs in a given network, without leaving a combinatorial framework.

TOPOLOGICAL COMPLEXITY OF CONFIGURATION SPACES

Andrea Bianchi Universität Bonn

Topological complexity was introduced by Michael Farber in the early 2000s in the context of topological robotics, and measures the minimal instability that a motion planner over a space must have. We address the problem of determining, or at least bounding, the topological complexity of unordered configuration spaces of aspherical surfaces. Our lower bounds rely on a general theorem on topological complexity of aspherical spaces proved in 2015 by Grant, Lupton and Oprea, whereas upper bounds rely on explicit, geometric constructions of motion planners. This is a joint work with David Recio-Mitter.

TOPOLOGICAL COMPLEXITY AND EFFICIENCY OF MOTION PLANNING ALGORITHMS

Zbigniew Błaszczyk Adam Mickiewicz University

I will discuss a variant of Farber's topological complexity, defined for smooth compact Riemannian manifolds, which takes into account only motion planners with the lowest possible "average length" of the output paths.

The talk is based on joint work with J. G. Carrasquel Vera.

A COMPARISON BETWEEN BREDON AND RELATIVE COHOMOLOGICAL DIMENSIONS

Arturo Espinosa Adam Mickiewicz University of Poznań

We recover a classic notion of relative cohomology of groups, and use it to compare said cohomological dimension with the Bredon cohomological dimension of a group with respect to a semifull family (i.e, a family of subgroups closed by conjugation and finite intersections) generated by a single subgroup. This is a joint work with Z. Błaszczyk and J.G. Carrasquel-Vera.

TOPOLOGICAL COMPLEXITY OF SYMPLECTIC MANIFOLDS

Mark Grant University of Aberdeen

Topological complexity is a numerical homotopy invariant defined by Farber as part of his topological approach to robot motion planning problems. While it is closely related to the more classical Lusternik-Schnirelmann (LS) category, it is in general harder to compute.

I will present recent joint work with Stephan Mescher, in which we prove that the topological complexity of any closed symplectically atoroidal manifold is equal to twice its dimension. The condition "symplectically atoroidal" means that the symplectic class vanishes on images of tori. This result is the analogue for topological complexity of a theorem of Oprea and Rudyak from 1999, which stated that the LS category of any closed symplectically aspherical manifold equals its dimension.

Whereas those authors employed the notion of category weight, we use the notion of TC weight due to Farber and myself. Our proof involves a careful analysis of the Mayer-Vietoris sequence for fibred joins of the free path fibration, and differential forms on the free loop space.

THE THIN LOOP SPACE

Sadok Kallel American University of Sharjah

The "thin loop" space (or thin fundamental group) intervenes when computing holonomies of smooth fiber bundles with connection. We verify that for a finite simplicial complex X and for piecewise linear loops on X , the thin loop space is of the same homotopy type as the space of continuous loops. This gives a topological group model for the loop space. The analogous result is not true for higher loop spaces. This work is joint with Moncef Ghazel.

A DIGITAL HOPF FIBRATION

Gregory Lupton Cleveland State University

I will present a progress report on some work joint with John Oprea and Nick Scoville. An n -dimensional digital image is a finite subset of the integer lattice in \mathbb{R}^n , together with an adjacency relation. For instance, a 2-dimensional digital image is an abstraction of an actual digital image consisting of pixels. Our work consists of developing notions and techniques from homotopy theory in the setting of digital images. In an extensive literature, a number of authors have introduced concepts from topology into the study of digital images. In particular, there have been attempts to study digital fibrations. The notions of digital fibration in the literature, however, do not seem satisfactory from a homotopy point of view. Indeed, most of the constructs most useful in homotopy theory, such as path and loop spaces, are absent from the literature. We put forward a digital analogue of the Hopf fibration. This talk will be elementary, and will include a survey of the basics on topological notions in the setting of digital images.

REALIZATION PROBLEM FOR REEB GRAPHS

Łukasz Patryk Michalak Adam Mickiewicz University

The Reeb graph $\mathcal{R}(f)$ of a smooth function $f: M \rightarrow \mathbb{R}$ with finitely many critical points on a closed manifold M is defined by contracting the connected components of level sets of f . Sharko and Masumoto–Saeki showed that each graph with the so-called good orientation is the Reeb graph of a function on a closed surface. In this talk we focus on the problem of realization of a graph as the Reeb graph of a function on a given manifold. We need to study the Reeb number $\mathcal{R}(M)$ which is the maximal number of cycles among all Reeb graphs of functions on M . It turns out that $\mathcal{R}(M)$ is equal to the corank of fundamental group of M what has been also showed by Gelbukh for orientable manifolds. We give a complete characterization of Reeb graphs for surfaces and a realization up to homeomorphism of graphs for higher-dimensional manifolds.

PERSISTENT HOMOLOGY OF PHASE TRANSITIONS

Francesco Vaccarino Politecnico di Torino

Persistent homology analysis is applied to the study of the phase transitions undergone by the so-called mean-field XY model and by the ϕ^4 lattice model, respectively. For both models, the relationship between phase transitions and the topological properties of certain submanifolds of configuration space are exactly known. It turns out that these a priori known facts are clearly retrieved by persistent homology analysis of dynamically sampled submanifolds of configuration space. Joint work with I. Donato, M. Gori, M. Pettini, G. Petri, S. De Nigris and R. Franzosi. Phys, Rev. E 93, 052138 (2019)

TOPOLOGICAL COMPLEXITY OF A CONNECTED SUM OF REAL PROJECTIVE SPACES

Lucile Vandembroucq University of Minho Portugal

The topological complexity of a connected sum of real projective planes, that is, a nonorientable surface of genus greater than or equal to 2, is known to be maximal. In this talk we will see that the analogous result holds for connected sums of higher dimensional real projective spaces. This is a joint work with D. Cohen.

A COMBINATORIAL APPROACH TO L-S CATEGORY AND TOPOLOGICAL COMPLEXITY

Jose Antonio Vilches Universidad de Sevilla

The goal of this talk is presenting discrete versions of Lusternik-Schnirelmann category and topological complexity in the simplicial context. Both invariants are defined inspired by its continuous counterparts, considering that a natural notion of homotopy between simplicial maps is belonging to same contiguity class. It allows us to establish links, under a geometrical point of view, with the strong simple homotopy type introduced by J. Barmak and G. Minian, where the key notion of strong collapse plays a central role. Main results of both notions will be presented, pointing out similarities and differences with the topological (continuous) case. Taking into account the hardness of computing L-S category or topological complexity in the continuous case, our combinatorial approach can be useful to establish bounds to estimate these topological invariants. It is interesting to point out that these combinatorial versions not only depend on the topology of the considered complex, but also they are linked to the chosen combinatorial structure (triangulation).

Joint work with D. Fernández-Ternero, E. Macías-Virgós and E. Minuz.

STABLE COMPONENTS OF DIRECTED SPACES

Krzysztof Ziemianski Polish Academy of Sciences

Directed spaces are objects that can be used for modeling behavior of concurrent programs. A directed space is a topological space, whose points represent possible states of a program, with a family of distinguished paths, which represent possible executions. Directed spaces allow for analysis of concurrent programs with means of algebraic topology. One of the problems of directed algebraic topology is that many classical, non-directed homotopy invariants of topological spaces have no satisfactory directed counterparts. In my talk, I will consider decompositions of directed spaces into a finite number of disjoint subsets that satisfy some “stability” axioms. I will show that a large class of directed spaces admits the “coarsest” decomposition into stable components. Next, I will define the component category for a system of stable components; this category is enriched in the homotopy category. I hope that such defined component categories will play a part of homotopy groups in directed algebraic topology.

33

Geometric Topology, Manifolds, and Group Actions

Organizers:

Światosław Gal (University of Wrocław)

Carlo Petronio (Università di Pisa)

Mattia Mecchia (Università degli Studi di Trieste)

Krzysztof M. Pawłowski (Adam Mickiewicz University, Poznań)

(DE)CONSTRUCTING ACTIONS ON PRODUCT MANIFOLDS WITH AN ASYMMETRIC FACTOR

Zbigniew Błaszczyk Adam Mickiewicz University

I will discuss transformation groups of manifolds of the form $M \times S^n$, where M is an asymmetric manifold, i.e. it does not admit any non-trivial action of a compact group. The starting point is the following question: $M \times S^n$ is clearly not asymmetric, but does it admit non-diagonal actions? We will see that the answer typically is “yes”; in fact, provided that $n \geq 2$, there exist infinitely many distinct non-diagonal effective circle actions on such products, and a similar result holds for cyclic groups of prime order.

The talk is based on joint work with M. Kaluba.

SOLVABLE COMPACT CLIFFORD-KLEIN FORMS

Maciej Bocheński University of Warmia and Mazury

Let G/H be a non-compact homogeneous space of a Lie group G . The space G/H admits a solvable compact Clifford-Klein form if there exists a discrete, solvable subgroup F of G such that F acts properly discontinuously, freely and co-compactly on G/H . In this talk I will discuss conditions under which a semisimple homogeneous space does not admit solvable compact Clifford-Klein forms. For example 3-symmetric spaces and almost all symmetric spaces do not admit solvable compact Clifford-Klein forms. This generalizes the well known non-existence theorem of Benoist for nilpotent compact Clifford-Klein forms.

CONCORDANCE OF CERTAIN 3-BRAIDS AND GAUSS DIAGRAMS

Michael Brandenbursky Ben Gurion University

Let $\alpha = \sigma_1^2 \sigma_2^{-2}$ be a braid in B_3 , where B_3 is the braid group on 3 strings and σ_1, σ_2 are the standard Artin generators. Using Gauss diagram formulas I will show that for each n not divisible by 3 the knot which is represented by the closure of the braid α^n is algebraically slice if and only if n is odd. As a consequence, I will deduce some properties of Lucas numbers.

REPRESENTING AND CLASSIFYING COMPACT PL 4-MANIFOLDS VIA REGULAR 5-COLORED GRAPHS

Maria Rita Casali University of Modena and Reggio Emilia (Italy)

The use of regular edge-colored graphs to combinatorially represent closed PL-manifolds of arbitrary dimension, has given rise during the last decades to several results in geometric topology, especially in dimension three and four.

Quite recently, the theory has been extended so as to represent by means of $(d+1)$ -colored graphs (i.e. $(d+1)$ -valent multigraphs endowed with a proper edge-coloration by means of the color set $\{0, 1, \dots, d\}$) ANY compact PL d -manifold, via the associated *singular d -manifold* obtained by capping off each boundary component by a cone.

In this talk we present the above representation method and some related properties and constructions, with a special focus on the 4-dimensional case. In particular, we show how to draw, directly from the planar diagram of a framed link (L, c) , a 5-colored graph representing the 4-manifold obtained by adding to the 4-disk a 2-handle along (L, c) (and whose boundary is the 3-manifold arising by Dehn surgery on (L, c)).

The representation of all compact PL d -manifolds via regular graphs enables also to define the PL invariant *generalized regular genus*, which extends the classical *regular genus* for closed PL d -manifolds. In dimension 4, some results about the classification of singular 4-manifolds (or, equivalently, compact PL 4-manifolds) via generalized regular genus are presented. Finally, we will hint to related achievements concerning the so called *G-degree* of edge-colored graphs, which is a crucial quantity in the context of colored tensor models, as an approach to quantum gravity.

CR STRUCTURES OF ONCE-PUNCTURED TORUS BUNDLES

Alex Casella Florida State University

The Cauchy-Riemann geometry (CR in short) is modelled on the three sphere and the group of its biholomorphic transformations. In 2008 Falbel makes use of ideal triangulations to show that the figure eight knot complement admits a (branched) CR structure. This three manifold belongs to a larger class of important three manifolds that are fiber bundles over the circle, with fiber space the once-punctured torus. In this talk we introduce the audience to these manifolds and show that almost every once-punctured torus bundle admits a (branched) CR structure.

A JOURNEY THROUGH LOOP BRAID GROUPS

Celeste Damiani University of Leeds

The study of loop braid groups has been widely developed during the last twenty years, in different domains of mathematics and mathematical physics. They have been called with several names such as motion groups, groups of permutation-conjugacy automorphisms, braid-permutation groups, welded braid groups, untwisted ring groups, ...and others! We will give a glance on how this richness of formulations carries open questions in different areas.

BIHARMONIC HYPERSURFACES IN EUCLIDEAN SPACES

Ram Shankar Gupta Guru Gobind Singh Indraprastha University, New Delhi, India

Chens conjecture on biharmonic submanifolds is an interesting area in differential geometry. We present some results on biharmonic hypersurfaces in Euclidean space with constant norm of second fundamental form.

COMPUTATIONAL APPROACH TO PROPERTY (T)

Marek Kaluba Adam Mickiewicz University

During the talk we will briefly discuss the the results of Ozawa which make the computational approach to the Kazhdan property (T) possible. It is known that property (T) is equivalent to positivity of the element $\Delta^2 - \lambda\Delta$ in the full group $*$ -algebra, where Δ is group Laplacian associated to a generating set. In Ozawa's formulation, positivity is equivalent to the existence of a sum-of-squares decomposition of $\Delta^2 - \lambda\Delta$ in the real group ring. This in turn is equivalent to the feasibility of a certain problem of (semi-definite) optimisation. We will describe the algorithm encoding the optimisation problem, and how an (imprecise) numerical solution can be turned into a mathematical proof. Since the problem for $\text{Aut}(F_n)$, the automorphism group of the free group on n generators is out of reach of the method (due to its size, even for $n = 4$) we will show how to use the representation theory of finite groups to obtain a smaller, equivalent problem. This leads to constructive (alas) computer-assisted proof that $\text{Aut}(F_5)$ has Kazhdan's property (T).

Property (T) for $\text{Aut}(F_n)$ has been a long-standing open problem and, as observed by Lubotzky and Pak, the positive resolution leads to better understanding of the effectiveness of the product replacement algorithm commonly used in computational group theory.

BOUNDEDNESS OF GROUPS AND APPLICATIONS

Jarosław Kędra University of Aberdeen

I discuss boundedness properties of groups and discuss various applications (to Hamiltonian dynamics and to finite simple groups). For example, I will show that the group of automorphisms of a regular tree does not admit an effective Hamiltonian action on a closed symplectic manifold. This is a recent joint work with Assaf Libman and Ben Martin.

FIBRINGS OF 3-MANIFOLDS AND RELATED GROUPS

Dawid Kielak Bielefeld University

A key step in Agol's proof of Thurston's virtually fibred conjecture is the following theorem of Agol: if G is the fundamental group of a 3-manifold with vanishing Euler characteristic, and G is residually finite rationally solvable (RFRS), then the manifold virtually fibres. I will discuss what virtual fibring means for groups, and then I will show how to drop the assumption of G being the fundamental group of a 3 manifold.

ENTROPY AND QUASIMORPHISMS

Michał Marcinkowski Uniwersytet Wrocławski & Regensburg Universität

Let S be a compact oriented surface and let $\text{Diff}(S, \text{area})$ be the group of area preserving diffeomorphisms of S . On $\text{Diff}(S, \text{area})$ we have interesting conjugacy invariant norm coming from dynamics: the entropy norm. During the talk I will explain how to construct certain quasimorphisms on $\text{Diff}(S, \text{area})$ and how to use them to prove that the entropy norm is unbounded.

NONEXISTENCE OF SMOOTH ONE FIXED POINT ACTIONS ON SPHERES

Piotr Mizerka Adam Mickiewicz University in Poznań

In this talk we focus on one fixed point actions of finite groups on spheres. According to the work of Laitinen, Morimoto, Oliver and Pawałowski, a finite group G has a smooth effective one fixed point action on some sphere if and only if G is an Oliver group. For some finite Oliver groups G of order up to 216, and for $G = A_5 \times C_n$ for $n = 3, 5, 7$, we present a strategy of excluding of smooth effective one fixed point G -actions on low-dimensional spheres. We mention also the recently obtained results for the case of $G = GL(3, 2)$.

IDEAL SIMPLICIAL VOLUME

Marco Moraschini University of Pisa

Simplicial volume is a homotopy invariant of compact manifolds introduced in 1982 by Gromov in his seminal paper “Volume and Bounded Cohomology”. Roughly speaking, the simplicial volume measures the complexity of a manifold in terms of real singular chains.

In this talk, we will define the *ideal simplicial volume*, a variation of the ordinary simplicial volume for compact manifolds with boundary. The main difference between ideal simplicial volume and the ordinary simplicial volume of a manifold M is that this new invariant measures the minimal size of possibly ideal triangulations of M “with real coefficients”, since ideal simplices are now allowed to appear in representatives of the fundamental class.

After having discussed the main properties of the ideal simplicial volume, we will see that for manifolds whose boundary components all have an amenable fundamental group, the ideal simplicial volume coincides with the classical one.

Finally, if we have enough time, we will discuss the precise computation of the ideal simplicial volume of an infinite family of hyperbolic 3-manifolds with geodesic boundary, for which the exact value of the classical simplicial volume is not known.

This is a joint work with Roberto Frigerio

REMARKS ON SMOOTH ACTIONS ON SPHERES

Masaharu Morimoto Graduate School of Natural Science and Technology,
Okayama University

Let G be a finite group. For a natural number m , a smooth G -action on a manifold is called an **m -fixed-point action** if the associated G -fixed point set consists of exactly m points. A smooth G -action is also called an **odd-fixed-point action** if the G -action is an m -fixed-point action for some odd integer m . Various authors have studied one-fixed-point G -actions and two-fixed-point G -actions on spheres, which are related to the Smith equivalence problem of tangential representations at G -fixed points. In the present talk, we give new observations on smooth actions on spheres, for example we discuss the dimension of spheres which admit smooth odd-fixed-point actions of the symmetric group on five letters as well as related groups.

ON THE COMPLEXITY OF NON-ORIENTABLE SEIFERT FIBRE SPACES

Michele Mulazzani Università di Bologna

We deal with Seifert fibre spaces, which are compact 3-manifolds admitting a foliation by circles. This definition is more general than Seifert original one, since these manifolds are locally modeled either on fibered solid tori or on fibered solid Klein bottles, and so the singular fibres are either isolated (as in Seifert definition) or form surfaces properly embedded in the manifold.

We introduce a combinatorial description of Seifert fibre spaces and state a classification theorem, up to fibrewise homeomorphism, in all the possible cases: orientable, non-orientable, closed, with boundary. Moreover, we compute potentially sharp upper bounds for their complexity in terms of the invariants of the combinatorial description, extending to the non-orientable case results by Fominykh and Wiest for the orientable case with boundary and by Martelli and Petronio for the closed orientable case.

This is a joint work with Sergei Matveev (Chelyabinsk State University), Alessia Cattabriga (University of Bologna) and Timur Nasybullov (Catholic University of Leuven Kulak).

FINITE GROUPS WITH THE BORSUK-ULAM PROPERTY

Ikumitsu Nagasaki Kyoto Prefectural University of Medicine

Elementary abelian groups have the Borsuk-Ulam property; i.e, Borsuk-Ulam type theorems hold for elementary abelian groups. In previous researches, it is known that almost finite groups other than elementary abelian groups do not have the Borsuk-Ulam property. In this talk, we show that the remained finite groups also do not have the Borsuk-Ulam property. Consequently a finite group G has the Borsuk-Ulam property if and only if G is an elementary abelian group.

FOUR-MANIFOLDS AS BRANCHED COVERS

Riccardo Piergallini Università di Camerino

The talk will concern some old and new results about representing four-manifolds in terms of branched coverings. In particular, it will be discussed how the most known representation theorem of closed smooth four-manifold as covers of the four-sphere can be extended to the cases of bounded, open, and topological four-manifolds, as well as to branched covers of different basic four-manifolds in the closed smooth case.

ON TWISTED COHOMOLOGY GROUPS OF THE AUTOMORPHISM GROUPS OF FREE GROUPS

Takao Satoh Tokyo University of Science

The study of cohomology groups of the automorphism groups of free groups has been developed with those of the mapping class groups of surfaces. In this talk, based on Morita's and Kawazumi's previous works on the first Johnson homomorphisms, we focus on low-dimensional twisted cohomology groups of the automorphism groups of free groups. The purpose of this talk is to show that there exist non-trivial unstable second cohomology classes. On the other hand, we also discuss an analogue of the theory of the Johnson homomorphisms to the ring of $SL(2, \mathbb{C})$ -representations of free groups.

