Set Theory and Topology Session - Abstracts

17-20 September 2018

Marek Balcerzak

Ideal convergent subseries and rearrangements of series in Banach spaces

Let I be a 1-shift-invariant ideal on $\mathbb{N}$ with the Baire property. Assume that a series $sum x_n$ with terms in a real Banach space X is not unconditionally convergent. We show that the sets of I-convergent subseries and of I-convergent rearrangements of a given series are meager in the respective Polish spaces. A stronger result, dealing with I-bounded partial sums of a series, is obtained if X is finite-dimensional. We apply the main theorem to series of functions with the Baire property, from a Polish space to a separable Banach space over
$mathbbR$, under the assumption that the ideal I is analytic or coanalytic.

Also, we discuss two methods of coding subseries of $\sum x_n$: by the choice of a sequence of increasing indices, or by the choice of parameters $t(n) \in \{0,1\}$ multiplied by x_n. In the second approach, we study the Haar measure on $\{0,1\}^{\mathbb{N}}$ of the set of I-convergent subseries of a divergent series in a Banach space.

References

[1] M. Balcerzak, M. Popławski, A. Wachowicz, *The Baire category of ideal convergent subseries andrearrangements*, Topology Appl. **231** (2017), 219–230.

[2] M. Balcerzak, M. Pop
lawski, A. Wachowicz, *Ideal convergent subseries in Banach spaces*, Quaest. Math., in press.

Taras Banakh

Haar-I sets in Polish groups

Piotr Borodulin-Nadzieja

Secret connections between Banach spaces and analytic P-ideals

Many classical Banach spaces can be constructed by a certain procedure from families of finite sets. In a similar way, from families of finite sets, one can construct classical analytic P-ideal. Using this simple remark we may define (potentially) new examples of Banach spaces, motivated by analytic P-ideals and vice versa. We will discuss those new examples and connections between Banach spaces and analytic P-ideals defined by the same families.

Gianluca Basso

A Lelek-like disconnected compact metric space

Given a class of compact metric spaces and continuous surjections, the theory of projective Fraisse limits reduces the problem of finding a universal and approximately homogeneous space for said class to a combinatorial problem: checking whether amalgamation holds for a class of finite structures and epimorphisms. We reverse such approach by proving that an interesting class of finite structures is Fraisse limits and characterizing the compact metric space to which it gives rise. Such space, while disconnected, shares some similarities with the Lelek fan. We then attempt to find the optimal class of spaces and maps for which it is universal and approximately homogeneous. This is joint ongoing work with R. Camerlo.

Raphael Carroy

The open graph dichotomy and the second level of the Borel hierarchy

I will explain how variants of the open graph dichotomy can be used to obtain various descriptive-set-theoretical dichotomies at the second level of the Borel hierarchy. This shows how to generalise these dichotomies from analytic metric spaces to separable metric spaces by working under the axiom of determinacy.

David Chodounský

Splitting Chains

Splitting chains are linearly ordered splitting families in $P(\omega)/fin$. We will discuss the existence of these objects.

Vincenzo Dimonte

Generalized Descriptive Set Theory under I0

Generalized Descriptive Set Theory is the study of "simple" subsets of spaces of the form 2^κ, where κ is an uncountable cardinal. The classic way to approach this is to consider κ to be regular, but we are going to introduce a new line of research where
kappa is a cardinal of cofinality ω. In this setting, all the independence vagaries of the classical case disappear, and we can prove results analogous to the 2^ω case, like the Suslin and Silver dichotomies. We argue that this is because the tree-structure of subsets of 2^κ and 2^ω is very much alike, and, as a consequence, we show that postulating I0 will yield determinacy-like results on the space 2^κ (for example proving the perfect set property for all projective sets). Finally, we notice that many results can be transferred to a generality of non-separable topological spaces. Joint work with Luca Motto Ros and Xianghui Shi.

Mirna Dzamonja

Weak versions of compactness and their productivity

Tychonoff topology of the product of topological spaces was introduced with the purpose of making the product of compact spaces compact. We are interested in compactness-like properties that are preserved by more generous products, such as the κ-box product for an uncountable κ. This is closely related to large cardinal notions. For example, what is the large cardinal strength of the notion of square compactness studied by Hajnal and Juhasz in the 1970s? It is known to be between the weak and the strong compactness. We introduce some topological and measure-theoretic ideas to study the question.

The work presented is joint research with David Buhagiar from the University of Malta.