EXAMPLES OF FOUR-DIMENSIONAL GEOMETRIC TRANSITION

Andrea Seppi University of Luxembourg

Roughly speaking, a geometric transition is a deformation of geometric structures on a manifold, by "transitioning" between different geometries. In his PhD thesis, J. Danciger introduced a new such transition, which enables to deform from hyperbolic structures to Anti-de Sitter structures, going through another type of real projective structures called "half-pipe", and provided conditions for a compact 3-manifold to admit a geometric transition of this type. In this talk, I will explain the construction of examples of finite-volume geometric transition in dimension 4. This is joint work with S. Riolo.

SUFFICIENT CONDITION TO BE A BORSUK-ULAM GROUP

Toshio Sumi Kyushu University

In 1991, Wasserman proposed a Borsuk-Ulam group. This corresponds with an isovariant map, which is an extension of the Borsuk-Ulam theorem. He also showed that many finite groups including solvable groups are Borsuk-Ulam groups. We are interested in simple finite Borsuk-Ulam group, since an extension of a Borsuk-Ulam group by a Borsuk-Ulam group is also a Borsuk-Ulam group. Nagasaki and Ushitaki (2013) showed that projective special linear groups $PSL(2, F)$ over a finite field F are Borsuk-Ulam groups. I will talk about how to apply linear programming for their expression and give new examples of simple Borsuk-Ulam groups.

HOW TO PRESENT ALL COMPACT TOPOLOGICAL 4-MANIFOLDS

Daniele Zuddas Universität Bayreuth

In the smooth or PL category, a compact 4-manifold can be presented by handle decompositions and triangulations. These tools are not available for non-smoothable topological 4-manifolds. So, it is natural to ask how to present topological 4-manifolds by means of a finite amount of combinatorial data. In this talk, we show how the problem in the TOP category can be reduced to the smooth category by smoothing a topological 4-manifold in the complement of a point (by Quinn's theorem) and then taking a certain compact smooth chunk of it, which is large enough to contain a locally flat 3-sphere, then allowing us to reconstruct the given manifold by cutting along the 3-sphere and capping by a ball. We give a certificate for such embedded 3-sphere. This is a joint work with Mike Freedman.

34

Stochastic Analysis and Nonlocal PDEs

Organizers:

Krzysztof Bogdan (Wrocław University of Science and Technology)

Enrico Priola (University of Turin)

SYMMETRY PROPERTIES FOR LONG-RANGE PHASE COEXISTENCE MODELS

Serena Dipierro University of Western Australia

We discuss some recent results on nonlocal phase transitions modelled by the fractional Allen-Cahn equation, also in connection with the surfaces minimising a nonlocal perimeter functional. In particular, we consider the “genuinely nonlocal regime” in which the diffusion operator is of order less than 1 and present some rigidity and symmetry results.

ON THE DUNKL HEAT KERNEL

Jacek Dziubański Uniwersytet Wrocławski

Dunkl operators were introduced by C.F. Dunkl in 1989. They are differential-difference operators on \mathbf{R}^N associated with a finite group generated by reflections. During the talk we shall derive estimates for the Dunkl kernel and for the integral kernels of the Dunkl heat and Poisson semigroups. Further we shall discuss applications of these estimates.

This is the first talk in a series of two devoted to harmonic analysis in the Dunkl setting. The second one will be given by A. Hejna in the Harmonic Analysis session. Each lecture is self-contained.

The talk is based on the joint articles with J.-Ph. Anker and A. Hejna.

ABSOLUTE CONTINUITY OF THE LAW FOR THE TWO DIMENSIONAL STOCHASTIC NAVIER-STOKES EQUATIONS

Benedetta Ferrario Università di Pavia

We consider the two dimensional Navier-Stokes equations in vorticity form; this is a stochastic nonlinear equation where the nonlinearity is of local type. We add a stochastic forcing term given by a Gaussian noise, white in time and coloured in space.

First, we improve the known results on existence and uniqueness of a solution in order to get a solution process continuous in time and space. Then we show that the solution $\xi(t, x)$ (evaluated at fixed points in time and space) is locally differentiable in the Malliavin calculus sense and that its image law is absolutely continuous with respect to the Lebesgue measure on \mathbb{R} .

This is based on a joint work with M. Zanella.

PROGRESSIVE INTRINSIC ULTRA CONTRACTIVITY FOR NONLOCAL SCHRÖDINGER OPERATORS

Kamil Kaleta Wrocław University of Science and Technology

We give sharp two-sided large time estimates of the heat kernel for a wide class of non-local Schrödinger operators with confining potentials, which are based on generators of Lévy processes. We identify a new useful regularity property of compact semigroups, which is weaker than asymptotic intrinsic ultracontractivity. It means that the space-time regularity of the semigroup essentially improves as the time parameter diverges to infinity. This is a joint work with René Schilling.

ON SOME INTEGRO-DIFFERENTIAL OPERATOR WHICH EXTENDS TO A GENERATOR OF A FELLER SEMIGROUP

Victoria Knopova TU Dresden

We consider an integro-differential operator

$$Lf(x) = b(x) \cdot \nabla f(x) + \int_{\mathbb{R}^d \setminus \{0\}} \left(f(x+u) - f(x) - \nabla f(x) \cdot u 1_{|u| \leq 1} \right) N(x, du),$$

defined on the space $C_\infty^2(\mathbb{R}^d)$ of twice continuously differentiable functions with vanishing at infinity derivatives. The drift $b \in \mathbb{R}^d$ is assumed to be bounded and Hölder continuous, and the Lévy-type kernel $N(x, du)$ is a sum of an α -stable like part and a lower order perturbation.

We show that under certain regularity assumptions the extension of $(L, C_\infty^2(\mathbb{R}^d))$ is the generator of a Feller semigroup $(P_t)_{t \geq 0}$.

The talk is based on the on-going work with A. Kulik and R. Schilling.

ON THE DOMAIN OF FRACTIONAL LAPLACIANS AND RELATED GENERATORS OF FELLER PROCESSES

Franziska Kuehn Institut de Mathématiques de Toulouse

In this talk we study the domain of the generator of stable processes, stable-like processes and more general pseudo- and integro-differential operators which naturally arise both in probability theory, as infinitesimal generators of Lévy- and Feller processes, and in analysis. We present conditions on the symbol of the operator ensuring that certain (variable order) Hölder and Hölder-Zygmund spaces are in the domain. The statements are obtained by means of probabilistic methods; in particular, we will investigate the small time asymptotics of generalized moments of a Feller process $(X_t)_{t \geq 0}$,

$$\lim_{t \rightarrow 0} \frac{\mathbb{E}^x f(X_t) - f(x)}{t}$$

for functions f which are not necessarily differentiable.

ON THE NONLOCAL DIRICHLET PROBLEM

Łukasz Leżaj Wrocław University of Science and Technology

We prove a classical representation formula for distributional solutions of the nonlocal Dirichlet generated by translation-invariant integrodifferential operators of the form

$$\mathcal{L}u(x) = \lim_{\varepsilon \rightarrow 0} \int_{|y| > \varepsilon} (u(x+y) - u(x)) v(y) dy,$$

where $v: \mathbb{R}^d \setminus \{0\} \mapsto [0, \infty)$ is a radial, non-increasing function satisfying $\int_{\mathbb{R}^d} (1 \wedge |x|^2) v(x) dx < \infty$, and study the question under which assumptions distributional solutions are twice differentiable in the classical sense. Sufficient conditions and counterexamples are provided.

REGULARIZING PROPERTIES FOR SOME STOCHASTIC DIFFERENTIAL EQUATIONS WITH DELAY

Federica Masiero Università di Milano Bicocca

In this talk we consider stochastic differential equations (SDEs) with delay in the state. It is well known that the Ornstein-Uhlenbeck transition semigroup doesn't have regularizing properties, such as the strong Feller property. So in general, the associated Hamilton-Jacobi-Bellman (HJB) equation cannot be solved by a classical fixed point argument. In this talk we consider SDEs with delay and with a special dependence on the past trajectory, and we show regularizing results for the transition semigroup of such SDEs. We consider controlled SDEs with delay and we solve in mild sense the associated HJB equation as a first step to solve the stochastic controlled problem related. The talk is based on a joint work (in progress) with G. Tessitore.

KERNEL ESTIMATES FOR ELLIPTIC OPERATORS WITH SINGULAR AND DISCONTINUOUS COEFFICIENTS

Giorgio Metafuno University of Salento, Lecce

We consider a family of second order elliptic operators which includes the Schroedinger operator with inverse square potential, and prove optimal upper bounds for the heat kernel. We prove also lower bounds and deduce two side-estimates for the Green function.

THE DIRICHLET PROBLEM FOR THE p -FRACTIONAL LAPLACE EQUATION

Giampiero Palatucci University of Parma

In this talk we consider nonlinear integro-differential operators of differentiability order $s \in (0, 1)$ and summability growth $p > 1$, whose model is the fractional p -Laplacian $(-\Delta)_p^s$,

$$(-\Delta)_p^s u(x) := \text{p. v.} \int_{\mathbb{R}^n} \frac{(u(x) - u(y))^{p-2} (u(x) - u(y))}{|x - y|^{n+sp}} dy.$$

We present several results for the corresponding weak supersolutions, as comparison principles, a priori bounds, lower semicontinuity, weak Harnack estimates, and many others. We then discuss the good definition of (s, p) -superharmonic functions, and we introduce the fractional counterpart of the celebrated Perron method in nonlinear Potential Theory.

This talk is based on a series of papers in collaboration with Agnese Di Castro (Sapienza University of Rome), Janne Korvenpää (Aalto University) and Tuomo Kuusi (Oulu University).

INFINITE DIMENSIONAL GRADIENT ESTIMATES FOR PERTURBED ORNSTEIN–UHLENBECK SEMIGROUPS IN CONVEX SETS

Diego Pallara University of Salento

Let X be a separable Hilbert space endowed with a non-degenerate centred Gaussian measure γ and let λ_1 be the maximum eigenvalue of the covariance operator associated with γ . The associated Cameron–Martin space is denoted by H . For a sufficiently regular convex function U and a convex set $\Omega \subseteq X$, we set $\nu = e^{-U}\gamma$ and we consider the semigroup $(T_\Omega(t))_{t \geq 0}$ generated by the self-adjoint operator defined via the quadratic form

$$(\psi, \varphi) \mapsto \int_{\Omega} \langle D_H \psi, D_H \varphi \rangle_H d\nu,$$

where φ, ψ belong to $D^{1,2}(\Omega, \nu)$, the Sobolev space defined as the domain of the closure of the gradient operator along H in $L^2(\Omega, \nu)$.

We prove pointwise gradient estimate for $(T_\Omega(t))_{t \geq 0}$. In particular, we show that

$$|(D_H T_\Omega(t)f)(x)|_H^p \leq e^{-p\lambda_1^{-1}t} (T_\Omega(t)|D_H f|_H^p)(x),$$

for any $p \in [1, +\infty)$ and f smooth enough. We deduce some relevant consequences of the previous estimate, such as logarithmic Sobolev inequality, Poincaré

'e inequality and some improving summability properties of $(T_\Omega(t))_{t \geq 0}$. Finally we investigate on the asymptotic behaviour of $T_\Omega(t)$ as $t \rightarrow +\infty$.

The results have been obtained in collaboration with L. Angiuli and S. Ferrari.

EXTENSION THEOREM FOR NONLOCAL OPERATORS

Katarzyna Pietruska-Pałuba University of Warsaw

The talk is based on the paper:

Krzysztof Bogdan, Tomasz Grzywny, Katarzyna Pietruska-Pałuba, Artur Rutkowski, *Extension theorem for nonlocal operators*, arXiv:1710.05880

We are concerned with the probabilistic method of solving the Dirichlet problem in a domain $D \subset \mathbf{R}^d$ for a nonlocal (Lévy) operator with external boundary data g . The weak solution is given by the harmonic extension of g (via the Poisson kernel). We establish a Hardy-Stein-type identity between a weighted Sobolev semi-norm of g and another weighted Sobolev semi-norm of the harmonic extension.

SELBERG DISTRIBUTIONS AND INTEGRAL REPRESENTATIONS IN DUNKL THEORY

Margit Rösler Paderborn University, Germany

In a manuscript dating back to the 1980ies, but published only a few years ago in the arXiv, I.G. Macdonald proposed a one-parameter extension of the classical theory of hypergeometric functions of matrix argument. His framework includes multivariable generalizations of classical Sonine integrals between Bessel functions of certain different indices which are based on Kadell's generalization of the Selberg integral. The involved

multivariable Bessel functions can be identified with Bessel functions of Dunkl type associated with a root system of type B, and their integral representations, which involve measures with Selberg densities, are closely related to integral representations for Dunkl's intertwining operator. In this talk we discuss distributional extensions of these Selberg measures and their implications for integral representations in Dunkl theory.

The talk is based on joint work with Michael Voit.

EXTENSION, TRACE AND REGULARITY FOR NONLOCAL DIRICHLET FORMS

Artur Rutkowski Wrocław University of Science and Technology

We consider nonlocal quadratic forms

$$\mathcal{E}_D[u] = \int_{(D^c \times D^c)^c} (u(x) - u(y))^2 \nu(x, y) dx dy,$$

where $D \subseteq \mathbb{R}^d$ is an open set, and ν is a Lévy-type kernel. We solve the extension and trace problem, i.e., we obtain a class of functions $g: D^c \rightarrow \mathbb{R}$ which first, can be extended to a function $u: \mathbb{R}^d \rightarrow \mathbb{R}$ such that $\mathcal{E}_D[u] < \infty$, and second, are traces (restrictions to D^c) of such functions.

We then use these results to prove that the form \mathcal{E}_D is a regular Dirichlet form, which leads to a class of Hunt processes.

The talk is based on joint papers with Krzysztof Bogdan, Tomasz Grzywny and Katarzyna Pietruska-Pańuba.

FUNDAMENTAL SOLUTION FOR SUPER-CRITICAL NON-SYMMETRIC LÉVY-TYPE OPERATORS

Karol Szczypkowski Wrocław University of Science and Technology

In the talk I will present the results of the paper [4], which is a sequel to [2]. The aim is to construct the fundamental solution p^κ to the equation $\partial_t = \mathcal{L}^\kappa$, where under certain assumptions the operator \mathcal{L}^κ takes the form,

$$\mathcal{L}^\kappa f(x) := \int_{\mathbb{R}^d} (f(x+z) - f(x) - 1_{|z|<1} \langle z, \nabla f(x) \rangle) \kappa(x, z) J(z) dz.$$

In particular, $J: \mathbb{R}^d \rightarrow [0, \infty]$ is a Lévy density, i.e., $\int_{\mathbb{R}^d} (1 \wedge |x|^2) J(x) dx < \infty$. The function $\kappa(x, z)$ is assumed to be Borel measurable on $\mathbb{R}^d \times \mathbb{R}^d$ satisfying $0 < \kappa_0 \leq \kappa(x, z) \leq \kappa_1$, and $|\kappa(x, z) - \kappa(y, z)| \leq \kappa_2 |x - y|^\beta$ for some $\beta \in (0, 1)$.

We concentrate on the case when the order of the operator is positive and smaller or equal 1 (without excluding higher orders up to 2). The lack of the symmetry of the Lévy density $\kappa(x, z)J(z)$ in z variable may cause a non-zero *internal drift*, which reveals itself as a gradient term in the decomposition

$$\begin{aligned} \mathcal{L}^\kappa f(x) &= \int_{\mathbb{R}^d} (f(x+z) - f(x) - 1_{|z|<r} \langle z, \nabla f(x) \rangle) \kappa(x, z) J(z) dz \\ &+ \left(\int_{\mathbb{R}^d} z (1_{|z|<r} - 1_{|z|<1}) \kappa(x, z) J(z) dz \right) \cdot \nabla f(x). \end{aligned}$$

Our approach rests on imposing conditions on the expression

$$\int_{r \leq |z| < 1} z \kappa(x, z) J(z) dz.$$

We prove the uniqueness, estimates, regularity and other qualitative properties of p^κ . The result is new even for 1-stable Lévy measure $J(z) = |z|^{-d-1}$, cf. [3] and [1].

- [1] Zhen-Qing Chen and Xicheng Zhang, *Heat kernels for time-dependent non-symmetric stable-like operators*, preprint 2017, arXiv:1709.04614.
 - [2] Tomasz Grzywny and Karol Szczypkowski, *Heat kernels of non-symmetric Lévy-type operators*, preprint 2018, arXiv:1804.01313.
 - [3] P. Jin, *Heat kernel estimates for non-symmetric stable-like processes*, preprint 2017, arXiv:1709.02836.
 - [4] Karol Szczypkowski, *Fundamental solution for super-critical non-symmetric Lévy-type operators*, preprint 2018, arXiv:1807.04257.
-
-

CHAOTIC ORBITS FOR NONLOCAL EQUATIONS AND APPLICATIONS TO ATOM DISLOCATION DYNAMICS IN CRYSTALS

Enrico Valdinoci Università di Milano

We consider a nonlocal equation driven by a perturbed periodic potential. We construct multibump solutions that connect one integer point to another one in a prescribed way. In particular, heteroclinic, homoclinic and chaotic trajectories are constructed. This result regarding symbolic dynamics in a fractional framework is part of a study of the Peierls-Nabarro model for crystal dislocations. The associated evolution equation can be studied in the mesoscopic and macroscopic limit. Namely, the dislocation function has the tendency to concentrate at single points of the crystal, where the size of the slip coincides with the natural periodicity of the medium. These dislocation points evolve according to the external stress and an interior potential, which can be either repulsive or attractive, depending on the relative orientations of the dislocations. For opposite orientations, collisions occur, after which the system relaxes exponentially fast.

INTERACTING PARTICLE SYSTEMS OF CALOGERO-MOSER-SUTHERLAND TYPE IN THE FREEZING REGIME

Michael Voit Technische Universität Dortmund

Interacting particle systems of CMS type on the real line are described via multivariate Bessel processes on Weyl chambers; they are closely related e.g. with Dyson's Brownian motion and several random matrix models. We present several limit theorems when the coupling constants, i.e. the drift parts, tend to infinity. For the root systems of type A and B, the limits are described in terms of the zeros of classical Hermite and Laguerre polynomials. Depending on the form of the starting points of the processes, the limit results as well as the techniques of the proofs are different; some use saddle point methods and asymptotics of the associated Bessel functions, some others use the associated SDEs. Some results were also derived earlier via the tridiagonal random matrix models of Dumitriu and Edelman.

The talk is based on recent common work with Sergio Andraus and Jeannette Woerner.

35

Financial Mathematics

Organizers:

Zbigniew Palmowski (Wrocław University of Science and Technology)

Marzia De Donno (University of Parma)

INDIFFERENCE PRICING OF LIFE INSURANCE CONTRACTS VIA BSDEs UNDER PARTIAL INFORMATION

Alessandra Cretarola University of Perugia - Department of Mathematics and Computer Science

In this paper we investigate the pricing problem of a pure endowment contract when the insurer has a limited information on the mortality intensity of the policyholder. The payoff of this kind of policies depends on the residual life time of the insured as well as the trend of a portfolio traded in the financial market, where investments in a riskless asset, a risky asset and a longevity bond are allowed. We propose a modeling framework that takes into account mutual dependence between the financial and the insurance markets via an observable stochastic process, which affects the risky asset and the mortality index dynamics. Since the market is incomplete due to the presence of basis risk, in alternative to arbitrage pricing we use expected utility maximization under exponential preferences as evaluation approach, which leads to the so-called indifference price. Under partial information this methodology requires filtering techniques that can reduce the original control problem to an equivalent problem in complete information. Using stochastic dynamics techniques, we characterize the value function in terms of the solution to a quadratic-exponential backward stochastic differential equation.

FLUCTUATION IDENTITIES FOR OMEGA-KILLED MARKOV ADDITIVE PROCESSES AND DIVIDEND PROBLEM

Irmina Czarna Wrocław University of Science and Technology

We solve the exit problems for an one-sided Markov additive process (MAP) which is exponentially killed with a bivariate killing intensity $\omega(\cdot, \cdot)$ dependent on the present level of the process and the present state of the environment. Moreover, we analyze respective resolvents. All identities are given in terms of new generalizations of classical scale matrices for the MAP. We also remark on a number of applications of the obtained identities to (controlled) insurance risk processes. In particular, we show that our results can be applied to the so-called Omega model, where bankruptcy occurs at rate $\omega(\cdot, \cdot)$ when the surplus process becomes negative. Finally, we consider the Markov modulated Brownian motion (MMBM) and present the results for the particular choice of piecewise intensity function $\omega(\cdot, \cdot)$.

DOUBLE CONTINUATION REGIONS FOR AMERICAN AND SWING OPTIONS WITH NEGATIVE DISCOUNT RATE IN LEVY MODELS

Marzia De Donno University of Parma

We study perpetual American call and put options in an exponential Levy model. We consider a negative effective discount rate which arises in a number of financial applications including stock loans and real options. In this case, a double continuation region may appear. We compute the price of the option and identify the two critical prices. We also generalize this result to multiple stopping problems of Swing type, that is, when successive exercise opportunities are separated by i.i.d. random refraction times.

THE VALUE OF INFORMATIONAL ARBITRAGE

Claudio Fontana Paris Diderot University (Paris VII)

In the context of a general semimartingale model of a complete market, we aim at answering the following question: How much is an investor willing to pay for learning some inside information that potentially allows to achieve arbitrage? If such a value exists, we call it the value of informational arbitrage. In particular, we are interested in the case where the inside information yields arbitrage opportunities but not unbounded profits with bounded risk. In the spirit of Amendinger et al. (2003), we provide a general answer to the above question by relying on an indifference valuation approach. To this effect, we establish some new results on models with inside information and study optimal investment-consumption problems in the presence of initial information and arbitrage, also allowing for the possibility of leverage. We characterize when the value of informational arbitrage is universal, in the sense that it does not depend on the preference structure.

OPTIONS PORTFOLIO SELECTION

Paolo Guasoni Dublin City University

We develop a new method to optimize portfolios of options in a market where European calls and puts are available with many exercise prices for each of several potentially correlated underlying assets. We identify the combination of asset-specific option payoffs that maximizes the Sharpe ratio of the overall portfolio: such payoffs are the unique solution to a system of integral equations, which reduce to a linear matrix equation under suitable representations of the underlying probabilities. Even when implied volatilities are all higher than historical volatilities, it can be optimal to sell options on some assets while buying options on others, as hedging demand outweighs demand for asset-specific returns.

ON HARTMAN-WATSON DISTRIBUTION

Jacek Jakubowski Institute of Mathematics, University of Warsaw

In my talk I present new results on the family of Hartman-Watson distributions, which is defined using the special function u . I present a new representation of u , and a new form of the probability density function of the fundamental functional $A_t = \int_0^t e^{2B_u} du$, especially important for the problem of pricing Asian options (see, e.g. [1], [3]). Integral relations of convolution type between Hartman-Watson distributions and modified Bessel functions I_0 and K_0 will be presented.

The talk is based on the joint paper with M. Wiśniewolski ([2]).

Bibliography

- [1] Geman H., Yor M., Bessel processes, Asian options, and perpetuities, *Math. Finance* 3 (1993), 349-375.
 - [2] Jakubowski J., Wiśniewolski M., *Another look at the Hartman-Watson distributions*, submitted
 - [3] Lyasoff A., Another look at the integral of exponential Brownian motion and the pricing of Asian options, *Finance and Stochastics* 20(4), (2016)
-

BSDE'S WITH RANDOM HORIZON AND QUADRATIC HEDGING

Mariusz Niewęłowski Warsaw University of Technology

We consider BSDE's on random interval driven by general martingale of the form

$$Y_t = \eta + \int_t^\sigma (a(Y_u)dN_u + g(Y_u)d\langle M \rangle_u) + \int_t^\sigma \psi_u dM_t + L_\sigma - L_t$$

where N is a bounded counting process, M is a martingale and L is a martingale orthogonal with M and σ is bounded stopping time. We prove existence and uniqueness of solutions for such BSDE's. We show that one can construct solution by solving corresponding recursive system of BSDE on random intervals and piecing them together appropriately. This generalizes BSDE's considered by Carbone et.al and El Karoui and Huang. Then we prove that under some Markovianity assumption solution of the above BSDE are associated with system of Cauchy problems. This results are then applied to quadratic hedging problems i.e. risk-minimization of claims dividend process of general form.

FAIR VALUATION OF LÉVY-TYPE DRAWDOWN-DRAWUP CONTRACTS WITH GENERAL INSURED AND PENALTY FUNCTIONS

Zbigniew Palmowski Wrocław University of Science and Technology

We consider some insurances policies related to drawdown event of log-returns for an underlying asset modeled by a spectrally negative geometric Lévy process. We consider four contracts which insure against a drawdown fall. The first one is an insurance contract where the protection buyer pays a constant premium until the drawdown of fixed size of log-returns occurs. In return at the drawdown epoch he/she receives an insured amount which may depend on a drawdown level. Next policy is an extension of the previous one by additional cancellation feature which allows the investor to terminate contract earlier. In this case, the investor is obligated to pay a fee when he/she terminates the contract. The fee is assumed to be a function of the drawdown process. The last two contracts extend the previous ones by an additional termination feature connected with the drawup process. We focus on two problems: calculating the fair premium for the basic contracts and identifying the optimal rule for the polices with cancellation feature. To do this we solve some two-sided exit problems related to drawdown and drawup of spectrally negative Lévy processes, which is of independent mathematical interest. We also heavily rely on the theory of optimal stopping.

The talk is based on joint papers with Joanna Tumilewicz.

ROBUST FINANCE AND MARTINGALE OPTIMAL TRANSPORT - FROM THEORY TO NUMERICS

Jan Obłój University of Oxford

A robust approach to pricing and hedging does not postulate an a priori probability measure but rather computes no-arbitrage bounds on prices implied by other market instruments. One such problem leads to a version of the classical optimal transportation problem with an additional martingale constraint. I will outline the main contributions in the field and then go on to describe two recent, and ongoing, efforts. First, I will present work on characterising the structure of such transport plans in higher (and infinite) dimensions. Second, I will present our study of numerical methods for such problems.

CAPITAL ALLOCATIONS FOR CLASSICAL AND SET-VALUED RISK MEASURES

Emanuela Rosazza Gianin University of Milano Bicocca

To address future uncertainty about their net worth, firms, insurances and in general portfolio managers are often imposed to hold a so called risk capital, that is, an amount of riskless assets in order to hedge themselves. This fact then raises the issue of how to share all this immobilized capital in an a priori fair way among the different lines or business units. As risk capital is commonly accepted in the literature to be modeled through the use of risk measures, capital allocation problems in risk management and the theory of risk measures are naturally linked. Starting from Deprez and Gerber's work on convex risk premiums, Tsanakas (2009) defines a Capital Allocation Rule (C.A.R) for Gateaux differentiable risk measures inspired to the game theoretic concept of Aumann and Shapley value, and studies its properties for some widely used classes of convex risk measures, also providing explicit formulas. His analysis leaves anyway substantially open the case of general non Gateaux-differentiable risk measures (although he treats the case of distortion exponential risk measures, but it is easy to find other meaningful examples of convex and quasi-convex non Gateaux differentiable risk measures) as well as the study of quasi-convex risk measures, whose importance is well recognized in the literature. In the present work we propose a family of C.A.R. for (convex and quasi-convex) real-valued risk measures based on the dual representation theorems and on subdifferentials, study their properties and show that they reduce to Tsanakas' one, when we assume Gateaux differentiability. In the meantime, we discuss the suitability of the use of quasi-convex risk measures for capital allocation purposes. We then focus on capital allocations for set-valued or vector-valued risk measures and premia, by extending the C.A.R. above to this more general framework.

Based on joint works with Francesca Centrone.

ESTIMATION OF THE STOCHASTIC LEVERAGE EFFECT USING THE FOURIER TRANSFORM METHOD

Simona Sanfelici University of Parma

We define a non-parametric estimator of the integrated leverage effect as the covariance between the asset logarithmic returns and its volatility. In Curato and Sanfelici (2015), a consistent estimator of the leverage effect has been introduced through a pre-estimate of the Fourier coefficients of the volatility. This is a novel approach compared to the ones present in the literature which use a pre-estimate of the spot volatility path. The Fourier methodology used in this work allows to obtain a consistent and asymptotically normally distributed estimator without any manipulation of the dataset in the time domain. The finite sample properties of the estimator are investigated in a simulation study also in the presence of microstructure noise.

ASYMMETRIC INFORMATION IN SEQUENTIAL GAMES

Krzysztof Szajowski Faculty of Pure and Applied Mathematics, Wrocław University of Science and Technology

Non-zero-sum stopping game models are applied in various circumstances. An important, not fully explored case is the competition between players with different knowledge or asymmetrical rights to use information. The considered non-zero-sum Dynkin game between two players admits the various accuracy of process observation which determines the pay-offs. It also means asymmetry in the strategies they have. We are investigating the

existence of a game solution in the sense of the Nash equilibrium point. The natural conditions (situations) at which the game has value will be shown. With additional conditions, you can construct strategies in equilibrium. The discussion refers to the recently obtained results of Lempa & Matomäki (2013), Grün (2013) and Skarupski & Szajowski (2017).

RUIN PROBABILITY FOR THE INSURER-REINSURER MODEL WITH EXPONENTIAL CLAIMS

Marek Teuerle Wrocław University of Science and Technology

In this work we consider a two-dimensional risk process in which two companies split each claim and premium in a fixed proportion. Such a model can describe the capital in time for two companies that have a proportional reinsurance contract running between them. Using purely stochastic arguments including a change of probability measure we derive a joint infinite-time ruin probability in the case where claims are exponentially distributed. Our result simplifies Theorem 2 from Avram, Palmowski and Pistorius [Insurance: Mathematics and Economics 42 (2008) 227]. In the talk we will give a further possible extensions of the results and its application in approximations of the ruin probability for the considered model with claims distributions other than the exponential one.

RECENT ADVANCES IN ELECTRICITY PRICE FORECASTING: A 2018 PERSPECTIVE

Rafał Weron Wrocław University of Science and Technology

A variety of methods and ideas have been tried for electricity price forecasting over the last two decades, with varying degrees of success. In this talk I will provide a short overview of the recent advances.

36

Differential Inclusions and Controlled Systems

Organizers:

Mariusz Michta (University of Zielona Góra)

Irene Benedetti (University of Perugia)

Jerzy Motyl (University of Zielona Góra)

Tiziana Cardinali (University of Perugia)

ON TIMOSHENKO BEAM PROBLEMS

Grzegorz Bartuzel Warsaw University of Technology

We present the concise construction of the Green functions for the family of complex valued boundary value problems to the fourth order equation which comes from the description of the deflection of the timoshenko beam subject to the general boundary data. The obtained representation of the Green functions can be used for further study of properties of these functions, for example we compute their directional derivatives which in turn allows to investigate the problem of sufficient conditions on assuming real values for response of the real loads.

A LOCAL MINIMUM THEOREM AND ITS APPLICATIONS

Pasquale Candito Università Mediterranea di Reggio Calabria

The aim of this talk is to give a survey on a local minimum for smooth functionals obtained by using the nonsmooth critical point theory. In particular, the existence of at least two or three solutions for a quasilinear elliptic inclusion is shown.

EIGENVALUE PROBLEMS FOR FREDHOLM OPERATORS WITH SET-VALUED PERTURBATIONS

Antonio Iannizzotto University of Cagliari

We study the following inclusion problem in abstract Banach spaces, depending on two parameters $\lambda, \varepsilon \in \mathbf{R}$:

$$(1) \quad Lx - \lambda Cx + \varepsilon \phi(x) \ni 0, \quad x \in \partial\Omega.$$

Here L is a Fredholm linear operator of index zero between two Banach spaces E, F , while C is another bounded linear operator, ϕ is a locally compact, upper semicontinuous set-valued map of CJ -type, and Ω is an open subset of E containing the origin. For $\varepsilon = 0$, problem (1) reduces to a linear eigenvalue problem, while for $\varepsilon \neq 0$ it involves a set-valued perturbation. In this work, based on a special degree theory for set-valued maps, we prove some persistence results for both eigenvalues and eigenvectors under a small perturbation. As a consequence, we prove that whenever $\ker(L)$ is odd-dimensional and the set of trivial solutions (i.e., solutions with $\lambda = \varepsilon = 0$) of (1) is compact, then there exists a bifurcation point, namely a trivial solution around which non-trivial solutions cluster. Applications are given to ordinary differential inclusions.

Work in collaboration with P. Benevieri (University of São Paulo).

L.S.C. DIFFERENTIAL INCLUSIONS WITH P-LAPLACIAN

Salvatore Angelo Marano University of Catania (Italy)

The existence of solutions to lower semi-continuous closed-valued differential inclusions with p -Laplacian is investigated under various growth conditions. Proofs exploit the Bressan-Colombo-Fryszkowski continuous selection theorem and fixed point arguments. Some consequences are then pointed out.

STOCHASTIC DIFFERENTIAL INCLUSIONS AND SET-VALUED STOCHASTIC DIFFERENTIAL EQUATIONS

Mariusz Michta Faculty of Mathematics, Computer Science and Econometrics,
University of Zielona Gora, Poland

In the deterministic case set-valued (or multivalued) differential equations and differential inclusions represent two approaches that were developed due to investigations of dynamic systems having velocities not uniquely determined by the state of the system. Investigations in the area of set-valued differential equations were initiated by F. S. De Blasi and F. Iervolino and next developed by many other authors. The theory of differential inclusions were developed among others in connections with optimal control problems and it constitutes nowadays the important branch of set-valued analysis. A new direction in the extension of these studies are set-valued stochastic differential equations and stochastic differential inclusions. It is natural to ask whether there are any connections between them. In the talk we present results on such connections in stochastic case. Namely, we shall show that every solution to stochastic inclusion is a continuous selection of a multivalued solution to an associated set-valued stochastic equation and that the attainable sets generated by solutions to a stochastic inclusion are contained in values of multivalued solutions to an appropriate set-valued stochastic equation. Moreover, some topological properties of solutions sets to stochastic inclusions will be stated. Finally, we shall provide several remarks on possible extensions and the applicability of presented results.

ORDER CONVEX SELECTIONS OF SET-VALUED FUNCTIONS AND THEIR APPLICATIONS TO OPTIMAL CONTROL

Jerzy Motyl University of Zielona Góra

Let X be a Banach space while (Y, \preceq) a Banach lattice. We consider the class of “upper separated” set-valued functions $F : X \rightarrow 2^Y$ and investigate the problem of the existence of order-convex selections of F . First, we present results on the existence of order convex selections of upper separated multifunctions and we will discuss some applications of obtained selection results to the theory of differential and stochastic inclusions like the existence, stability or lower-upper bounds of their solutions.

In the second part of the talk we will discuss the applicability of obtained results to some deterministic and stochastic optimal control problems. Some examples will be presented also.

References

- [1] J. Motyl, Carathéodory convex selections of set-valued functions in Banach lattices, *Topological Methods in Nonlinear Analysis* 43 (2014), 1-10
 - [2] J. Motyl, Stochastic retarded inclusion with Carathéodory-upper separated multifunctions, *Set-Valued and Variational Analysis* 24 (2016), 191-205
 - [3] J. Motyl, Carathéodory-convex selections of multifunctions and their applications, *Journal of Nonlinear and Convex Analysis* 18 (2017), 535-551
-

DIFFERENTIAL INCLUSIONS WITH IMPULSES IN ABSTRACT SPACES AND APPLICATIONS

Paola Rubbioni University of Perugia

In a recent joint work with Tiziana Cardinali, we investigate the existence of solutions for differential inclusions in Banach spaces subject to impulses, which applies to a model from mathematical biology.

We show the existence of solutions for an impulsive Cauchy problem driven by the ordinary semilinear integro-differential inclusion in a Banach space

$$\frac{dv}{dt}(t) \in A(t)v(t) + F\left(t, v(t), \int_0^t \frac{e^{s/T}}{T} v(s) ds\right)$$

where $\{A(t)\}_{t \in [0, T]}$ generates an evolution system, F is a given multivalued map and the impulsive conditions are given by the equations

$$v(t_i^+) = v(t_i) + I_i(v(t_i)), \quad i = 1, \dots, p,$$

with $I_i : E \rightarrow E$, $i = 1, \dots, p$.

Then, by means of the existence result, we provide the controllability of an impulsive integro-differential process with nonlocal feedback controls driven by the parametric integro-differential equation

$$\frac{\partial u}{\partial t}(t, z) = -b(t, z)u(t, z) + g\left(t, u(t, z), \int_0^t \frac{e^{s/T}}{T} u(s, z) ds\right) + \omega(t, z)$$

where: $u : [0, T] \times [0, 1] \rightarrow \mathbb{R}$, $b : [0, T] \times [0, 1] \rightarrow \mathbb{R}$, and $g : [0, T] \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are given functions; the control ω is subject to the condition

$$\omega(t, \cdot) \in W(u(t, \cdot)).$$

In a biological setting, $u(t, z)$ and $b(t, z)$ represent the density of a population and the removal coefficient (including death and migration) at time t and position z ; g is the population development law, affected by a distributed delay which accounts for a memory-effect expressed by the Volterra integral operator; the parameter T measures the relevance of the delay. The set of functions $W(u(t, \cdot)) \subset L^2([0, 1])$ depends on the weighted values of $u(t, \cdot)$ all over the habitat normalized to $[0, 1]$; that is we consider a control strategy with a nonlocal nature.

The impulsive action is given by equations

$$u(t_i^+, z) = u(t_i, z) + \mathcal{I}_i(u(t_i, z)), \quad i = 1, \dots, p.$$

Sometimes the role of functions \mathcal{I}_i is the regulation of the process; for example, in practical pest management, the pesticide is not periodically used, but the human control acts on the pest population only if it overcomes prescribed thresholds at fixed times. In our result we can take completely arbitrary impulse functions.

NONLOCAL SEMILINEAR DIFFERENTIAL PROBLEMS IN BANACH SPACES WITHOUT COMPACTNESS

Valentina Taddei Department of Sciences and Methods for Engineering, University of Modena and Reggio Emilia

An existence result is provided for semilinear differential equations in a Banach space. A wide family of nonlocal boundary value problems is treated, including periodic, anti-periodic, weighted mean value and multipoint conditions. The study is based on an approximation solvability method. Advanced topological methods are used as well as a Scorza Dragoni-type result. The compact embedding of the Banach space into a suitable other space is the unique amount of compactness which is needed in this discussion. The solutions are located in bounded sets and they are limits of functions with values in finitely dimensional spaces. The conclusions are original also in the single-valued setting. Applications to reaction-diffusion models are given.

RELAXATION OF NON LOCAL SUPREMAL FUNCTIONALS

Elvira Zappale Università degli Studi di Salerno

Recently nonlocal functionals in the form of double integrals have attracted much attention due to their natural appearance in models of peridynamics. I will present recent developments on analogous models for homogeneous supremal functionals in the nonlocal setting, precisely,

$$\operatorname{ess\,sup}_{(x,y) \in \Omega \times \Omega} W(u(x), u(y)), \quad u \in L^\infty(\Omega; \mathbb{R}^m)$$

with $\Omega \subset \mathbb{R}^n$ be a bounded, open set and $W : \mathbb{R}^m \rightarrow \mathbb{R}^m$ is a continuous density.

Lower semicontinuity properties and relaxation of such functionals will be shown in connection with separated level convexity properties of a suitably modified version of W . The obtained results exploit and enlighten the strong connection existing between nonlocal supremal functionals and nonlocal indicator ones, the related analysis thus amounting to the study of weak closures of a class of non-local inclusions.

References

- [1] J. Bellido, C. Mora-Corral, Lower semicontinuity and relaxation via Young measures for nonlocal variational problems and applications to peridynamics, *SIAM J. Math. Anal.* 50(1) (2018), 779- 809.
- [2] J. Bevan, and P. Pedregal, A necessary and sufficient condition for the weak lower semicontinuity of one-dimensional non-local variational integrals, *Proc. R. Soc. Edinburgh Sect. A* 136(4) (2006), 701-708.
- [3] C. Kreisbeck, and E. Zappale, Lower semicontinuity and relaxation of nonlocal supremal functionals, In preparation.
- [4] J. Munoz, Characterization of the weak lower semicontinuity for a type of nonlocal integral functional: the n-dimensional case, *J. Math. Anal. Appl.* 360(2) (2009), 495-502.
- [5] P. Pedregal, Nonlocal variational principles, *Nonlinear Anal.* 29(12) (1997), 1379-1392.
- [6] P. Pedregal, Weak lower semicontinuity and relaxation for a class of non-local functionals, *Rev. Math. Complut* 29 (2016), 485-495.