Grażyna Horbaczewska

On sets which can be moved away from sets of a certain family

An operation which assigns to an arbitrary family of sets the class of sets which can be translated away from every set from the fixed family is considered in abelian groups. Assuming CH it is proven that on the real line meager sets can be defined as sets "shiftable" from the family of strong measure zero sets ($\mathcal{K} = \mathcal{SMZ}^*$). A similar result is shown for Lebesgue null sets and strongly meager sets ($\mathcal{N} = \mathcal{SM}^*$).

Aleksandra Kwiatkowska

Permutation groups

We discuss a number of results on infinite permutation groups, that is, closed subgroups of the symmetric group on a countable set, or equivalently, automorphism groups of countable structures. We will focus on ample generics, where a group G has ample generics if for every n, the diagonal conjugacy action of G on G^n has a comeager orbit, on similarity classes, and on topological generators of permutation groups. For example, we show that for a permutation group G, under mild assumptions, for every n and an n-tuple $\bar{f}$ in G, the countable group generated by
$barf$ is discrete, or precompact, or the conjugacy class of $\bar{f}$ is meager. Finally, we will present results on conjugacy classes and on similarity classes of automorphism groups of structures equipped with a linear order, such as the ordered random graph, the ordered rational Urysohn metric space, or the ordered random poset, and many other extremely amenable permutation groups. This is joint work with Maciej Malicki.

Sandra Meuller

Large Cardinals in the Stable Core

The Stable Core $\mathbb{S}$, introduced by Sy Friedman in 2012, is a proper class model of the form $(L[S], S)$ for a simply definable predicate S. He showed that V is generic over the Stable Core (for $\mathbb{S}$-definable dense classes) and that the Stable Core can be properly contained in HOD. These remarkable results motivate the study of the Stable Core itself. In the light of other canonical inner models the questions whether the Stable Core satisfies GCH or whether large cardinals is V imply their existence in the Stable Core naturally arise. We answer these questions and show that GCH can fail at all regular cardinals in the Stable Core. Moreover, we show that measurable cardinals in general need not be downward absolute to the Stable Core, but in the special case where $V = L[\mu]$ is the canonical inner model for one measurable cardinal, the Stable Core is in fact equal to $L[\mu]$.

This is joint work with Sy Friedman and Victoria Gitman.

Gianluca Paolini

On a Cardinal Invariant Related to the Haar Measure Problem

In a recent work of Tsaban et al., given a metrizable profinite group G, a cardinal invariant of the continuum $\mathfrak{fm}(G)$ was introduced, and a positive solution to the Haar Measure Problem for G was given under the assumption that $\mathrm{non}(\mathcal{N}) \leq \mathfrak{fm}(G)$. We prove here that it is consistent with ZFC that there is a metrizable profinite group G_* such that $mathrmnon(\mathcal{N}) > \mathfrak{fm}(G_*)$, thus demonstrating that the strategy of Tsaban et al. does not suffice for a general solution to the Haar Measure Problem.

Viera Sottova

The role of ideals in topological selection principles

Piotr Szewczak

Products of Luzin-type sets with combinatorial properties

A topological space is Menger if for any sequence $\mathcal{U}_1, \mathcal{U}_2, \dots$ of open covers of the space, there are finite sets $\mathcal{F}_1 \subseteq \mathcal{U}_1, \mathcal{F}_2 \subseteq \mathcal{U}_2, \dots$ such that the union $\bigcup_{n\in\mathbb{N}} \mathcal{F}_n$ covers the space. If we can request that the sets $\mathcal{F}_1, \mathcal{F}_2, \dots$ are singletons, then the space is Rothberger. A subset of the real line, of cardinality at least $\mathrm{cov}(\mathcal{M})$, is $\mathrm{cov}(\mathcal{M})$-Luzin if its intersection with any meager subset of the real line has cardinality strictly smaller than $\mathrm{cov}(\mathcal{M})$. Each $\mathrm{cov}(\mathcal{M})$-Luzin set is Rothberger, and thus Menger. In 2003, assuming $\mathrm{cov}(\mathcal{M}) = \mathfrak{c}$, Bartoszyński, Shelah, and Tsaban constructed two $\mathrm{cov}(\mathcal{M})$-Luzin sets whose all finite powers are Rothberger but its product space is not Menger. We show that such sets exist, assuming $\mathrm{cov}(\mathcal{M}) = \mathrm{cof}(\mathcal{M})$ and $\mathrm{cov}(\mathcal{M})$ is regular. Our proof, in contrast to the topological construction of Bartoszyński, Shelah, and Tsaban, is purely combinatorial. We apply this result to local properties of function spaces with pointwise convergence topology. This is a joint work with Grzegorz Wiśniewski.