The talk is based on a work in progress with Carolin Kreisbeck Universiteit Utrecht, Mathematisch Instituut, Budapestlaan 6, 3584 CD Utrecht, The Netherlands e-mail: c.kreisbeck@uu.nl

37

Stochastic Models of Molecular Anomalous Diffusion and Fractional Dynamics

Organizers:

Krzysztof Burnecki (Wrocław University of Science and Technology)

Carlo Manzo (Universitat de Vic – Universitat Central de Catalunya)

Gianni Pagnini (Basque Center for Applied Mathematics)

Aleksander Weron (Wrocław University of Science and Technology)

ERGODICITY SURFACES FOR GAUSSIAN PROCESSES

Michał Balcerek Wrocław University of Science and Technology

In this talk I will introduce a rigorous statistical test for Gaussian process based on two statistics related to ergodicity, namely autocovariance function (ACF). I will demonstrate some analysis of the distribution of the ACF and present the construction of some statistical test regarding considered model. While constructing such test I will create the, so called, ergodicity surfaces.

ANALYSIS OF RECEPTOR–G PROTEIN INTERACTIONS AT CELL SURFACE HOT SPOTS

Krzysztof Burnecki Wrocław University of Science and Technology

G-protein-coupled receptors mediate the biological effects of many hormones and neurotransmitters and are important pharmacological targets. They transmit their signals to the cell interior by interacting with G proteins. However, it is unclear how receptors and G proteins meet, interact and couple. Here we analyse the concerted motion of G-protein-coupled receptors and G proteins on the plasma membrane and provide a quantitative model that reveals the key factors that underlie the high spatiotemporal complexity of their interactions. To this end we use hidden Markov models, the Lucy–Richardson algorithm and various statistical test. We find hot spots on the plasma membrane, at least partially defined by the cytoskeleton and clathrin-coated pits, in which receptors and G proteins are confined and preferentially couple. These findings shed new light on the dynamic interactions that control G-protein-coupled receptor signalling.

Literature

T. Sungkaworn, M-L. Jobin, K. Burnecki, A. Weron, M.J. Lohse, D. Calebiro, Single-molecule imaging reveals receptor–G protein interactions at cell surface hot spots, *Nature* **550**, 543-547 (2017).

ANOMALOUS DIFFUSION – ε -ERGODICITY TESTING APPROACH VIA DYNAMICAL FUNCTIONAL

Hanna Loch-Olszewska Wrocław University of Science and Technology

The problem of ergodicity of the process is an essential issue for the real-life processes - together with the stationarity it guarantees satisfying the Boltzmann hypothesis, i.e. that the temporal and ensemble averages are asymptotically equal. Hence, for an ergodic process, the observation of many realizations in one time point can be equivalent to observing single trajectory over a long time horizon. Lately, in various fields of physics, biology and related sciences, the non-ergodic systems that show the anomalous behaviour have been noted.

A crucial practical question is how long trajectories one needs to observe in an experiment in order to claim the ergodicity breaking of the sample. Based on the empirical estimator $F(n)$ for the dynamical functional $D(n)$, defined as a Fourier transform of the n -lag increments of the process, one can verify so-called ε -ergodicity - the convergence of the estimator to some pre-defined interval $(-\varepsilon, \varepsilon)$. The current work is focused on the dynamical functional for the increments of fractional Brownian motion - the explicit formulas for $D(n)$ and $\widehat{D}(n)$ are derived and the empirical sensibility of the $F(n)$ estimator to Hurst exponent is shown.

ERGODICITY BREAKING FROM HETEROGENEOUS DIFFUSION AT THE PLASMA MEMBRANE

Carlo Manzo Universitat de Vic - Universitat Central de Catalunya

We have studied the motion of a transmembrane receptor involved in the capture of pathogens, exhibiting anomalous diffusion with signatures of weak ergodicity breaking. Through the study of mutated forms of the receptor, we have been able to correlate the receptors motion to its molecular structures, lateral organization and interactions, thus establishing a link between nonergodicity and biological function. In addition, we have quantitatively interpreted the receptor dynamics through a stochastic model of random motion with random diffusivity on scale-free media, and we are attempting to gain further insight into the molecular causes of this complex diffusion. Our work highlights the role of heterogeneity in cell membranes and proposes a connection with function regulation.

CENTRE-OF-MASS LIKE SUPERPOSITION OF ORNSTEIN-UHLENBECK PROCESSES: A PATHWAY TO NON-AUTONOMOUS STOCHASTIC DIFFERENTIAL EQUATIONS AND TO FRACTIONAL DIFFUSION

Gianni Pagnini BCAM - Basque Center for Applied Mathematics

" We consider an ensemble of Ornstein-Uhlenbeck processes featuring a population of timescales and a population of noise amplitudes that characterize the heterogeneity of the ensemble. We show that the centre-of-mass like variable corresponding to this ensemble is statistically equivalent to a process driven by a stochastic differential equation with time-dependent drift and a white noise. In particular, the time scaling and the density function of such variable are driven by the population of timescales and of noise amplitudes, respectively. Moreover, we show that this variable is equivalent in distribution to a process build by the product of a Gaussian process times a non-negative independent random variable. This last result establishes a connection with the so-called generalized gray Brownian motion and suggests application to modelling fractional anomalous diffusion.

STATISTICAL TOOLS FOR ANOMALOUS DIFFUSION RECOGNITION

Agnieszka Wyłomańska Wrocław University of Science and Technology

Anomalous diffusion in crowded fluids, e.g., in cytoplasm of living cells, is a frequent phenomenon. A common tool by which the anomalous diffusion of a single particle can be classified is the time-averaged mean square displacement (TAMSD). However there are also different statistics that can be useful in this problem. A validation of anomalous diffusion processes for single-particle tracking data is of great interest for experimentalists. In this presentation we demonstrate statistical methods useful in the anomalous diffusion property recognition. One of the example is the rigorous statistical test based on TAMSD for classical anomalous diffusion process, namely fractional Brownian motion (FBM) or visual test based on the dynamical functional which is useful in the problem of differentiation between FBM and continuous time random walk, the second classical model of anomalous diffusion. We demonstrate also the role of codifference, the general measure of dependence, adequate for processes with infinite variance, in the problem of the anomalous diffusion property exhibition.

ASYMPTOTIC PROPERTIES OF LEVY WALKS

Tomasz Żórawik Wrocław University of Science and Technology

Levy walks are useful models of anomalous diffusion and have found a number of applications in physics and biology. We present some results regarding properties of their limit processes in a ballistic regime. We will focus on probability distributions and aging property. In particular we will show that despite a complicated memory structure and the appearance of α -stable processes in the definition of the limit process its probability density function is given by an elementary function in a 3D case. In 2D the result for PDF is more complicated and it requires a fractional derivative and hypergeometric functions. We also discuss a simulation algorithm for these processes. It turns out that there is a perfect agreement between statistics obtained via Monte Carlo simulations and our theoretical findings. Some results concerning path properties of the limit processes will be discussed as well.

38

Probability Measures – Structures, Identification and Characterizations

Organizers:

Jacek Wesółowski (Politechnika Warszawska)

Mauro Piccioni (Sapienza Università di Roma)

ASYMPTOTICS OF KENDALL RANDOM WALKS, CORRESPONDING RENEWAL THEORY AND WILLIAMSON TRANSFORM

Barbara Jasiulis-Gołdyn Mathematical Institute, University of Wrocław

The origin of generalized convolutions comes from paper of Kingman (J. G. C. Kingman, Random Walks with Spherical Symmetry, Acta Math. 109(1), 11-53, 1963) about spherically symmetric random walks. Instead of classical convolution corresponding to sum of independent random elements, we consider commutative and associative binary operation \diamond described by Urbanik in the series of papers (K. Urbanik, Generalized convolutions I-V, Studia Math., 23(1964), 217-245, 45(1973), 57-70, 80(1984), 167-189, 83(1986), 57-95, 91(1988), 153-178) and developed by Bingham (N. H. Bingham, On a theorem of Kłosowska about generalized convolutions, Coll. Math. 48(1), 117-125, 1984) in the context of regularly varying functions. The wealth of generalized convolutions gives us hope for their widespread application in the future to describe various phenomena. The simplest example is the classical convolution corresponding to sum of independent random elements. Another example is the maximum convolution that leads us to the extreme value theory. Generalized extreme value distributions already today are widely used for modeling phenomena that are rare, but whose occurrence leads to great losses. Now imagine that we use other binary operations to model phenomena. This is a natural turn of events, after all, not everything can be described with the addition or operation of the maximum. We focus on the Kendall convolution Δ_α case, i.e. algebra with

$$\delta_1 \Delta_\alpha \delta_1 = \pi_{2\alpha},$$

where $\pi_{2\alpha}(dy) = 2\alpha y^{-2\alpha-1} \mathbf{1}_{[1,\infty)}(y) dy$. For $x \in [0, 1]$ we define

$$\delta_x \Delta_\alpha \delta_1 = x^\alpha \pi_{2\alpha} + (1-x^\alpha) \delta_1.$$

We consider extremal Markov sequences of the Kendall type with unit step distribution $\mathcal{L}(S_1) = \nu \in \mathcal{P}_+$ defined in the following way:

$$X_0 = 1, \quad X_1 = Y_1, \quad X_{n+1} = M_{n+1} [\mathbf{1}(\xi_n > \rho_{n+1}) + \theta_{n+1} \mathbf{1}(\xi_n < \rho_{n+1})],$$

where

$$M_{n+1} = \max\{X_n, Y_{n+1}\}, \quad \rho_{n+1} = \frac{\min\{X_n, Y_{n+1}\}^\alpha}{\max\{X_n, Y_{n+1}\}^\alpha},$$

and sequences $(Y_k) \sim i.i.d.(\nu)$, $(\xi_k) \sim i.i.d.(U([0, 1]))$, $(\theta_k) \sim i.i.d.(\pi_{2\alpha})$ are independent. Then $\{X_n : n \in \mathbb{N}\}$ has transition probabilities:

$$P_{k,n}(x, A) = P(S_n \in A | S_k = x) := \delta_x \Delta_\alpha \nu^{\Delta_\alpha(n-k)}(A),$$

given by the Kendall convolution Δ_α . We present the renewal theory for the stochastic processes. We prove an analogue of the Fredholm theorem for all generalized convolutions algebras. Using the technique of regularly varying functions we prove the Blackwell theorem for renewal processes as well as the limit theorems for Kendall random walks. The main mathematical tool in the Kendall convolution algebra, corresponding to characteristic function, is the Williamson transform. Talk is based on two papers: [1] (M. Arendarczyk, B.H. Jasiulis-Gołdyn, E.A.M. Omeý, Asymptotic properties of Kendall random walks, submitted, 2018.) and [2] (B.H. Jasiulis-Gołdyn, K. Łukaszewicz, J.K. Misiewicz, E.A.M. Omeý, Renewal theory for extremal Markov sequences of the Kendall type, submitted, <https://arxiv.org/pdf/1803.11090.pdf>, 2018.).

This work is a part of project “First order Kendall maximal autoregressive processes and their applications”, which is carried out within the POWROTY/REINTEGRATION programme of the Foundation for Polish Science co-financed by the European Union under the European Regional Development Fund.

INDEPENDENCE CHARACTERIZATIONS FOR WISHART AND KUMMER RANDOM MATRICES

Bartosz Kołodziejek Warsaw University of Technology

In this talk, I will present recent developments regarding the so-called independence characterizations with special emphasis on two characterizations of Wishart and Kummer random matrices. These results generalize the corresponding one dimensional results in the literature.

The talk is based on a joint work with Agnieszka Piliszek (Warsaw).

A RANDOM WALK IS COMPLETELY DETERMINED BY DISTRIBUTIONS OF $\max(X_n, 0)$

Mateusz Kwaśnicki Wrocław University of Science and Technology, Poland

With the methods of the theory of holomorphic functions, we prove the theorem stated in the title of this talk. This result resolves a conjecture of L. Chaumont and R. Doney, who gave a proof under various additional assumptions. An equivalent formulation asserts that a random walk is completely determined by the distributions of the running maxima $\max(0, X_1, X_2, \dots, X_n)$, or, in a yet another form, by the ascending ladder-height process. Similar theorem holds true for Lèvy processes.

The result described above was first given, without a proof, by I.V. Ostrovskii and A.M. Ulanovskii in 1989. The second author of this work provided a proof in 1992, but it contained an essential gap.

THE MULTIVARIATE INVERSE GAUSSIAN DISTRIBUTIONS, THE EDGE REINFORCEMENT PROCESS AND THE DISERTORI-SPENCER-ZINBAUER INTEGRAL

Gerard Letac Université de Toulouse

Sabot and Tarrès have explicitly computed the integral

$$ST_n(y) = \int \exp(-\langle x, y \rangle) (\det M_x)^{-1/2} dx$$

where M_x is a symmetric matrix of order n with fixed non positive off-diagonal coefficients $-w_{ij}$ and with diagonal $(2x_1, \dots, 2x_n)$. The domain of integration is the part of R^n for which M_x is positive definite. The result is simple although the proof is involved. This is a less daunting reformulation of a Disertori-Spencer-Zinbauer integral. These integrals occur in the study of the reinforced Markov chain on a graph when the edges are the (ij) 's such that $w_{ij} > 0$. In this non homogeneous chain, the more you use an edge, the more the probability to use it again in the future increases. We calculate more generally for $b_1 > 0, \dots, b_n > 0$ the integral

$$GST_n(y) = \int \exp\left(-\langle x, y \rangle - \frac{1}{2} b^* M_x^{-1} b\right) (\det M_x)^{-1/2} dx.$$

This creates a family of distributions on R^n with striking properties like stability by marginalization and (up to a translation) stability by conditioning. This is joint work with Jacek Wesółowski.

- DISERTORI, M., SPENCER, T. AND ZIRNBAUER, M. (2010) Quasi-diffusion in a 3D super-symmetric hyperbolic sigma model. *Communications in Mathematical Physics*, **300** (2), 435-486.

- SABOT, C. AND TARRÈS, P. (2015) Edge-reinforced random walk, vertex-reinforced jump process and the supersymmetric sigma model. *J. Eur. Math. Soc.* **17** (9), 2353-2378.

THE GEOMETRY OF RANDOM EIGENFUNCTIONS

Domenico Marinucci Department of Mathematics, University of Rome Tor Vergata

We shall review some recent results concerning the asymptotic behaviour (in the high-frequency regime) of geometric functionals on the excursion sets of random spherical eigenfunctions. We shall show in particular how the Lipschitz-Killing curvatures (equivalently, the Minkowski functionals) evaluated on these excursion sets are dominated by a single component, corresponding to their projection on the so-called Wiener chaos of order 2. This component disappears for the excursion set corresponding to a zero threshold, where the asymptotic behaviour is hence different (the so-called Berry cancellation phenomenon). A similar behaviour can also be established for random eigenfunctions on the torus (arithmetic random waves).

The talk is based on some joint works with Valentina Cammarota, Giovanni Peccati, Maurizia Rossi and Igor Wigman.

MATRIX VALUED GAMMA DISTRIBUTIONS, THEIR GROUP PROPERTY AND RELATED STOCHASTIC PROCESSES

Krzysztof Podgorski Lund University

In this talk we introduce a singular matrix-variate gamma distribution which is a natural extension of a singular Wishart distribution and complement the class of non-singular matrix valued gamma distribution. We present their fundamental properties including its density and characteristic functions. For the so extended class of matrix-variate gamma distributions we study in full detail the infinite divisibility and some related group properties both from the historical and structural perspectives and provide some new results on the topic. Using these properties we present a method of constructing matrix valued stochastic processes. Moreover, we introduce a (singular) matrix-variate Laplace distribution which is obtained by compounding the matrix normal distribution with the (singular) matrix-variate gamma distribution. The work is jointly with Tomasz J. Kozubowski and Stepan Mazur.

MULTIVARIATE CONDITIONAL HAZARD RATES AND A SYMMETRY CONDITION FOR VECTORS OF NON-NEGATIVE RANDOM VARIABLES

Fabio Spizzichino Department of Mathematics, Sapienza University of Rome

We consider n non-negative random variables X_1, \dots, X_n , defined on a same probability space (Ω, \mathcal{F}, P) , and assume their joint probability distribution to be absolutely continuous. Under such assumption, one can introduce the notion of *multivariate conditional hazard rate* (m.c.h.r.) function. The latter can be seen as a direct extension of the common concept of hazard rate function of a scalar non-negative random variable. Such a definition arises as a natural one when X_1, \dots, X_n are, respectively, interpreted as the lifetimes of n units C_1, \dots, C_n which start working simultaneously as components in a same system. On the other hand, m.c.h.r. functions are related to the more general concepts of *stochastic intensity* and *compensator*, dealt with in the theory of point processes. Let

$j \in [n] \equiv \{1, \dots, n\}$, $0 \leq t_1 < \dots < t_k < t$, $I \subset [n]$ such that $|I| = k$ and $j \notin I$, and consider the “dynamic” history of failures and survivals in the time interval $[0, t]$ defined by

$$\mathbf{h}_t \equiv \{X_i = t_i, i \in I; X_r > t, r \notin I\}.$$

The m.c.h.r. is a function $\lambda_j(t|I; t_1, \dots, t_k)$ of time t indicating the instantaneous risk at t of the unit C_j , conditionally on the observation of \mathbf{h}_t . The symbol $\lambda_j(t|\emptyset)$ denotes the instantaneous risk of C_j , conditionally on the event of no failure up to t .

Formal definitions and basic references will be recalled in the talk.

The family formed by all the m.c.h.r. functions is a tool apt to describe the joint probability distribution for (X_1, \dots, X_n) . Such a description is equivalent, but alternative, to the one based on the joint density function. Such a tool is useful in the study of several different aspects related with vectors of random lifetimes. In particular, it has been used in the definition of some concepts of stochastic comparison between two random vectors and of some concepts of stochastic dependence. It is also specially convenient to describe *dynamic* models of dependence. See e.g. [2] and references cited therein.

In my talk I will point out a somehow different context where the m.c.h.r. functions can turn out to be useful. More precisely, I will first concentrate attention on the characterization of the property of exchangeability for the random vector (X_1, \dots, X_n) . Based on results obtained in [1] and [3], I will then discuss the role of the family of the m.c.h.r. functions in the computation of the probability of an event of the form

$$(X_{1:n} = X_{\pi(1)}, \dots, X_{n:n} = X_{\pi(n)}),$$

where $X_{1:n}, \dots, X_{n:n}$ denote the *order statistics* of (X_1, \dots, X_n) and $\pi \equiv (\pi(1), \dots, \pi(n))$ is a permutation of $\{1, 2, \dots, n\}$.

Such computations are, in particular, needed for determining the *structure signature* and *probability signature* of a system (see, e.g., again [3]). Under the condition

$$P(X_{1:n} = X_{\pi(1)}, \dots, X_{n:n} = X_{\pi(n)}) = \frac{1}{n!}, \forall \pi, \quad (38.1)$$

the probability signature does coincide with the structure signature and, in such a case and reliability analysis of the system may become significantly simpler. The condition in (38.1) is trivially true in the case of exchangeability and, more generally, it can still be seen as a natural condition of symmetry. A simple condition, sufficient for (38.1) but weaker than exchangeability, will be obtained in terms of the m.c.h.r. functions.

- [1] E. De Santis, Y. Malinovsky, F. Spizzichino. The use of multivariate conditional hazard rates in the study of non-transitivity aspects of stochastic precedence. *Work in progress*, 2018.
- [2] M. Shaked, J. G. Shanthikumar. Multivariate conditional hazard rate functions: an overview. *Appl. Stoch. Models Bus. Ind.*, 31, 285-296, 2015
- [3] F. Spizzichino. Reliability, Signature, and Relative Quality Functions of Systems under Time-Homogeneous Load-Sharing . *Submitted*.

39

Dependence Modelling and Copulas

Organizers:

Piotr Jaworski (University of Warsaw)

Carlo Sempì (Università del Salento)

Fabrizio Durante (Università del Salento)

ON COPULA-ITO PROCESSES

Piotr Jaworski University of Warsaw

Let a multivariate stochastic process X_t be a solution of a system of stochastic differential equations and C_t be a family of corresponding copulas. We will study the existence of a tangent vector field to the curve $t \rightarrow C_t$. Basing on the above we will derive the necessary conditions for a copula C to be a stationary copula of some process X_t .

COUPLING OF TWO DIMENSIONAL ITÔ DIFFUSIONS

Marcin Krzywda Jagiellonian University

We study two dimensional self-similar Itô diffusion processes, whose margins are Brownian Motions, given by a specified set of stochastic differential equations.

We give the characterisation of the cumulative distribution functions of the solution of our equation in terms of an elliptic partial differential equation. Furthermore we give conditions on the existence of a self-similar solution.

As a corollary we characterise our model in terms of copulae and conclude with examples of solutions for particular families of copulae.

The talk is based on the joint work with Prof Piotr Jaworski.

- [1] Jaworski P., Krzywda M. *Coupling of Wiener Processes by Copulas*, Stochastic & Probability Letters, 83, pp. 2027-2033, 2013
 - [2] Sempì C. *Coupled Brownian Motion*, Advances in Intelligent and Soft Computing, 77, pp. 569-574, 2010
-

ASYMMETRIC LINKAGES: MAXMIN VS. REFLECTED MAXMIN COPULAS

Blaž Mojškerc Faculty of Economics, University of Ljubljana

In this paper we introduce some new copulas emerging from shock models, motivated by applications, i.e. the reflected maxmin copulas with dependent endogenous shocks and their extension to multivariate case. Our main goal is to show that reflected maxmin copulas exhibit conceptually better characteristics as the original maxmin copulas. An important evidence for that is the iteration procedure of the reflected maxmin transformation which we prove to be always convergent and give many properties of it that are useful in applications. Even more, using this result we give also the limit of the iteration procedure of the maxmin transformation thus answering a question proposed in [1]. The third main result of ours is the multivariate (dependent) reflected maxmin copula which is conceptually simpler than the standard version of it. In all our copulas the idiosyncratic and systemic shocks are combined via asymmetric linking functions as opposed to Marshall copulas where symmetric linking functions are used.

The paper is a result of joint work with D. Kokol Bukovšek, T. Košir and M. Omladič.

- [1] F. Durante, M. Omladič, L. Oražem, N. Ružić, *Shock models with dependence and asymmetric linkages*, Fuzzy Sets and Systems, **323** (2017), 152–168.

[2] T. Košir, M. Omladič, *Reflected maxmin copulas and modelling quadrant subindependence*, preprint.

EXTENDED MARSHALL-OLKIN MODELS AND JOINT LIFE INSURANCE PRICING

Sabrina Mulinacci Department of Statistics, University of Bologna

We introduce the Extended Marshall-Olkin models and related copulas for modelling the joint residual lifetimes of two individuals in a couple. These models are based on the combination of two approaches: the absolutely continuous copula based approach, where the copula is used to capture dependences based on environmental factors shared by the two lives, and the approach based on the classical Marshall-Olkin construction, where the association is given by accounting for a fatal event causing the simultaneous death of the two lives. The properties of the Extended Marshall-Olkin model are studied and the model is applied to a sample of censored residual lifetimes of couples of insureds extracted from a dataset of annuities contracts of a large Canadian life insurance company. This is a joint work with Fabio Gobbi and Nikolai Kolev.

DETECTION OF BLOCK-EXCHANGEABLE STRUCTURE IN LARGE-SCALE CORRELATION MATRICES

Johanna G. Neslehova McGill University

Correlation matrices are omnipresent in multivariate data analysis. When the number d of variables is large, the sample estimates of correlation matrices are typically noisy and conceal underlying dependence patterns. In this talk, I will consider the case when the variables can be grouped into K clusters with exchangeable dependence; an assumption often made in applications in finance and econometrics. This partial exchangeability property is also often implicitly present in dependence models that aim to reduce dimensionality through clustering. Under this partial exchangeability condition, the corresponding correlation matrix has a block structure and the number of unknown parameters is reduced from $d(d-1)/2$ to at most $K(K+1)/2$. I will propose a robust algorithm based on Kendall's rank correlation to identify the clusters from data. No prior knowledge of the clusters, their number or composition is assumed, and no assumptions on the marginal distributions are made except continuity. The algorithm also outputs an estimator of the Kendall rank correlation matrix that has a block-structure. This estimator will be seen to perform considerably better than the sample Kendall rank correlation matrix when $K < d$. Even in the unstructured case $K = d$, though there is no gain asymptotically, the new estimator can be much more efficient in finite samples. When the data are elliptical, the results extend to linear correlation matrices and their inverses. The procedure will be illustrated on financial stock returns. This talk is based on joint work with Thierry Duchesne and Samuel Perreault.

APPLICATIONS OF COPULAS FOR A MAXMIN SYSTEM TO ORDER STATISTICS AND TO RELIABILITY THEORY

Matjaž Omladič University of Ljubljana, Institute of Mathematics, Physics, and Mechanics

In this talk we present a joint work with M. Vidmar. One of the objectives are to extend the work of M. O. and N. Ružić (2016), and of F. Durante, M. O., L. Oražem, and N. Ružić (2017). In these papers a new line of investigation was started introducing maxmin copulas, closely related to Marshall copulas, but allowing for asymmetric linkages. In this joint work in progress we go beyond the bivariate case or even the usual d -variate case. On a standard probability space, let $\mathbf{X} = (X_i)$ be a random vector in \mathbb{R}^n , for some integer n , and C a copula thereof. For any nonempty subset M of $[n]$ we denote $\vee_M \mathbf{X} := \max\{X_i; i \in M\}$ and $\wedge_M \mathbf{X} := \min\{X_i; i \in M\}$. We choose a nonempty set \mathcal{C} of nonempty subsets M of $[n]$ for which we compute $\vee_M \mathbf{X}$ and a nonempty set \mathcal{D} of nonempty sets M of $[n]$ for which we compute $\wedge_M \mathbf{X}$. We call the $(|\mathcal{C}| + |\mathcal{D}|)$ -dimensional random vector $X^{\vee\wedge} := ((\vee_M \mathbf{X})_{M \in \mathcal{C}}, (\wedge_M \mathbf{X})_{M \in \mathcal{D}})$ a *maxmin system*. Under certain technical conditions we provide an expression (in terms of C and the marginals $\{F_i, i \in [n]\}$) for the copula of this random vector. Besides the obvious applications to shock models there is an important application of this approach to order statistics. Another important application is the approach to reliability theory as studied by the group gathered around J.-L. Marichal.

COMPARISONS RESULTS FOR INACTIVITY TIMES OF COHERENT SYSTEMS HAVING DEPENDENT COMPONENTS BASED ON THEIR REPRESENTATIONS THROUGH DISTORTION FUNCTIONS

Franco Pellerrey Politecnico di Torino

Coherent systems, i.e., multicomponent systems where every component monotonically affects the working state or failure of the whole system, are among the main objects of study in reliability analysis. Consider a coherent system with possibly dependent components having lifetime T , and assume we know that it failed before a given time $t > 0$. Its inactivity time $t - T$ can be evaluated under different conditional events. In fact, one might just know that the system has failed and then consider the inactivity time $(t - T | T < t)$, or one may also know which ones of the components have failed before time t , and then consider the corresponding system's inactivity time under this condition. For all these cases, a representation of the distribution function of system inactivity time based on the notion of distortion functions (which, in turn, make use of copulas) are provided. Making use of these representations, new stochastic comparison results for inactivity times of systems under the different conditional events are shown. - Talk based on a joint work with M. Longobardi (Università di Napoli) and J. Navarro (Universidad de Murcia)

CONDITIONAL COVARIANCE MATRICES: NORMALITY AND ELLIPTICITY INVARIANTS

Marcin Piłera Jagiellonian University

We show an analytical formula for the conditional covariance matrix of elliptically distributed random vector when the conditioning is based on the values of linear combination of the marginal random variables. We show that one could introduce an invariant depending solely on the conditioning set which greatly simplifies the

calculations. Also, we show that this invariant could be used to construct a statistical framework for financial applications.

FAILURE DEPENDENT PROPORTIONAL HAZARD MODEL

Tomasz Rychlik Insitute of Mathematics, Polish Academy of Sciences

The failure dependent proportional hazard model describes behavior of random lifetimes of components working in a technical system. The components have initially independent and identically distributed lifetimes with a given form of failure rate. After each consecutive component failure, the still operating components work further independently, but their common failure rates change abruptly at the failure moments due to increase of external load acting on each individual component. It is assumed that the new failure rate amounts to the old one multiplied by a fixed constant dependent on the working circumstances of the system. We present some inequalities for distributions of system lifetimes working in the failure dependent proportional hazard regime.

FROM INDEPENDENCE TO STOCHASTIC DEPENDENCE IN THE TARGET-BASED UTILITY THEORY

Fabio Spizzichino Department of Mathematics, Sapienza University of Rome

Let \mathcal{C} denote a space of possible *consequences* for a set of economic decisions. As well known, in the field of choices under uncertainty, the decision maker is supposed to choose a decision out of a predetermined set A of possible decisions $a \in A$ and any decision gives rise to a *prospect*, namely to a \mathcal{C} -valued random variable X_a . On such a purpose, a utility function $u : \mathcal{C} \rightarrow \mathbb{R}$ over the space \mathcal{C} should be specified in advance and the decision a is to be taken in order to maximize the expected value of the utility, namely $\mathbb{E}[u(X_a)]$. We concentrate attention on the case when $\mathcal{C} \equiv \mathbb{R}$, where consequences can be interpreted as monetary amounts. I will initially show in which sense such a setting can be equivalently formulated in terms of a *Target Based* approach ([1]; see also [2]), under the condition that the function u is bounded and right-continuous. In practice, a Target is a random variable T , stochastically independent of each prospect, and with distribution function corresponding to an affine transformation of the utility function u . Such an approach reveals to be useful in giving easy probabilistic interpretations of some economic properties.

It might be interesting, however, to consider a more general setting, which can be obtained by allowing some form of dependence for the pairs (T, X_a) . In this respect, I will first focus on the possibility of paradoxical conclusions related with the lack of *transitivity property* of the notion of *stochastic precedence* (see also [3]). I will then point out related properties of those special cases of dependent targets defined by the condition that all the pairs (T, X_a) , with $a \in A$, do share a same connecting copula. In fact, the latter can be seen as the most direct generalization of the case of stochastic independence for (T, X_a) .

- [1] R. Bordley, M. LiCalzi (2000). Decision analysis using targets instead of utility functions. *Decisions in Economics and Finance*, 23-53
- [2] F. Spizzichino (2018). On the probabilistic meaning of copula-based extensions of fuzzy measures. Applications to target-based utilities and multi-state reliability systems. *Fuzzy Sets and Systems*, in print.
- [3] E. De Santis, F. Fantozzi, F. Spizzichino (2015). Relations between stochastic orderings and generalized stochastic precedence. *Probab. Engin. Inform. Sciences*, 9, 3, 329-343.

CoVAR-BASED PORTFOLIO SELECTION: CONDITIONS ON CONDITIONING VAR

Anna Zalewska Warsaw University of Technology

We consider the portfolio optimization problem with the risk measured by the modified conditional value-at-risk (VaR) under the normality assumption. This risk measure, as given by G. Girardi and A. T. Ergün, is based on the stress event of the chosen asset being at most at the opposite of its VaR level. The optimization problem can be represented as minimizing VaR under two linear constraints, with the significance level given by an implicit equation involving copula - the third constraint. The solvability depends on the chosen significance levels.

40

Challenges and Methods of Modern Statistics

Organizers:

Małgorzata Bogdan (University of Wrocław)

Fabrizio Ruggeri

Piotr Zwiernik

USING MACHINE LEARNING TECHNIQUES TO DISCOVER RESPONSE-RELATED COMMUNITIES IN THE BRAIN

Damian Brzyski Indiana University Bloomington

Classical regression methods treat covariates as a vector and estimate a corresponding vector of regression coefficients. In medical applications, however, regressors in a form of multidimensional arrays can be often met. For example, one may be interested in identifying regions of the brain associated with an outcome of interest based on MRI images. Turning such image array into a vector is an unsatisfactory solution, since it destroys the inherent spatial structure of the image and could be very challenging from the computational point of view. In my talk, I will present an alternative approach – the regularized matrix regression – where the matrix of regression coefficients is defined as a solution to the specific optimization problem. The method, called Sparsity Inducing Nuclear Norm Estimator (SpINNER), simultaneously imposes two types of penalties on the matrix – the nuclear and the LASSO-type norm – to encourage the low rank of the solution and its entry-wise sparsity. Our software allows for the automatic selection of the weights defining the optimal trade-off between two considered types of penalties and the alternating direction method of multipliers (ADMM) was used to build the fast and efficient numerical solver. SpINNER has been applied to investigate associations between structural connections in the brain and HIV disease-related outcomes. Our approach outperforms others methods in the estimation accuracy in the considered situation – when the response-related entries (representing brain connections) are arranged in well-connected communities.

A MULTIVARIATE NEGATIVE-BINOMIAL MODEL FOR DIFFERENTIAL GENE-EXPRESSION ANALYSIS OF CORRELATED RNA-SEQ DATA

Tomasz Burzykowski Hasselt University

We propose a model for differential gene-expression analysis using exon counts obtained by RNA-Seq. The model assumes that exon counts of a particular gene of an individual sample jointly follow a multivariate negative-binomial distribution. Additional correlation between exon counts obtained for, e.g., individual samples within the same pair or cluster is taken into account by including a cluster-level normal random effect into the model. The model can also be interpreted as a hierarchical one, with individual-specific gamma-distributed random effects and cluster-specific normally-distributed random effects. Conditionally on the random effects, exon counts are assumed to be independent and Poisson-distributed. The model is applicable in the analysis of correlated data of different types: paired, clustered, or longitudinal. An interesting feature is that it provides explicit expressions for correlation between exon counts at different levels (within the same individual, between individuals, etc.). The performance of the model is evaluated by using a simulation study and an analysis of two real datasets: a paired RNA-Seq experiment for 24 patients with clear-cell renal cell carcinoma and longitudinal RNA-Seq experiment for 29 patients with Lyme disease.

BAYESIAN DYNAMIC TENSOR REGRESSION

Roberto Casarin University Ca' Foscari of Venice

Multidimensional arrays (i.e. tensors) of data are becoming increasingly available and call for suitable econometric tools. We propose a new dynamic linear regression model for tensor-valued response variables and

covariates that encompasses some well known multivariate models as special cases. For dealing with the over-parametrization and over-fitting issues due to the curse of dimensionality, we exploit a suitable parametrization based on the parallel factor (PARAFAC) decomposition which enables to achieve both parameter parsimony and to incorporate sparsity effects. Our contribution is twofold: first, we provide an extension of multivariate econometric models to account for both tensor-variate response and covariates; second, we show the effectiveness of proposed methodology in defining an autoregressive process for time-varying real economic networks. Inference is carried out in the Bayesian framework combined with Monte Carlo Markov Chain (MCMC). We show the efficiency of the MCMC procedure on simulated datasets, with different size of the response and independent variables, proving computational efficiency even with high-dimensions of the parameter space. Finally, we apply the model for studying the temporal evolution of real economic networks.

AN ALGEBRAIC ESTIMATOR FOR LARGE SPECTRAL MATRICES (JOINT WORK WITH MATTEO BARIGOZZI)

Matteo Farnè University of Bologna

We present a method to estimate a large p -dimensional spectral matrix assuming that the data follow a dynamic factor model with a sparse residual. In specific, we apply a nuclear norm plus l_1 norm heuristics to any kernel input estimate at each frequency. We assume that the latent eigenvalues scale to p^α , $\alpha \in [0, 1]$, and the sparsity degree scales to p^δ , with $\delta \leq \frac{1}{2}$ and $\delta \leq \alpha$. We prove that the algebraic recovery of latent rank and sparsity patterns is guaranteed if the smallest latent eigenvalue λ_r and the minimum residual nonzero entry in absolute value \min_S are large enough across frequencies. The identifiability of the underlying matrix recovery problem requires the absolute convergence of latent and residual filters and a limited discrepancy among the eigenvectors of the factorial coefficients and the sparsity patterns of the residual coefficients across lags. The consistency of the input is derived via an appropriate weak dependence assumption both on factors and residuals in the sense of Wu and Zaffaroni (2017). The recovery quality directly depends on the ratio $\frac{p^\alpha}{\sqrt{T}}$, where T is the sample length, and the magnitude of T is required to be $p^{3\delta}$ or larger. In a wide simulation study, we stress the crucial role of λ_r and \min_S across frequencies, highlighting the conditions which cause our method to fail.

MULTIPLE TESTING AND PENALIZATION IN HIGH-DIMENSIONAL LINEAR MODEL SELECTION

Konrad Furmańczyk SGGW

We propose a new model selection algorithm (called “LassoSD”), which combines a stepdown multiple testing procedure with penalized methods. First, we use the Lasso to discard a substantial part of irrelevant predictors. Next, on the support of the Lasso we perform null hypothesis testing (similarly to the Bonferroni correction or the Holm method) to find significant predictors in the model. We state nonasymptotic probabilistic inequalities, that upper bounds the model selection error of LassoSD in the high-dimensional linear model, i.e. the number of predictors can be (much) larger than the sample size. We also investigate properties of the algorithm on simulated data sets and compare them to competitive algorithms. This talk is based on the joint work with Wojciech Rejchel (Nicolaus Copernicus University)

ON COMPUTING MAXIMUM LIKELIHOOD ESTIMATORS OF VARIANCE COMPONENTS IN LINEAR MIXED MODELS

Mariusz Grządziel Wrocław University of Environmental and Life Sciences

The likelihood function, as well as the REML likelihood function, in linear mixed models may have multiple local maxima; compare [GDP12] and [HH14]. There are several approaches for computing its global maximum that can be applied in some special models: the branch-and-bound type algorithm proposed by Lavine et al. [LBH15] and the algebraic methods in which the problem of computing the ML (or REML) estimator reduces to finding all roots of a certain polynomial (or system of polynomial equations); compare [GDP12] and [Grz16]. Their efficiency will be illustrated on examples.

References

[GDP12] Gross, E., Drton, M. and Petrović, S., Maximum likelihood degree of variance component models, *Electronic Journal of Statistics*, 6, 993–1016.

[Grz16] Grządziel, M. (2016), On the maximum likelihood degree of linear mixed models with two variance, preprint arXiv:1608.08789v2 [math.ST].

[HH14] Henn, L. and Hodges, J. S. (2014), Multiple local maxima in restricted likelihoods and posterior distributions for mixed linear models, *International Statistical Review*, 82, 90–105.

[LBH15] Lavine, M., Bray, A. and Hodges, J. S. (2015), Approximately exact calculations for linear mixed models, *Electronic Journal of Statistics*, 9, 2293–2323.

BRAIN CONNECTIVITY-INFORMED ADAPTIVE REGULARIZATION FOR GENERALIZED OUTCOMES

Jarosław Harezlak Indiana University, Bloomington, IN, USA

One of the challenging problems in the brain imaging research is a principled incorporation of information from different imaging modalities in regression models. Frequently, data from each modality is analyzed separately using, for instance, dimensionality reduction techniques, which result in a loss of information. We propose a novel regularization method, griPEER (generalized ridgified Partially Empirical Eigenvectors for Regression) to estimate the association between the brain structure features and a scalar outcome within the generalized linear regression framework. griPEER provides a principled approach to use external information from the structural brain connectivity to improve the regression coefficient estimation. Our proposal incorporates a penalty term, derived from the structural connectivity Laplacian matrix, in the penalized generalized linear regression. We address both theoretical and computational issues and show that our method is robust to the incomplete structural brain connectivity information. griPEER is evaluated via extensive simulation studies and it is applied in classification of the HIV+ and HIV- individuals.

STOCHASTIC APPROXIMATION EM FOR LOGISTIC REGRESSION WITH MISSING VALUES

Wei Jiang Ecole Polytechnique

Logistic regression is a common classification method in supervised learning. Surprisingly, there are very few solutions for performing it and selecting variables in the presence of missing values. We propose a stochastic

approximation version of the EM algorithm based on Metropolis-Hasting sampling, to perform statistical inference for logistic regression with incomplete data. We suggest a complete approach, including the estimation of parameters and their variance, derivation of confidence intervals, a model selection procedure, and a method for prediction on test sets with missing values. The method is computationally efficient, and its good coverage and variable selection properties are demonstrated in a simulation study where we contrast its performances to other methods. We then illustrate the method on a dataset of severely traumatized patients from Paris hospitals to predict the occurrence of hemorrhagic shock, a leading cause of early preventable death in severe trauma cases. The aim is to consolidate the current red flag procedure, a binary alert identifying patients with a high risk of severe hemorrhage. The methodology is implemented in the R package `misaem`.

ANALYSIS OF LANGEVIN MONTE CARLO VIA CONVEX OPTIMIZATION

Błażej Miasojedow Institute of Mathematics, Polish Academy of Sciences

We provide new insights on the Unadjusted Langevin Algorithm. We show that this method can be formulated as a first order optimization algorithm of an objective functional defined on the Wasserstein space of order 2. Using this interpretation and techniques borrowed from convex optimization, we give a non-asymptotic analysis of this method to sample from logconcave smooth target distribution. Our proofs are then easily extended to the Stochastic Gradient Langevin Dynamics, which is a popular extension of the Unadjusted Langevin Algorithm. Finally, this interpretation leads to a new methodology to sample from a non-smooth target distribution, for which a similar study is done. The talk is based on the joint work with Alain Durmus (ENS Saclay) and Szymon Majewski (IMPAN).

PARTICLE MCMC WITH POISSON RESAMPLING

Wojciech Niemiro University of Warsaw

We introduce a new version of particle filter in which the number of “children” of a particle at a given time has a Poisson distribution. As a result, the number of particles is random and varies with time. An advantage of this scheme is that descendants of different particles can evolve independently. This makes it easy to parallelize computations. Moreover, particle filter with Poisson resampling is readily adapted to the case when a hidden process is a continuous time, piecewise deterministic semi-Markov process. We show that the basic techniques of particle MCMC (Andrieu et al., JRSS (B) 2010), namely particle independent Metropolis-Hastings, particle Gibbs Sampler and its version with backward sampling of ancestors (Lindsten et al., JMLR 2014), work under our Poisson resampling scheme. We prove that versions of these algorithms, suitably modified to our setup, preserve the target (posterior) distribution on the space of trajectories of the hidden process.

HIGH-DIMENSIONAL MODEL SELECTION VIA GENERALIZED INFORMATION CRITERION

Wojciech Rejchel University of Warsaw

Model selection is a fundamental challenge, if one works with data sets that contains much more predictors than the sample size. In many practical problems finding a small set of significant predictors is as important as accurate estimation or prediction. The screening-selection algorithm is presented that is based on minimization of the empirical risk with the lasso penalty in the first step and with the generalized information criterion in the second step. I prove model selection consistency of this procedure in a wide class of models containing generalized linear models, quantile regression and support vector machines. The quality of the procedure is also investigated in numerical experiments.

LOCAL INTERPRETABILITY OF MACHINE LEARNING MODELS

Mateusz Staniak Institute of Mathematics, University of Wrocław

In [1] authors proposed a method (LIME) of explaining predictions generated by *black boxes* (complex machine learning models) by fitting a simple, interpretable model locally around an observation of interest. This is done by simulating a set of observations that are similar (close) to the given observation for so called *local exploration*, and then fitting a chosen interpretable model to this new dataset. To ensure the simplicity and interpretability of the model, a penalty term can be added to the fitting procedure. Later, in [2] authors showed that LIME can be thought of as a special case of a more general method of explaining single predictions called Shapley values.

In this talk I will show how we adapted LIME methodology to numerical data and regression problems, as opposed to classification problems from image and text data analysis which were the focus of the original article and present a new method of explaining single predictions called Break Down. Both of these method were introduced in [3]. To aid understanding of a model, visualization techniques can be applied. We implemented some of them in an R packages `live` and `breakDown`, which I will use to present examples. Model visualization is a growing field in statistics as exemplified by [4] and the use of its ideas boosts the advantages of LIME methodology.

References

- [1] Tulio Ribeiro, Singh, Guestrin, “*Why Should I Trust You?*”: *Explaining the Predictions of Any Classifier*, Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, 2016.
 - [2] Lundberg, Lee, *A unified approach to interpreting model predictions*, ArXiv e-prints, 2017.
 - [3] Biecek, Staniak, *Explanations of model predictions with live and breakDown packages*, submitted, 2018.
 - [4] Wickham, Cook, Hofmann, *Visualizing Statistical Models: Removing the Blindfold*. Statistical Analysis; Data Mining 8(4), 2015.
-

INDEPENDENCE VIA CROSS-COVARIANCE OPERATORS

Zoltan Szabo Ecole Polytechnique

Hilbert-Schmidt independence criterion (HSIC, also called distance covariance) is among the most popular and efficient techniques in statistics and data science to measure the dependence of random variables. The idea of HSIC is to take the Hilbert-Schmidt norm of the cross-covariance operator of “rich” non-linear features of the random variables. Surprisingly, very little is known about when HSIC characterizes independence. I will provide a complete answer to this question. [Joint work with Bharath K. Sriperumbudur. Preprint: <http://www.cmap.polytechnique.fr/~zoltan.szabo/publications/szabo18characteristic2.pdf>, <http://jmlr.org/papers/v18/szabo18a.html> (soon). Code: <https://bitbucket.org/szoli/ite-in-python/>.]

ON THE PROPERTIES OF SIGN ESTIMATOR DERIVED FROM HARD THRESHOLDED LASSO AND HARD THRESHOLDED BASIS PURSUIT

Patrick Tardivel Mathematics Institute, Wrocław University

In the high-dimensional linear model $Y = X\beta^* + \varepsilon$, when the number of observations is lower than the number of explicative variables, we aim at estimating $\text{sign}(\beta^*)$. It is well known that the irrepresentable condition is a necessary and “almost” sufficient condition to recover exactly $\text{sign}(\beta^*)$ with the LASSO sign estimator. In this article, in a first step, we provide a new result about the irrepresentable condition: the probability to recover $\text{sign}(\beta^*)$ with the LASSO is smaller than 1/2 once the irrepresentable condition does not hold. Consequently, there is an issue to provide a sign estimator able to recover $\text{sign}(\beta^*)$ under a weaker assumption than the irrepresentable condition. In a second step, we show that sign estimators derived from hard thresholded LASSO and hard thresholded basis pursuit only need identifiability condition to recover exactly $\text{sign}(\beta^*)$. Because the identifiability condition is a weaker condition than the irrepresentable condition, these sign estimators are theoretically better than the LASSO sign estimator. Finally, the irrepresentability and identifiability curves, function of the signal sparsity, show that the gap between the irrepresentable condition and the identifiable condition is huge. That is the reason why sign estimators derived from hard thresholded LASSO and hard thresholded basis pursuit outperform the LASSO sign estimator. Numerical experiments illuminate our theoretical results and show that sign estimators derived from hard thresholded LASSO, and hard thresholded basis pursuit perform well even if the columns of X are extremely correlated.

DOUBLE ASYMPTOTICS FOR GOODNESS-OF-FIT TESTS

Jacek Wesolowski Politechnika Warszawska

Consider the classical Pearson chi-square statistic with number of classes $m(n)$ tending to infinity together with the size of the sample n . We will show that under wide assumptions it is asymptotically normal though in a special (boundary) case of $m(n) = n^2$ the asymptotics is of the Poissonian type. The approach is via martingale clt in the normal case and via a submartingale clt related to the so-called “principle of conditioning” in the Poissonian case. The Poissonian case has been recently extended to other popular goodness-of-fit statistics which are asymptotically chi-square as long as the number of classes remains constant: the log-likelihood ratio statistic and power divergence statistics of Cressie and Read. The talk is based on a joint work with Greg Rempala (Ohio State Univ., Columbus).

A PERMUTATION TEST FOR THE TWO-SAMPLE RIGHT-CENSORED MODEL

Grzegorz Wyłupek Institute of Mathematics, University of Wrocław

The paper presents a novel approach to solving a classical two-sample problem with right-censored data. As a result, an efficient procedure for verifying equality of the two survival curves is developed. It generalizes, in a natural manner, a well known standard, that is, the log-rank test. First, the system of Laguerre polynomials in combination with the weighted log-rank statistics are used to define the components, which span the space of the general alternatives. Next, an auxiliary statistic, which is the sum of squares of the finite numbers of those components is defined. Finally, a flexible selection rule chooses the number of summands from the data at hand. Under the null hypothesis, the new test statistic has an asymptotic chi-square distribution with one degree of freedom, while the corresponding test is consistent for a wide range of the alternatives. On the other hand, to control the actual Type I error rate when sample sizes are finite, permutation approach is employed for the inference. An extensive simulation study shows that the new testing procedure improves upon classical solutions and selected popular recent developments in the field. An analysis of the real data sets confirm that findings.

41

Recent Advances in Numerical Modeling for Differential Problems

Organizers:

Zbigniew Bartoszewski (Gdańsk University of Technology)

Raffaele D'Ambrosio (University of L'Aquila)

IMPLICIT-EXPLICIT GENERAL LINEAR METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

Michał Braś AGH University of Science and Technology, Kraków

Many practical problems in science and engineering are modeled by large systems of ordinary differential equations (ODEs) which arise from discretization in space of partial differential equations (PDEs) by finite difference methods, finite elements or finite volume methods, or pseudospectral methods. For such systems there are often natural splittings of the right hand sides of the differential systems into two parts, one of which is non-stiff or mildly stiff, and suitable for explicit time integration, and the other part is stiff, and suitable for implicit time integration. The efficient solution can be provided by implicit-explicit (IMEX) schemes.

In present research we consider the class of general linear methods (GLMs) for ordinary differential equations. We construct IMEX GLMs of order $p = 1, \dots, 4$ with desired stability properties. We assume A -stability of implicit part of IMEX scheme and we search for methods with large regions of absolute stability. Next, we apply constructed methods to a series of test problems.

This is a joint work with A. Cardone, G. Izzo, Z. Jackiewicz and P. Pierzchała.

CONTINUOUS STIFFLY ACCURATE TWO-STEP RUNGE-KUTTA METHODS AND THEIR APPLICATION TO SINGULARLY PERTURBED DELAY DIFFERENTIAL EQUATIONS

Zbigniew Bartoszewski Faculty of Applied Physics and Mathematics, Gdańsk University of Technology

Many singularly perturbed delay differential equations are stiff and cannot be solved by explicit methods. There are a number approaches which allow to deal with this kind of challenging and numerically demanding problems like (adaptive) Bakhvalov meshes, (adaptive) Shishkin meshes, exponentially fitted difference schemes, just to name a few of them.

We propose using continuous stiffly accurate two-step Runge-Kutta methods (CSATSRK methods in short) to solve singularly perturbed delay differential equations. Numerical experiments has shown that they can be successfully applied to many such problems and give accurate results. My talk will include the description of construction of CSATSRK methods and presentation of the results of numerical experiments on a number of singularly perturbed delay differential equations.

NUMERICAL CONSERVATION ISSUES FOR STOCHASTIC HAMILTONIAN PROBLEMS

Raffaele D'Ambrosio University of L'Aquila

In this talk we aim to analyze conservation properties of numerical methods for stochastic differential equations (SDEs) that exhibit a-priori known qualitative behaviors. Such properties are sometimes hidden behind conditional stability issues of the numerical methods and do not have to be artificially conveyed in the numerical scheme. We especially consider stochastic Hamiltonian problems of Ito type, for which a linear drift of the expected energy is visible along the exact dynamics (differently from the deterministic case, for which the energy is preserved). We aim to study the behaviour of stochastic Runge-Kutta methods in order to understand their natural ability in retaining the same energy behaviour also along the numerical solutions. The analysis is driven through epsilon-expansions of the solutions, where epsilon is the amplitude of the stochastic fluctuation.

ADAPTED DISCRETIZATION OF PARTIAL DIFFERENTIAL EQUATIONS GENERATING PERIODIC WAVEFRONTS

Beatrice Paternoster University of Salerno, Department of Mathematics

The talk focuses on the numerical solution of advection-reaction-diffusion problems by adapted finite difference schemes. In other terms, the numerical scheme is developed in order to exploit the a-priori knowledge of the qualitative behaviour of the solution, gaining advantages in terms of efficiency and accuracy with respect to classical schemes already known in literature, which mostly rely on algebraic polynomials. The adaptation is carried out by the so-called trigonometrical fitting technique for the space-discretization, giving rise to a system of ODEs whose vector field contains both stiff and non-stiff terms. Due to this mixed nature of the vector field, an Implicit-Explicit (IMEX) method is employed for the time-integration. The coefficients of the introduced numerical scheme depend on unknown parameters which have to be properly estimated: such an estimate is performed by an efficient offline minimization of the leading term of the local truncation error. The effectiveness of this problem-oriented approach is shown through a rigorous theoretical analysis and selected numerical experiments.

42

Computational Mathematics: Discrepancy and Complexity

Organizers:

Leszek Plaskota (University of Warsaw)

Giancarlo Travaglini (Università di Milano-Bicocca)

Grzegorz Wasilkowski (University of Kentucky)

Stefano De Marchi (University of Padova)

DISCREPANCY FOR CONVEX BODIES WITH ISOLATED FLAT POINTS

Luca Brandolini Università degli Studi di Bergamo

We consider the discrepancy of the integer lattice with respect to the collection of all translated copies of a dilated convex body having a finite number of flat, possibly non-smooth, points in its boundary. We estimate the L^p norm of the discrepancy with respect to the translation variable as the dilation parameter goes to infinity.

COMPUTATION OF DISCREPANCY OF A BARRIER OPTIONS PRICE IN A COMPLETE MARKET

Salvatore Cuomo University of Naples Federico II

Discrepancy theory deals with the study of particular functions, discrepancy functions, measuring the differences between a real and selected statistical model; it finds application in many sectors of applied mathematics, in particular in statistics, engineering, physics; here we consider this kind of problem in a financial context, i.e. the price of a barrier option in a complete market. Barrier options are a particular kind of exotic options, whose pay-off function depends on a set of numerical constraints, named barriers; under the risk-neutral measure, the price of a barrier option can be expressed in terms of actualized expectation of the pay-off function (no-arbitrage price). In this context we estimate the discrepancy function between the no-arbitrage price and its Monte Carlo estimator. Monte Carlo (MC) methodologies are largely used in option pricing, especially for options with a very complex financial structure; they approximate high dimensional integral, whose integrand is the product of a density function and a generic function; from a computational point of view, a Monte Carlo procedure can be divided into three steps: i) drawing a set of numbers from the density function; ii) evaluation of the function in the points found at the point i); iii) calculation of the sample average obtained at the previous point. The main goals of our framework are mainly two: a) providing an estimation of the error of a Monte Carlo approach; b) determination of an optimal cardinality of the sample, in order to improve the performance of the Monte Carlo estimator.

INTEGRATION ON MANIFOLDS WITH MAPPED LOW-DISCREPANCY POINTS AND GREEDY MINIMAL k_s -ENERGY POINTS

Giacomo Elefante University of Fribourg

To integrate with the Quasi-Monte Carlo method (qMCM) on two-dimensional manifolds we consider two sets of points. The first is the set of mapped low-discrepancy sequence by a measure preserving map: low-discrepancy points are the best choice to integrate functions through qMCM in the unit cube $[0, 1]^d$ but to use them to integrate functions on a manifold we need to preserve their uniformity with respect to the Lebesgue measure.

The second points set is the greedy minimal Riesz s -energy points extracted from a suitable discretization of the manifold.

We chose greedy minimal energy points since thanks to the Poppy-seed Bagel Theorem we know that the class of points with minimal Riesz s -energy, under suitable assumptions, are asymptotically uniformly distributed with respect to the normalized Hausdorff measure.

On the other hand, we do not know if the greedy extraction produce a set of points that are still a good choice to integrate functions with the qMCM on manifolds.

Through theoretical considerations, by showing some properties of these points and by numerical experiments, we attempt to answer to these questions.

NORMS OF THE LATTICE POINT DISCREPANCY

Bianca Gariboldi Università degli Studi di Bergamo

Let $\mathcal{D}(r\Omega - x)$ be the discrepancy between the volume and the number of integer points in $r\Omega - x$, a dilated by a factor r and a translated by a vector x of a convex body Ω in \mathbb{R}^d . In this talk I will present some results about estimates of the mixed $L^p(L^2)$ norms of $\mathcal{D}(r\Omega - x)$ for fixed values of H and $R \rightarrow +\infty$, and also asymptotic ones when $H \rightarrow +\infty$:

$$\left\{ \int_{T^d} \left(\frac{1}{H} \int_R^{R+H} \left| \sum_{k \in \mathbb{Z}^d} \chi_{r\Omega-x}(k) - r^d |\Omega| \right|^2 dr \right)^{p/2} dx \right\}^{1/p}.$$

I will compare them with some results about estimates of the L^p norms of $\mathcal{D}(r\Omega - x)$ for $R \rightarrow +\infty$:

$$\left\{ \int_{\mathbb{R}} \int_{T^d} \left| \sum_{k \in \mathbb{Z}^d} \chi_{r\Omega-x}(k) - r^d |\Omega| \right|^p dx d\mu(r-R) \right\}^{1/p},$$

where μ is a Borel measure compactly supported on the positive real axis.

These results are joint work with L. Colzani and G. Gigante.

- [1] L. Brandolini, L. Colzani, G. Gigante and G. Travaglini, “ L^p and Weak- L^p estimates for the number of integer points in translated domains”, *Math. Proc. Cambridge Philos. Soc.* 159 (2015) 471–480.
- [2] L. Colzani, B. Gariboldi, G. Gigante, “Mixed $L^p(L^2)$ norms of the lattice point discrepancy”, arXiv:1706.04419 to appear in *Trans. Amer. Math. Soc.*
- [3] L. Colzani, B. Gariboldi, G. Gigante, “ L^p norms of the lattice point discrepancy”, arXiv:1805.06520.
- [4] M. Huxley, “A fourth power discrepancy mean”, *Monatsh. Math.* 73 (2014) 231–238.

VARIANCE OF LATTICE POINT COUNTING IN THIN ANNULI

Giacomo Gigante Università degli Studi di Bergamo

Y. G. Sinai proved in *Poisson distribution in a geometric problem*, *Adv. Soviet Math.* **3** (1991), 199–214, that the number of integer points in the plane inside a thin annulus of fixed area λ , of random shape and large random radius, with a suitable definition of randomness, converges in distribution to a Poisson random variable with parameter λ . The probabilistic proof does not exhibit a specific annulus. Indeed P. Major observed in *Poisson law for the number of lattice points in a random strip with finite area*, *Probab. Theory Related Fields* **92** (1992), 423–464, that the number of integer points in the circular annulus $\{r - 1/4r < |x| \leq r + 1/4r\}$ in the plane does not converge to a Poisson distribution when r varies randomly and uniformly in $[a_1L, a_2L]$ and L goes to $+\infty$. On the other hand, a translation of the annulus breaks the symmetry, and the situation changes. Indeed Z. Cheng, J. L. Lebowitz, P. Major proved in *On the number of lattice points between two enlarged and randomly shifted copies of an oval*, *Probab. Theory Related Fields* **100** (1994), 253–268, that if Ω is a convex set in the plane with

a smooth boundary with positive curvature, then the expectation and variance for the number of integer points in a shifted annular region of radius r and thickness c/r

$$[(r + c/(2r))\Omega - x] \setminus [(r - c/(2r))\Omega - x],$$

where x is uniformly distributed in the unit square, are both asymptotic to the area of the annulus $2c|\Omega|$ as c is fixed and $r \rightarrow +\infty$. Since the mean and the variance of a Poisson distribution coincide, this is consistent with the conjecture that this random variable converges in distribution to a Poisson random variable. Indeed these authors briefly mention higher dimensional analogues. In a joint work with L. Colzani and B. Gariboldi, we prove these higher dimensional analogues via Fourier analysis.

COMPLEXITY OF STOCHASTIC INTEGRATION PROBLEMS

Stefan Heinrich University of Kaiserslautern

We study the complexity of stochastic integration with respect to an isonormal process defined on a bounded Lipschitz domain $Q \subset \mathbf{R}^d$. This concept generalizes the stochastic integral with respect to the Wiener process on intervals to higher dimensional domains. We consider integration of functions from Sobolev classes $W_p^r(Q)$ and analyse the complexity in both the deterministic and randomized settings. We establish matching upper and lower bounds for the n -th minimal error, this way determining the complexity of the problem. It turns out that the stochastic integration problem is closely related to approximation of the embedding of $W_p^r(Q)$ into $L_2(Q)$.

SOME UPPER AND LOWER COMPLEXITY BOUNDS FOR ORDINARY DIFFERENTIAL EQUATIONS

Boleslaw Kacewicz AGH University of Science and Technology

We discuss two-sided complexity bounds for some initial- or boundary-value problems in ordinary differential equations. We give examples of problems for which proving good upper bounds is a challenging issue, as well as such for which lower bounds are non-trivial. The talk is based on recently published results.

EXPONENTIAL TRACTABILITY OF LINEAR MULTIVARIATE PROBLEMS

Peter Kritzer RICAM, Austrian Academy of Sciences

We consider approximations of compact linear multivariate operators defined over Hilbert spaces. We are particularly interested in studying the information complexity of the problem, which is the minimal amount of information needed to have an approximation with an error of at most $\varepsilon > 0$. The notion of tractability is used to describe how the information complexity depends on the dimension d of the problem and the error threshold ε . In this talk, we present necessary and sufficient conditions on various kinds of tractability, in particular exponential tractability. These conditions are mainly given in terms of sums of certain functions depending on the singular values of the multivariate problem. They do not require the ordering of these singular values which in many cases is difficult to achieve.

The talk is based on joint work with F.J. Hickernell (Illinois Institute of Technology) and H. Wozniakowski (Columbia University/University of Warsaw)

ANALYTIC NOISE MODEL FOR EFFICIENT APPROXIMATION OF ITÔ INTEGRALS

Paweł Morkisz AGH University of Science and Technology in Kraków

In the talk we present results on approximation of stochastic integrals of the following form

$$\mathcal{I}(X, W) = \int_0^T X(t) dW(t),$$

where $T > 0$, $W = \{W(t)\}_{t \geq 0}$ is a standard one-dimensional Wiener process, and $X = \{X(t)\}_{t \in [0, T]}$ belongs to a class of progressively measurable stochastic processes that are Hölder continuous in the r th mean.

Inspired by increasing popularity of computations with low precision (used on Graphics Processing Units - GPUs and standard Computer Processing Units - CPUs for significant speedup), we introduce suitable analytic noise model of standard noisy information about X and W . In this model we show that the upper bounds on the error of the Riemann-Maruyama quadrature are proportional to $n^{-\rho} + \delta_1 + \delta_2$, where n is a number of noisy evaluations of X and W , $\rho \in (0, 1]$ is a Hölder exponent of X , and $\delta_1, \delta_2 \geq 0$ are precision parameters for values of X and W , respectively. Moreover, we show that the error of any algorithm based on at most n noisy evaluations of X and W is at least $C(n^{-\rho} + \delta_1)$. We also report numerical experiments performed on both CPU and GPU that confirm our theoretical findings. We also present some computational performance comparison between those two architectures.

TRACTABILITY PROPERTIES OF THE WEIGHTED STAR DISCREPANCY

Friedrich Pillichshammer Johannes Kepler University Linz

Tractability properties of various notions of discrepancy have been intensively studied in the last two decades. In this presentation we consider the so-called weighted star discrepancy which was introduced by Sloan and Woźniakowski in 1998. The subject of tractability is concerned with the dependence of the weighted star discrepancy on the dimension. Roughly speaking, the weighted star discrepancy is said to be tractable if the inverse of weighted star discrepancy does not explode exponentially with the dimension.

We show that the Halton sequence achieves strong polynomial tractability for the weighted star discrepancy for product weights $(\gamma_j)_{j \geq 1}$ under the mildest condition on the weight sequence known so far for explicitly constructive sequences. However, there is also a hidden problem in this result which shall be discussed.

ON EFFICIENT INTEGRATION OVER \mathbb{R}^d

Leszek Plaskota Institute of Applied Mathematics and Mechanics, University of Warsaw

While integrating over \mathbb{R}^d , one often makes a change of variables to get an equivalent problem over a unit cube, and then uses, e.g., quasi-Monte Carlo or sparse grid methods that are well suited and well developed for such kind of domains. This approach is very sensitive to the chosen change of variables. For instance, consider the integration problem with gaussian weight,

$$\mathcal{I}(f) = (2\pi)^{-d/2} \int_{\mathbb{R}^d} f(\mathbf{x}) \exp^{-\|\mathbf{x}\|_2^2/2} d\mathbf{x}, \quad \mathbf{x} = (x_1, \dots, x_d),$$

where the function f is ‘smooth’. Then the standard change $x_i = \sqrt{2} \operatorname{erf}^{-1}(2t_i)$, $1 \leq i \leq d$, is often very bad as in the resulting integral

$$\int_{[-\frac{1}{2}, \frac{1}{2}]^d} g(\mathbf{t}) d\mathbf{t}, \quad \mathbf{t} = (t_1, \dots, t_d),$$

the function g is not smooth enough. However, one can add smoothness and make the numerical integration much more efficient by applying a slightly different change; namely, $x_i = \sqrt{2a} \operatorname{erf}^{-1}(2t_i)$ with appropriately chosen $a > 1$. We discuss theoretical issues related to the aforementioned change of variables, including the optimal choice of a , and show results of numerical experiments.

This is a joint project with P. Kritzer, F. Pillichshammer, and G.W. Wasilkowski.

TREATING GEOSPATIAL COMPLEX DATA BY COMPRESSION AND REDUCED ORDER METHOD

Stefano De Marchi University of Padova

The modeling and analysis of data coming from distributed measurements of physical quantities and satellite images is the theme on which focus the ERA-PLANET project Geo-Essential, that involves a research group of the Department of Mathematics of the University of Padova. Because of the huge size that some of these datasets achieve, reduced models need to be devised. We first focus on novel reduced spatial models for satellite data that will form the basis for the final time-simulators. The key idea behind this study is that, once we are able to consider a reduced model for the image, we can then model the dynamics of the considered quantities (e.g. soil moisture). The reduced order spatial models proposed is the CATCH (Caratheodory-Tchakaloff) algorithm [1, 2]. The second approach relies on kernel-based approximation coupled with Reduced Order Methods (ROM) (see [3]). When dealing with huge data sets, one usually needs to consider a surrogate model of drastically smaller size, with the aim of dealing with computationally efficient yet sufficiently accurate modeling tools. To achieve this goal, we implement methods based on the work [4] and [5].

References

- [1] Piazzon, F., A. Sommariva and M. Vianello. 2017. “Caratheodory-Tchakaloff Least Squares.” Sampling Theory and Applications, IEEE Xplore Digital Library (2017).
- [2] Sommariva, A. and M. Vianello. 2018. “Nearly optimal nested sensors location for polynomial regression on complex”geometries. *Sampl. Theory Signal Image Process.* 17:95-101 (2018)

[3] De Marchi, S., and G. Santin. “Fast computation of orthonormal basis for RBF spaces through Krylov space methods.” BIT 55: 949-966 (2015)

[4] Haasdonk, B., and G. Santin. Greedy “Kernel Approximation for Sparse Surrogate Modelling.” Proceedings of the KoMSO Challenge Workshop on Reduced-Order Modeling for Simulation and Optimization (1017)

[5] Wirtz, D., N. Karajan and B. Haasdonk. “Surrogate modelling of multiscale models using kernel methods.” Int. J. Numer. Met. Eng. 101:1-28 (2015).

PATH-DEPENDENT VS PATH-INDEPENDENT ALGORITHMS FOR JUMP-DIFFUSION SDEs

Pawel Przybyłowicz AGH University of Science and Technology

In the talk we discuss strong global approximation of solutions of stochastic differential equations (SDEs) of the following form

$$\begin{cases} dX(t) = a(t, X(t))dt + b(t, X(t))dW(t) + c(t, X(t-))dN(t), & t \in [0, T], \\ X(0) = x_0, \end{cases} \quad (42.1)$$

where $x_0 \in \mathbf{R}$, $a, b, c : [0, T] \times \mathbf{R} \rightarrow \mathbf{R}$ satisfy certain regularity conditions, $W = \{W(t)\}_{t \in [0, T]}$ is a one-dimensional Wiener process and $N = \{N(t)\}_{t \in [0, T]}$ is a homogeneous Poisson process with intensity $\lambda > 0$.

We consider three classes of methods χ^{eq} , χ^{noneq} and χ^{noneq^*} . Algorithms from the class χ^{eq} use only equidistant sampling for $[W, N]$, methods from the second class χ^{noneq} rely on the nonequidistant but path-independent sampling for $[W, N]$, while algorithms that belong to χ^{noneq^*} might use adaptive (path-dependent) sampling with respect to trajectories of the driving processes $[W, N]$. In the latter case the sampling consists of sequence of stopping times. In each class we define suitable schemes, that are based on specific interpolation of the classical Milstein steps. Moreover, we establish their exact asymptotic errors. It turns out that the methods based on adaptive sampling with respect to the trajectories of $[W, N]$ are more efficient than those based on the equidistant/nonequidistant path-independent discretization. We also discuss optimality of the presented methods and report results of numerical experiments.

LATTICE POINTS IN CONVEX BODIES WITH ISOLATED FLAT POINTS. PART I: FOURIER ANALYTIC TOOLS

Giancarlo Travaglini Università di Milano-Bicocca

We introduce the Fourier analytic tools used in the lattice points problems considered in the second part of this seminar by L. Brandolini.

In particular we show how to obtain pointwise estimates for Fourier transforms of characteristic functions of convex bodies with isolated flat points, and how to prove average decay estimates in the case of arbitrary convex bodies.

INFINITELY-VARIATE INTEGRATION

Grzegorz Wasilkowski University of Kentucky

We present results on efficient approximation of integrals with infinitely many variables. We consider concepts of worst case truncation and superposition dimensions and show that, under modest error demands, these dimensions are small for functions from weighted tensor product spaces. We also present Multivariate Decomposition Method that is almost as efficient as quadratures for univariate problems. The presentation is based on papers co-authored with A. Gilbert, M. Gnewuch, M. Hefter, P. Kritzer, F. Y. Kuo, D. Nuyens, F. Pillichshammer, L. Plaskota, K. Ritter, I. H. Sloan, and H. Woźniakowski.

ABSOLUTE VALUE INFORMATION

Henryk Wozniakowski University of Warsaw

Typically, two kinds of information have been considered in Information-Based Complexity (IBC) for solving continuous problems: standard and arbitrary linear. Recently, due to Daubechies and others, new nonlinear information has been introduced in the context of phase retrieval, where it is assumed that only absolute values of linear functionals are available. We call it absolute value information (AVI). The purpose of this study is to establish the power of AVI for various IBC problems in comparison to results in the regular IBC settings. We show, in particular, that AVI is too weak to solve linear problems using standard information, but it is powerful enough to recover typical IBC results for linear information. However, in the latter case, the concept of algorithm error has to be slightly modified.

This is a joint project with Leszek Plaskota and Paweł Siedlecki.

43

Quantitative Techniques for Complex Real-Life Applications

Organizers:

Piotr Bilski (Warsaw University of Technology)

Francesca Guerriero (University of Calabria)

Luigi Di Puglia Pugliese (University of Calabria)

Janusz Granat (Warsaw University of Technology)

OPTIMIZING ROUTING OVER A WIRELESS TELECOMMUNICATION NETWORK: A CASE STUDY

Pasquale Avella Università del Sannio

We study a network design problem arising in the management of a wireless network. The aim is to route a traffic matrix, minimizing a measure of the network congestion while guaranteeing a prescribed quality of service. We formulate the problem using mixed-integer programming techniques. We devise presolve procedures to reduce the size of the MIP formulation and show that the proposed approach can efficiently solve real-life problems.

PARAMETRIC SINGLE-FAULT DIAGNOSIS FOR COMPLEX ANALOG CIRCUITS USING NODE OPTIMIZATION WITH PCA AND SVM

Adrian Bilski Warsaw University of Life Sciences

The aim of this paper is to bring the reader closer to the diagnostics of complex linear analog systems with parametric faults, using Support Vector Machine (SVM) as a tool for fault location. The method for finding minimal set of test nodes by using taboo search and Genetic algorithm as node selectors in conjunction with SVM network as the fault locator is proposed. A strategy for finding the optimal kernels and their parameters for particular System Under Test is also presented. The diagnostics were conducted on the three low-pass filters, selected based on the increasing number of parameters describing them. Characteristic points reduction is based on the statistical PCA method.

AUTOMATED SYSTEM FOR MAXIMIZING THE GAMEPLAY ATTRACTIVENESS ON THE EXAMPLE OF THE MEMORY GAME

Piotr Bilski Institute of Radioelectronics and Multimedia Technology, Warsaw University of Technology

The paper presents the methodology of assessing the gameplay attractiveness in the modern games. This feature would be used to connect the players performing the live streaming of their gameplay and the audience observing it. This way it would be possible to obtain the most interesting live action by introducing the obstacles and challenges and force the player to the maximum toil. The introduced attractiveness function can be used during the interaction with the audience, allowing them to participate in configuring the gameplay. Another application is the player profiling, which enables selecting the difficulty levels adaptively to the player's abilities and experience. The paper presents the theoretical background the system and the implementation details, regarding the automated gameplay configuration and the player's quality assessment. The methodology was verified on the Memory mobile game (run under the Android and iOS systems). Results show that the whole scheme may be used for a vast spectrum of streamed games.

SHORTEST PATH TOUR PROBLEM WITH TIME WINDOWS

Luigi Di Puglia Pugliese University of Calabria

The paper aims at studying a new variant of the shortest path tour problem, where time windows constraints are taken into account. This is the first work dealing with the shortest path tour problem with time windows. The problem is formally described and a theoretical analysis is carried out. We prove the problem belongs to the NP-hard class of complexity by reducing the knapsack problem to the problem at hand. An optimal solution approach is devised based on dynamic programming paradigm. Labelling algorithms are defined along with well-tailored pruning strategies based on cost and time. The correctness of the bounding strategies is proven and the empirical behaviour is analyzed in dept. Extensive computational experiments have been carried out on a significant set of test problems in order to evaluate the performance of the proposed approach. Sensitivity analysis is carried out by considering both algorithmic and instance parameters.

NETWORK AND GRAPH MODELS IN DATA ANALYSIS

Janusz Granat Warsaw University of Technology

Analysis and processing of Big Data is a significant challenge for developing new algorithms. Massive datasets are collected and analyzed in numerous domains. Extracting valuable information from Big Data requires utilization of their structures. In the paper we will focus network or graph models. In particular the algorithms for detection of anomalous activity in graphs will be presented. Moreover, the algorithms will work on dynamic graphs that are changed over time. Selected algorithms will be based on formulation of optimization problems. Examples of analysis of location data will be shown.

44

Multiscale Models in Mathematical Biology

Organizers:

Jacek Miękisz (University of Warsaw)

Luigi Preziosi (Politecnico di Torino)

Ryszard Rudnicki (Instytut Matematyczny PAN)

SOME SINGULARLY PERTURBED MODELS IN ECOLOGY AND EPIDEMIOLOGY

Jacek Banasiak University of Pretoria

Many predator-prey (and, similarly, epidemiological) systems can be written as multiple scale models due to a significant difference of the rates of the involved process - for instance, often the typical life spans of the prey and the predator differ widely. This allows for an approximate reduction of the complexity of the models by singular perturbation theory. We shall discuss some recent developments in the field including the existence of the delayed stability exchange leading to t-called canard solutions.

PREHISTORY OF MATHEMATICAL STOCHASTIC GENE REGULATION AND EXPRESSION: A DOZEN YEARS AGO OR SO.

Adam Bobrowski Politechnika Lubelska

I will discuss basic mathematical results related to modelling stochastic gene regulation and expression: singular perturbations (including the notion of a convex combination of Feller generators) and asymptotic behavior.

INFLUENCE OF TIME DELAY ON DYNAMIC OF GENES EXPRESSION MODELS

Marek Bodnar Uniwersytet Warszawski

There exist a number of genes that change their expression pattern in an oscillatory manner. In some cases these oscillations are stable and can be treated as molecular clocks as in circadian clock and the cell cycle. In 2002, Hirata *et al.* observed oscillations in the Hes1 system. In 2003 Monk proposed a very simple model of this system with time delay, see. Independently, Jensen *et al.* numerically studied the same model as proposed by Monk, and observed that sustained oscillations may be induced by time delay introduced to the system.

We will discuss the influence of time delay on the dynamics of gene expression models showing that sustained oscillation can be obtained in the deterministic model. We generalize classical model proposed by Monk adding some equations and we show that oscillations are possible, under some assumption, even if delay is not present. We also discuss the influence of time delay on the model dynamics

MICROWAVE INVERSE SCATTERING MEDICAL IMAGING VIA ITERATIVE REGULARIZATION IN BANACH SPACES

Claudio Estatico University of Genova, Department of Mathematics

Diagnostic capabilities of microwave imaging can be very useful in biomedical applications where the dielectric properties of human tissues have to be restored by means of minimally-invasive techniques. The mathematical model of this inverse problem leads to the solution of an ill-posed, nonlinear and implicit 3D integral equation.

After a brief introduction of the recent regularization theory in Banach spaces, in this talk we discuss a conjugate-gradient-based iterative regularization algorithm developed in L^p spaces, with $1 < p < +\infty$, in conjunction with an inexact-Newton solving scheme. The proposed method is applied to obtain the reconstruction of hemorrhagic brain strokes. We will show numerical simulations with two- and three-dimensional anatomically-realistic phantoms, as well as some preliminary experimental results.

This is a joint work with A. Fedeli, M. Pastorino and A. Randazzo.

-
- [1] T. Schuster, B. Kaltenbacher, B. Hofmann, K.S. Kazimierski; *Regularization Methods in Banach Spaces*. Radon Series on Computational and Applied Mathematics, vol. 10, De Gruyter, 2012.
 - [2] I. Bisio, C. Estatico, A. Fedeli, F. Lavagetto, M. Pastorino, A. Randazzo, A. Sciarrone; Brain Stroke Microwave Imaging by Means of a Newton-Conjugate-Gradient Method in L_p Banach Spaces, *IEEE Transactions on Microwave Theory and Techniques*, in press, 2018.
 - [3] P. Brianzi, F. Di Benedetto, C. Estatico, Preconditioned iterative regularization in Banach spaces, *Computational Optimization and Applications*, vol. 54, pp. 263-282, 2013.
-
-

ANALYSIS OF TWO MODELS OF DRUG RESISTANCE FOR LOW GRADE GLIOMAS

Urszula Forjś University of Warsaw

On the basis of Ollier *et al.* (2017) and Pérez-García *et al.* (2015) we consider two models reflecting different mechanisms of acquiring drug resistance which were developed to describe heterogeneous (consisting of sensitive and resistant cells) low grade gliomas. Moreover, we take into account possible restoring of drug sensitivity by resistant cells.

We will discuss how restoring of drug resistance influences dynamics of both models.

DYNAMICS IN AN ECOLOGICAL NICHE PAOLO FREGUGLIA, ARMANDO BAZZANI DISIM, UNIVERSITY OF L'AQUILA PFREGUGLIA@GMAIL.COM

Paolo Freguglia DISIM, University of L'Aquila, Italy

An ecological niche is a complex system composed by different interacting communities as a set of species. Our idea is to consider the ecological niches as a basins where some species go in and others go out and they modify the environment. In order to describe this ecological situation we apply a multiagent ago-antagonist model.

TIME DELAYS IN STOCHASTIC MODELS OF GENE REGULATION

Jacek Miękisz University of Warsaw

Simple stochastic models of gene regulation based on Markov jump processes will be presented. We take into account that protein production takes some time - we introduce time delays in our models. For small time delays, we derive approximate formulas for the expected value and the variance of the number of protein molecules in the stationary state. We discuss adiabatic limit of fast switching gene states as a simple example of a singular perturbation. Joint effects of both time delays and gene switching will be investigated.

KINETIC MODELS WITH NON-LOCAL SENSING FOR CHEMOTAXIS

Loy Nadia Politecnico di Torino

Cells move by run and tumble, that is a kind of dynamics in which the cell alternates runs over straight lines and re-orientations. This erratic motion may be influenced by external factors, like chemicals, nutrients or the extra-cellular matrix: the cell measures the external field and elaborates the signal eventually adapting its dynamics. We talk about *kinesis* if the stimuli influences the frequency of tumbling and the speed of cells and we talk about *taxis* if the choice of the velocity assumed after a re-orientation is biased by the external stimuli. The size of the cell and its way of scouting its neighborhood highly influences its dynamics.

We propose a kinetic transport equation implementing a velocity-jump process in which the transition probability takes into account a double bias, which acts, respectively, on the choice of the velocity and on the frequency and the speed. The double bias depends on a double non-local sensing of the external environment. We analyze how the size of the cell and the way of scouting the environment with respect to the variation of the external fields affect the cell population dynamics by recovering an appropriate macroscopic limit.

[1] A. Chauviere, N. Loy and L. Preziosi, *Kinetic models with non-local sensing for chemotaxis*, in preparation.

MODELLING OF PATIENT TRANSFER AND PATHOGEN TRANSMISSION WITHIN HEALTHCARE FACILITY NETWORK: REAL DATA STUDY

Monika Piotrowska University of Warsaw, Faculty of Mathematics, Informatics and Mechanics, Institute of Applied Mathematics and Mechanics

Recently multidrug-resistant Enterobacteriaceae (MDR-E) have become a major public health threat in many European countries, also in developed ones. While traditional infection control strategies primarily target the containment of intra-hospital transmission, there is growing evidence that the inter-hospital patient traffic is an important factor for the spread of MDR-E within healthcare systems.

We propose a network model, which reflects a patient traffic in healthcare system and thus it provides the framework for systematic study of transmission dynamics of MDR-E and the effectiveness of infection control strategies.

We base our study on anonymized real hospitalization data, which unfortunately are rather limited, due to the a strong emphasis on the privacy of patients. To reflect the complexity of the real hospital network connections and dynamics of patient transfers between healthcare facilities, we model both direct and indirect inter-hospital patient transfers. We show the importance of taking into account indirect patient transfers i.e. when patients stay in the society for a certain period of time between dismiss and next admission. In general, the model dynamics is based on the network structure. However, the spread of bacteria within the healthcare system is modelled separately by a different framework – systems of ordinary differential equations.

Acknowledgements: This work was supported by grant no. 2016/22/Z/ST1/00690 of National Science Centre, Poland within the transnational research programme JPI-EC-AMR (Joint Programming Initiative on Antimicrobial Resistance) entitled “Effectiveness of infection control strategies against intra- and inter-hospital transmission of Multidrug-resistant Enterobacteriaceae insights from a multi-level mathematical NeTwork model” (EmerGe-Net).

IDENTIFYING RADIATION DOSE THAT INDUCES ROBUST SYSTEMIC ANTI-TUMOR IMMUNITY: A MATHEMATICAL MODELING APPROACH

Jan Poleszczuk Nalecz Institute of Biocybernetics and Biomedical Engineering

Synergy of radiation and immune system is currently receiving significant attention in oncology as numerous studies have shown that cancer irradiation can induce strong antitumor immune responses. It remains unclear, however, what are the best radiation fractionation protocols to maximize the therapeutic benefits of this synergy.

We develop a novel mathematical models operating on the several scales that can be used to predict and dissect the complexity of the immune-mediated response at multiple tumor sites after applying focal irradiation and systemic immunotherapy. The models are expressed in terms of Markov chains (in order to describe T cell trafficking in the blood stream) and ordinary differential equations (in order to describe complex tumor-immune system interactions at multiple co-existing cancer sites).

MULTISCALE MODELS FOR STOCHASTIC REACTION NETWORKS

Grzegorz Rempala The Ohio State University

The formalism of stochastic reaction networks (SRNs) provides building blocks for number of models in mathematical biology both at molecular and population levels (e.g., gene transcription or epidemic outbreak). In particular the SRNs allow to naturally incorporate both delay and multi-scale phenomena. In the first case the resulting models may be often expressed in the language of queuing theory, in the second case they lead to stochastic diffusions and ODE/PDE approximations. In this talk I will provide a brief overview of the applications of SRNs to modeling biological systems emphasizing the recent work on multi-scaling for enzyme kinetics and non-Markovian stochastic queues.

ASYMPTOTIC PROPERTIES OF A STOCHASTIC GENE EXPRESSION MODEL

Ryszard Rudnicki Instytut Matematyczny PAN

We give a short introduction into the framework of piecewise deterministic Markov processes (PDMPs) [1]. We illustrate the abstract mathematical setting with a gene expression model [2] described by a switching system - a kind of PDMP. Then we present some new results concerning the long-time decomposition of substochastic semigroups [3] induced by such Markov processes and their applications to the above mentioned model. We also analyse the properties of the deterministic limit of our model when jump rates between active and inactive states tend to infinity.

[1] R. Rudnicki, M. Tyran-Kamińska, Piecewise Deterministic Processes in Biological Models, SpringerBriefs in Applied Sciences and Technology, 2017.

[2] R. Rudnicki, A. Tomski On a stochastic gene expression with pre-mRNA, mRNA and protein contribution, J. Theor. Biol., 2015

[3] K. Pichór, R. Rudnicki, Asymptotic decomposition of substochastic semigroups and applications, Stochastics and Dynamics, 2018.

STOCHASTIC MODELS OF GENE EXPRESSION WITH BURSTING

Marta Tyran-Kamińska Institute of Mathematics, Polish Academy of Sciences and University of Silesia

We study the dynamic behavior of stochastic models given in terms of piecewise deterministic Markov processes for gene expression in the presence of transcriptional/translational bursting. We treat this problem in generality with respect to the distribution of the burst size as well as the frequency of bursting. Our results are applicable to both inducible and repressible expression patterns in prokaryotes and eukaryotes.

PDE MODEL OF CELL CYCLE DYNAMICS AND CLUSTERING IN YEAST

Radosław Wieczorek University of Silesia

Autonomous oscillation and clustering of cell cycle in yeast cultures attracted recently a significant interest. The Response/Signaling feedback models have been proposed to explain this phenomenon. We apply a non-linear physiologically-structured PDE model of cell population to describe clustering in yeast cultures. The linear stability of stationary states will be discussed. We will present some results about clustering have been proven. In particular, in some cases the solutions of PDEc converge to traveling delta measures.

Poster Session

A RECONSTRUCTION SCHEME FOR $f(T)$ MODIFIED GRAVITY THROUGH HOLOGRAPHIC DARK ENERGY

Surajit Chattopadhyay Dept of Mathematics, Amity University, Kolkata

Inspired by the work of Bamba et al.(2012) the present work reports a study on the reconstruction of modified holographic Ricci dark energy (MHRDE) in the framework of modified gravity taken as $f(T)$ gravity. A correspondence between modified Chaplygin gas (MCG) and MHRDE has also been considered and thereafter the $f(T)$ gravity has been reconstructed via reconstruction of the Hubble parameter. The reconstructed equation of state (EoS) parameter obtained this way has been found to be able to cross the phantom boundary. In the next phase of the work a viable model of $f(T)$ gravity has been considered and MHRDE has been discussed in this modified gravity frame. The EoS parameter due to the torsion contribution obtained this way has been found to behave like quintessence. The transition of the universe from the dark matter (DM) dominated to dark energy (DE) dominated phase is apparent from this model. Also, the model is exhibiting DE domination of the current universe. Finally, the statefinder hierarchy has been discussed through the statefinder and snap parameters. The model has been found to be able to attain the Λ CDM fixed point in the statefinder trajectory.

FORMULAE OF CAUCHY-BINET TYPE FOR ANALOGUES OF FREDHOLM MINORS

Grażyna Ciecierska University of Warmia and Mazury in Olsztyn

We consider linear Fredholm operators between linear spaces. By exploiting terms of determinant systems for $A \in L(Y, Z)$ and $B \in L(X, Y)$, we provide a direct construction of a determinant system for AB . In our approach, we refer to the correspondence between any Fredholm operator and its determinant system. Since terms of a determinant system are analogues of the classical Fredholm minors, the obtained result yields Cauchy-Binet type formulae.

- [1] Buraczewski A., Determinant system for composite of generalized Fredholm operators, *Studia Math.* 34 (1970), 197–207.
 - [2] Karlin A., Rinott Y., A generalized Cauchy-Binet formula and applications to total positivity and majorization, *J. Mult. Anal.* 27 (1988), 284–299.
 - [3] Caracciolo S., Sokal A.D., Sportiello A., Noncommutative determinants, Cauchy-Binet formulae and the Capelli and Turnbull identities, *Electron. J. Comb.* 16 (2009), no. 1, 1–43.
 - [4] Knill O., Cauchy-Binet for pseudo-determinants, *Linear Algebra Appl.* 459 (2014), 197–207.
 - [5] Ciecierska G., Cauchy-Binet type formulas for Fredholm operators, *J. Appl. Math. Comput. Mech.* 16 (2017), no. 2, 43–54.
-

DIRICHLET FORMS OF NONLOCAL OPERATORS

Damian Fafała Wrocław University of Science and Technology

We consider an isotropic unimodal Lévy measure ν on \mathbb{R}^d , i.e. $\nu(dz) = \nu(|z|) dz$ where $\nu : [0, \infty) \rightarrow (0, \infty]$ is a nonincreasing function such that $\int_{\mathbb{R}^d} \nu(z) dz = \infty$ and

$$\int_{\mathbb{R}^d} (|z|^2 \wedge 1) \nu(z) dz < \infty.$$

For $x, y \in \mathbb{R}^d$ let $\nu(x, y) = \nu(|x - y|)$. For $u : \mathbb{R}^d \rightarrow \mathbb{R}$ and an open set $D \subset \mathbb{R}^d$ we study a quadratic form

$$\mathcal{E}_D[u] := \frac{1}{2} \iint_{\mathbb{R}^d \times \mathbb{R}^d \setminus D^c \times D^c} (u(x) - u(y))^2 \nu(x, y) dx dy.$$

We introduce basic properties of such form and give a way to construct a Hunt process using above form.

- [1] K. Bogdan, T. Grzywny, K. Pietruska-PaÅ, uba, A. Rutkowski *Extension theorem for nonlocal operators*. ArXiv e-prints, 2017.
- [2] Z.-Q. Chen and M. Fukushima. *Symmetric Markov processes, time change, and boundary theory*, volume 35 of *London Mathematical Society Monographs Series*. Princeton University Press, Princeton, NJ, 2012.
- [3] A. Fiscella, R. Servadei, and E. Valdinoci. Density properties for fractional Sobolev spaces. *Ann. Acad. Sci. Fenn. Math.*, 40(1):235–253, 2015.
- [4] M. Fukushima, Y. Oshima, and M. Takeda. *Dirichlet forms and symmetric Markov processes*, volume 19 of *De Gruyter Studies in Mathematics*. Walter de Gruyter & Co., Berlin, extended edition, 2011.

MITTAG-LEFFLER TYPE PROCESSES

Janusz Gajda University of Warsaw

We introduce a class of some Levy processes based on Mittag-Leffler special function. We provide various properties of this distribution and discuss procedures of the parameters estimation and simulation of sample paths.

A NOTE ON THE ARENS TRIADJOINT OF B-BIMORPHISMS

Omer Gok Yildiz Technical University

Let A be an Archimedean vector lattice. In this study, we give the definition of b-bimorphism and we prove that the Arens triadjoint of a b-bimorphism from $A \times A$ into A is a b-bimorphism. It is a generalization of biorthomorphisms.

DISCRIMINATING BETWEEN SCALED AND FRACTIONAL BROWNIAN MOTION VIA P-VARIATION STATISTICS

Aleksandra Grzesiek Wroclaw University of Science and Technology

The anomalous diffusion processes have found various applications in recent years. In the literature one can find different systems representing that kind of behaviour, like fractional Brownian motion or scaled Brownian motion. A most common tool for anomalous diffusion behaviour recognition is the time-averaged mean square displacement. However, this statistics has the same behaviour for fractional and scaled Brownian motions. That is the reason why there is a need to apply different approach in order to discriminate between those two anomalous diffusion processes. One of the possibility is the p-variation statistics which demonstrates different behaviour for fractional and scaled Brownian motions. We present a new formula for p-variation of scaled Brownian motion and compare it to the known formula of p-variation for fractional Brownian motion. Moreover, we show how to apply them to real trajectories analysis in order to recognize the proper anomalous diffusion model. The theoretical results are supported by simulations.

EXISTENCE AND REGULARITY OF MINIMIZERS IN THE ANISOTROPIC LEAST GRADIENT PROBLEM

Wojciech Górny University of Warsaw

Consider the anisotropic least gradient problem in two dimensions

$$\min\left\{\int_{\Omega}\phi(Du), \quad u \in BV(\Omega), \quad u|_{\partial\Omega} = f\right\}$$

where $\Omega \subset \mathbb{R}^2$ is an open bounded set with Lipschitz boundary, $f \in C(\partial\Omega)$, and ϕ is an anisotropic norm on \mathbb{R}^2 . As in the classical least gradient problem, existence and uniqueness of minimizers depend on the geometry on Ω . From here, as we have some additional interplay between Ω and ϕ , there are two different scenarios:

(1) The unit ball $B_{\phi}(0, 1)$ is strictly convex. In this case, we have existence and uniqueness of minimizers of the anisotropic least gradient problem for every boundary data $f \in C(\partial\Omega)$ if Ω is strictly convex. Furthermore, if Ω satisfies some form of uniform convexity, then we may obtain regularity estimates regardless of the regularity of ϕ ; in particular, if $\partial\Omega \in C^2$ and the curvature is bounded from below, then $f \in C^{0,\alpha}(\partial\Omega)$ implies $u \in C^{0,\alpha/2}(\Omega)$ (the same as in the isotropic case).

(2) The unit ball $B_{\phi}(0, 1)$ is not strictly convex. In this case existence of minimizers is obtained (for uniformly convex domains) using the regularity estimates from case (1). This gives us one minimizer with the same regularity estimates as in the previous paragraph. Furthermore, if the domain is not strictly convex, using a blow-up argument we prove existence of a minimizer with the correct trace, but it is only continuous up to the boundary. However, in both cases uniqueness of minimizers fails and the additional minimizers may fail to be $W^{1,1}(\Omega)$ or $SBV(\Omega)$ even for smooth boundary data.

ANGULAR MULTISELECTIVITY WITH SPHERICAL WAVELETS

Iłona Iglewska-Nowak West Pomeranian University of Technology, Szczecin

The spherical wavelet transform based on approximate identities is a powerful tool in spherical signal investigation, e.g., in geodesy or astronomy. It is a challenge to construct wavelets that are directional, i.e. not rotation-invariant, and have an adaptive angular selectivity. The problem of how to find a proper representation of distinct kinds of details of real images, ranging from highly directional to fully isotropic ones, was quite intensively studied for the case of signals over the Euclidean space. Here, we present the first attempt to deal with this task in the case of spherical signals.

SECOND ORDER EVOLUTION DIFFERENTIAL FUNCTIONAL EQUATIONS WITH INFINITE DELAY

Adrian Karpowicz Instytut Matematyki, UG

We consider a second order semilinear functional evolution equation with infinite delay in a Banach space. We prove the existence of mild solutions for this equation using the measure of noncompactness technique and the Schauder fixed point theorem.

SMOOTH FUNCTIONS AND FRACTIONAL SOBOLEV SPACES

Michał Kijaczko Politechnika Wroclawska

For $s \in (0, 1)$, $p \in [1, \infty)$ and an open set $\Omega \subset \mathbb{R}^d$ we define the fractional Sobolev space $W^{s,p}(\Omega)$ as

$$W^{s,p}(\Omega) := \{f \in L^p(\Omega) : \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} \frac{|f(x) - f(y)|^p}{|x - y|^{d+sp}} dx dy < \infty\}$$

and a norm

$$\|f\|_{W^{s,p}(\Omega)} = \left(\int_{\Omega} |f(x)|^p dx + \int_{\Omega} \int_{\Omega} \frac{|f(x) - f(y)|^p}{|x - y|^{d+sp}} dx dy \right)^{\frac{1}{p}}.$$

Using Whitney decomposition of open sets into cubes we prove the density of smooth functions in $W^{s,p}(\Omega)$, assuming that s, p and Ω are such that the Hardy inequality holds.

- [1] E. Di Nezza, G. Palatucci and E. Valdinoci *Hitchhiker's guide to the fractional Sobolev spaces*, arXiv:1104.4345, 2011
 - [2] E.M. Stein *Singular integrals and Differentiability Properties of Functions*, Princeton University Press, 1970
 - [3] Y. Zhou *Fractional Sobolev extension and imbedding*, Trans. Amer. Math. Soc. 367 (2015), 959-979
-

ESTIMATION OF AR MODELS PARAMETERS WITH INFINITE VARIANCE

Piotr Kruczek Wrocław University of Science and Technology, Faculty of Pure and Applied Mathematics

Among various techniques of modeling discrete real empirical data those based on time series approach represent constantly the most irreplaceable place. One can distinguish many time series models and most of them are usually based on classical autoregressive (AR) or moving average (MA) approach. The common assumption of the Gaussian distribution of the innovations, although crucial in time series theory, cannot be applied in many real-life examples like for instance when the data exhibit significant peaks. To overcome this problem one often uses stable processes. The stable distributions, which belong to the heavy-tailed class of distributions are due to the generalized central limit theorem natural extension of the Gaussian law. They arise when the evolution of a system or result of an experiment is influenced by a sum of many factors, which are not necessarily quantified by the finite variance. Lack of autocorrelation for heavy-tailed random sequences is the main obstacle in developing methods of parameter estimation of heavy-tailed AR time series. However, apart of autocorrelation there exist other measures of dependence especially suited for heavy-tailed distributions and processes. We discuss here two of them, namely autocovariation and fractional lower order covariance. Based on measures of dependence for stable sequences we develop rigorously a new YW method for estimation of AR parameters.

APPROXIMATIONS FOR KINETIC EQUATIONS OF SWARM FORMATION

Karolina Lademann University of Gdansk

We study Euler-type approximations along characteristics for the kinetic equations that describe swarm formations in the variable interaction rate case and prove existence, local error estimation and convergence for every considered scheme.

OPTIMAL FUNCTION SPACES FOR SOME CLASSICAL OPERATORS OF HARMONIC ANALYSIS

Zdeněk Mihula Charles University, Prague

Given an operator T defined on a function space X , the question then arises: what is its optimal function space Y such that $T : X \rightarrow Y$ is bounded? We answer this question for some classical operators of harmonic analysis, e.g. the Hardy-Littlewood maximal operator, the fractional maximal operator, the Hilbert transform and the Riesz potential, within the scope of rearrangement-invariant spaces.

HITTING TIMES OF SETS FOR LÉVY PROCESSES ON THE REAL LINE

Maciej Mišta Politechnika Wrocławska

The purpose of this thesis was to examine distributions of random variables that describe hitting times of the asymmetric Lévy process. There are given estimates of the tail function and asymptotic behavior. Moreover, there is given relationship between regular variation of the real part of the characteristic exponent and the imaginary part.

EXISTENCE AND UNIQUENESS OF SOLUTIONS OF A MODEL OF ELASTOPLASTIC PLATES COUPLED USING SEMIGROUPS OF NON-LINEAR OPERATORS

Ramiro Peñas Galezo Universidad del Atlántico

In this document we develop the equations of evolution and the initial and boundary conditions of a system of coupled plates subject to plastic deformation whose variational formulation uses the perfectly plastic Maggiani-Mora plates approach. The existence and uniqueness of solutions of this model are demonstrated from the results of Kato for Cauchy problems with differential inclusion using the theory of non-linear semigroups generated by accretive operators.

MAXIMUM LIKELIHOOD ESTIMATORS FOR DISCRETE MODELS

Tomasz Skalski Wrocław University of Science and Technology

The notion of Maximum Likelihood Estimators (MLE) and their existence in discrete exponential families will be discussed. The definition of the set of uniqueness is presented, in order to describe the existence of MLE. Among others, we will define the exponential random graph model. Finally, we are going to characterise the existence of MLE of the parameters in our model and its' special cases.

THE IDEAL CENTRE OF A VECTOR LATTICE

Damla Yaman Yildiz Technical University

In this work, we investigate the center of Banach lattices, Riesz spaces, and the space of regular operators and so on.

EFFICIENT ENCODINGS TO HYPERELLIPTIC CURVES OVER FINITE FIELDS

Amirmehdi Yazdani Kashani University of Kashan

Many cryptosystems are based on the difficulty of the discrete log-arithm problem in finite groups. In this case elliptic and hyperelliptic cryptosystems are more noticed because they provide good security with smaller size keys. Since these systems were used for cryptography, it has been an important issue to transform a random value in finite field into a random point on an elliptic or hyperelliptic curve in a deterministic and efficient method. In this paper we propose a deterministic encoding to hyperelliptic curves over finite field. For cryptographic desires it is important to have an injective encoding. In finite fields with characteristic three we obtain an injective encoding for genus two hyperelliptic curves.